Mémoires secrets sur l'...

ARITHMETICAL COLLECTIONS AND IMPROVEMENTS.

BEING

A COMPLETE SYSTEM OF PRACTICAL ARITHMETIC.

BY

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MDCCLXXIV.

TO THE

RIGHT HONOURABLE

The Lord BROWNLOW BERTIE,

One of the Repreſentatives in Parliament for the County of LINCOLN, &c. &c.

MY LORD,

THE utility of the ſcience treated upon in the following ſheets, muſt be well known to your Lordſhip, who has ſo often been a witneſs of the advantage and uncommon weight, arguments drawn from arithmetical computations carry with them in that houſe, whereof you are ſo worthy a member; ſo that if this performance be found equal to the uſefulneſs of the ſubject, it may juſtly be entitled to your Lordſhip's patronage.

Your aſſiduity in parliament for promoting the drainage of, and making roads through a lately inundated, though rich country, is and will be of ſuch great benefit, that ſucceeding ages, as well as the preſent, muſt reap the advantage of theſe

ſalutary works, and poſterity bleſs the time when a BERTIE graced the Britiſh ſenate.

That your Lordſhip may ſtill ſucceed in promoting the good of your native country, and long live the great ornament thereof, is the hearty wiſh of,

My LORD,

Your Lordſhip's

Moſt obedient, and

Moſt humble Servants,

The AUTHORS.

PREFACE.

THE book here preſented to the world is a regular ſyſtem of common arithmetic, adapted to the uſe of the gentleman and the ſcholar, as well as the man of buſineſs.

BOOK I.

NUMERATION, and the next four general rules, are enriched with many compendious methods and examples; and the rules of Practice very copious and extenſive.

The doctrine of Vulgar Fractions is here rendered more eaſy, conciſe, and uſeful, by the means of an eaſy axiom; Decimals are purſued through all the late improvements, in the management both of plain and circulating numbers; and the Extraction of the Roots, particularly the Cube, is done in a more eaſy manner than in any book of arithmetic, which has ever yet come under our inſpection.

BOOK II.

In which Proportion is treated on in a ſcientific manner, and adapted as well to the uſe of the young mathematician, as together with the rules of Practice applied to all branches of buſineſs; and the mercantile rules

are

are exemplified and diversified with great variety of curious examples.

In Exchange are exhibited Sir Isaac Newton's tables of the assay and real and standard-weight, and value, of most of the gold and silver coins in Europe; together with those of the conformity of weights and measures, by the Sieur John Larue, merchant, at Lyons; also the method of solving questions in the arbitration of exchange by a numerical equation.

BOOK III.

Contains the less useful, though most pleasant and delightful parts of arithmetic; viz. Alligation, medial, partial, and total; the Specific Gravity of Metals; the Rule of False, or Position; Progression both arithmetical and geometrical; also Variations, Combinations, and the method of filling Magic Squares: these, tho' they are done and accounted for better by Algebra, &c. yet may serve to open the mind, and excite the curiosity of youth to proceed to the most sublime and abstruse sciences.

To these are added Compound Interest, with the method of calculating the value of freehold estates at any rate of interest; also annuities in reversion, according to that late ingenious mathematician Mr. Thomas Simpson, F. R. S. from a set of tables calculated by him for that purpose. Also a collection of questions in Mensuration, with such directions as may enable any person to perform the measurement of most sorts of superficies and solids; and also some curious miscellaneous questions.

This

This treatise concludes with an Appendix, by Mr. Thomas Allen, teacher of the mathematics at Spalding, containing rules and examples for finding the sum of any given number of terms in certain progressions. A collection of problems concerning the maxima and minima of quantities, with the theorems annexed. And investigation of the sums of certain infinite series.

This work contains several hundred questions rationally solved; among which are all those in Clare's Introduction to Trade, &c. several from the Palladiums, Ladies Diaries, and other periodical books, as well as the most valuable and entertaining that could be found in other authors.

N. B. We were favoured with the Cribbage Question by Major Watson.

The algebraic demonstration of the rules here laid down are omitted for these three reasons; first, as arithmetical computations often carry their rationale along with them, the offering to prove a self-evident truth renders it more obscure. Secondly, the mathematicians are already acquainted with them. And thirdly, the young student is as little benefited by them as a pure English scholar would be by an exposition of the Bible in Greek.

What oversights may have escaped the authors, either in the press (from whence their residence is more than 100 miles) or otherwise, hope their readers will generously excuse; they having in the whole endeavoured to remove the difficulties, and render the passage easy and pleasant through this useful and delightful science.

The

The following algebraic figns or characters are very neceffary to be underftood, as being a much fhorter, better, and more fignificant way of expreffion, than by words at length.

Signs	*Names*	*Significations*
$+$	Plus or more.	The fign of addition; as $9 + 5$ is 9 more 5, and fignifies that the numbers 9 and 5 are to be added together.
$-$	Minus or	The fign of Subtraction; $8 - 3$, is 8 lefs 3, and fignifies that 3 is to be taken from 8.
$\times$	Multiplied into or by	The fign of Multiplication; as 7×5, is 7 multiplied into, or by 5.
$\div$	Divided by	The fign of Divifion; $8 \div 2$, is 8 divided by 2: alfo thus $\frac{8}{2}$ or 2) 8 (4, which fignifies the fame thing.
$=$	Equal to	The fign of Equality; $9 = 9$, or $9 + 6 = 15$, or $9 - 6 = 3$, that is 9 is equal to 9, or 9 more 6, is equal to 15, and 9 lefs 6, is equal to 3.
$:$ $::$	Is to So is	The Signs of Proportion, or Rule of Three; thus, $2 : 8 :: 6 : 24$ are to be read, as 2 is to 8, fo is 6 to 24.
$\div\!\div$		Continued Proportionals in Geometric Progreffion.
ⓒ	Involution.	Thus 27 ⓒ 3 or 27^3, fignifies that 27 is to be involved to the third power.
$\sqrt{}$	Extraction of the roots	Thus $\sqrt{4} = 2$, fignifies that the fquare root of 4 equals 2, or $\sqrt[5]{243} = 3$: viz. the furfolid or root, or the 5th power of $3 = 243$.

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Arithmetical

Arithmetical Collections

AND

IMPROVEMENTS.

BOOK I.

CHAP. I. SECT. I.

NUMERATION, or NOTATION, teacheth to read or expreſs any number when wrote down; and conſequently to write down any propoſed number, according to its true value.

TABLE.

Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thouſands.	Tens of thouſands.	Thouſands.	Hundreds.	Tens.	Units.
3	6	7	4	2	8	7	2	5
-	3	9	1	0	6	8	5	3
-	-	8	4	6	1	3	2	7
-	-	-	3	9	2	8	7	5
-	-	-	-	6	2	4	3	8
-	-	-	-	-	4	1	3	7
-	-	-	-	-	-	7	3	5
-	-	-	-	-	-	-	6	4
-	-	-	-	-	-	-	-	7
9	8	7	6	5	4	3	2	1

All figures in the firſt row towards the right-hand, are units; thoſe in the ſecond row tens; thoſe in the third hundreds; and thoſe in the fourth thouſands, &c.

A cypher, although by itſelf, it ſignifies nothing; yet being placed on the right-hand of any figure, augments the value of that figure ten times, by advancing it into a higher place than if the cypher had not been there. As 6 ſix, 60 ſixty, 600 ſix hundred, &c.

To every three figures are orderly repeated the denominations of units, tens, hundreds; ſo that he that can read three figures, may, with a little more inſtruction, be quickly able to read any number, how large ſoever. And to every three figures, the names units, and thouſands, are alternately applied.

Likewiſe to every ſix figures from the right-hand a new general name is given. As to the firſt ſix figures, the general name of units are given; to the ſecond ſix, the general name of millions; to the third ſix, billions; to the fourth trillions; to the fifth quadrillions, &c.

The whole art of figural notation is comprehended in the following table:

Quadrillions.		Trillions.		Billions.		Millions.		Units.	
Thouſ.	Units.	Thouſ.	Units.	Thouſ.	Units.	Thouſ.	Units.	Thouſ.	Units.
712	348	634	235	314	527	625	284	123	714
h t u	h t u	h t u	h t u	h t u	h t u	h t u	h t u	h t u	h t u

Read thus: Seven hundred twelve thouſand, three hundred and forty-eight quadrillions.

Six hundred thirty-four thouſand, two hundred thirty-five trillions.

Three hundred fourteen thouſand, five hundred twenty-ſeven billions.

Six hundred twenty-five thouſand, two hundred eighty-four millions.

One hundred twenty-three thouſand, ſeven hundred and fourteen (units.)

The following numbers are alſo expreſſed in words at length, 370084, three hundred ſeventy thouſand, and eighty-four.

418427900, four hundred eighteen million, four hundred twenty-ſeven thouſand, nine hundred.

6210003745, ſix thouſand two hundred ten million, three thouſand ſeven hundred forty-five.

41027308751,

41027308751, forty-one thousand twenty-seven million, three hundred eight thousand, seven hundred fifty-one.

293417604712, two hundred ninety-three thousand four hundred seventeen million, six hundred four thousand, seven hundred and twelve.

618002030694713, six hundred eighteen billions, two thousand thirty millions, six hundred ninety-four thousand, seven hundred and thirteen.

47038066250433251889411, forty-seven thousand thirty-eight trillions, sixty-six thousand two hundred fifty billions, four hundred thirty-three thousand two hundred fifty-one millions, eight hundred eighty-nine thousand, four hundred and eleven.

A TABLE of NUMERICAL CHARACTERS *used by the Romans.*

I	One.
V	Five.
X	Ten.
L	Fifty.
C	An hundred.
D or IↃ	Five hundred.
M or CIↃ	A thousand.
IↃↃ	Five thousand.
CCIↃↃ	Ten thousand.
IↃↃↃ	Fifty thousand.
CCCIↃↃↃ	A hundred thousand.
IↃↃↃↃ	Five hundred thousand.
CCCCIↃↃↃↃ	A million.

A line drawn over any number less than a thousand, intimates so many thousands; as $\overline{\text{LXX}}$ is 70,000; $\overline{\text{C}}$ is 100,000; and $\overline{\text{M}}$ a million.

I and X are sometimes placed before characters of greater value, namely, I before V or X, and X before L or C, in which case the value of I and X is to be subtracted from the value of the following character, as IV four, IX nine, XL forty, XC ninety.

V and L are never repeated, and none of the other characters above four times. Thus, IIII or IV; but V five, XXX thirty; but XL forty, LXXX eighty; but XC ninety, CCCC four hundred; but D five hundred.

In figures express, a million and a half South-sea bonds. Ninescore, and fourteen thousand, eight hundred sheep. Threescore and twelve thousand, thirteen hundred pounds of lead. Fifteen thousand, and fourscore million of stivers. One hundred and twenty thousand, two hundred and six millions, seventy thousand, seven hundred, and seven rials of plate. Three millions, and thirty-three thousand, and thirty pieces of eight. Four thousand, and forty hundred pounds, thirty-four shillings, and fourteen pence five farthings.

South-sea bonds - - - -	1500000
Sheep - - - - - -	194800
Lead - - - - - -	73300 lb.
Stivers - - - -	15080000000
Rials of plate - - -	120206070707
Pieces of eight - - -	3033030

l.	s.	d.
404001	15	$3\frac{1}{4}$

SECT.

SECT. II.

ADDITION.

ADDITION is a rule whereby ſeveral numbers are ſo connected and put together, that their aggregate or total amount, may be known.

Obſerve to place your numbers ſo, that each figure may ſtand directly underneath thoſe of the ſame value, viz. units under units, tens under tens, and hundreds under hundreds, &c. Then,

RULE,

Always begin your addition at the place of units, and add together all the figures that ſtand in that place; and if their ſum be under ten, ſet it down below the line underneath its own place; but if their ſum be more than ten, then you muſt ſet down only the overplus, or odd figure, above the ten (or tens) and ſo many tens as the ſum of thoſe units amount to, you muſt carry to the place of tens; adding them and all the figures that ſtand in the place of tens together, in the ſame manner as thoſe of the units were added; then proceed in the ſame order to the place of hundreds, and ſo from place to place till all is finiſhed.

1. In the biſſextile, or leap-year, how many days and hours?

			Hours.
January	31	- - - - =	744
February	29	- - - - =	696
March	31	- - - - =	744
April	30	- - - - =	720
May	31	- - - - =	744
June	30	- - - - =	720
July	31	- - - - =	744
Auguſt	31	- - - - =	744
September	30	- - - - =	720
October	31	- - - - =	744
November	30	- - - - =	720
December	31	- - - - =	744
Anſwer	366 days	- - =	8784

2. Find the number of chapters in the five books of Moses; after that the number of verses, and give their joint sum.

Ch	Gen.	Ver.	Ex.	Ver.	Lev.	Ver.	Num.	Ver	Deu	Ver.	Ch.
1		31		22		17		54		46	1
2		25		25		16		34		37	2
3		24		22		17		51		29	3
4		26		31		35		49		49	4
5		32		23		19		31		33	5
6		22		30		30		27		25	6
7		24		25		38		89		26	7
8		22		32		36		26		20	8
9		29		35		24		23		29	9
10		32		29		20		36		22	10
11		32		10		47		35		32	11
12		20		51		8		16		32	12
13		18		22		59		33		18	13
14		24		31		57		45		29	14
15		21		27		33		41		23	15
16		16		36		34		50		22	16
17		27		16		16		13		20	17
18		33		27		30		32		22	18
19		38		25		37		22		21	19
20		18		26		27		29		20	20
21		34		36		24		35		23	21
22		24		31		33		41		30	22
23		20		33		44		30		25	23
24		67		18		23		25		22	24
25		34		40		55		18		19	25
26		35		37		46		65		19	26
27		46		21		34		23		26	27
28		22		43				31		68	28
29		35		46		859		40		29	29
30		43		38				16		20	30
31		55		18				54		30	31
32		32		35				42		52	32
33		20		23				56		29	33
34		31		35				29		12	34
35		29		35				34			
36		43		38				13		959	
37		36		29							
38		30		31				1288			
39		23		43							
40		23		38							
41		57									
42		38		1213							
43		34									
44		34									
45		28									
46		34									
47		31									
48		22									
49		23									
50		26									
		1533									

	Chapters.	Verses.
Genesis - - -	50	1533
Exodus - - -	40	1213
Leviticus - - -	27	859
Numbers - - -	36	1288
Deuteronomy -	34	959
	187	5852
		187
Joint sum		6339

3. Decipher

3. Decipher the following numerical letters, and find their ſum, viz.

IV	4
VI	6
IX	9
IIX uncommon	8
XIII	13
XLV	45
LXXXI	81
XCVI	96
CXC	190
CD uncommon	400
DCC	700
MCL	1150
MDCXLVIII	1648
$\overline{\text{M}}$CCM uncommon	1000800
IↃↃIↃCCCLVII	5857
$\overline{\text{VI}}$CCLXXXX	6290
Anſwer,	1017297

I would adviſe the young accomptant, in long operations in addition, to point at every 60, carrying on the overplus; and when he hath caſt up the whole line, carry 6 to the next place for every point.

And to prove the work, begin at the top, and caſt it downwards, in the ſame manner as was done upwards, pointing on the other ſide the figures: and if the amount be the ſame both ways, it may be preſumed the work is right.

Or, when you have caſt up the whole, divide it into two or more parts, which caſt up ſeparately; then add the ſums of the ſaid parts together, which, if like the firſt ſum, the work may be adjudged to be right.

4. How much is A (born 16 years ago) older than B, who will come into the world fourteen years hence?

$$16 + 14 = 30 \text{ years, the anſwer.}$$

5. A perſon was 17 years of age 29 years ſince, and he will be drowned 23 years hence; pray in what year of his age will this happen?

$$17 + 29 + 23 = 69 \text{ years, the anſwer.}$$

SECT. III.

Addition of English Coins.

The leaſt piece of money uſed in England is a farthing.

And

Farth.				
4 =	1 penny.			
48 =	12 =	1 ſhilling.		
960 =	240 =	20 =	1 pound ſterling.	

---

	s.		d.	
And	5	:	–	is a crown.
	6	:	8	is a noble.
	10	:	–	is an angel,
	13	:	4	is a mark.

N, B. l., s., d., q. ſtand for Pounds. Shillings. Pence. Farthings. ‖ d. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ is wrote for 1, 2, 3 Farth. But moſt commonly,

Pence Tables to be got by heart.

d.	s.	d.	s.	d.	s.	d.	d.	s.	d.
12 =	1	72 =	6	20 =	1	8	70 =	5	10
24 =	2	84 =	7	30 =	2	6	80 =	6	8
36 =	3	96 =	8	40 =	3	4	90 =	7	6
48 =	4	108 =	9	50 =	4	2	100 =	8	4
60 =	5	120 =	10	60 =	5	–	110 =	9	2

Having placed the numbers to be added in this order, viz. pounds under pounds, ſhillings under ſhillings, and pence under pence, &c.

RULE,

Begin with the farthings, and for every four carry one penny, ſetting down the overplus under the farthings; then proceed to the pence, caſting up to 60, where make a dot, and ſo proceed to the top, ſetting down the odd pence

pence in their proper place; and carrying one for every 12 pence, and five for every dot (as 60 pence make five ſhillings;) ſo procced to the units place of the ſhillings, ſetting down the overplus above 10, for each of which carry one, and ſix for every dot to the angels; and as two angels make one pound, carry half the number of angels to the pounds, ſetting down the odd one, if it ſo happen; then caſt up the pounds, as before directed, in addition of whole numbers.

1. A nobleman going out of town, is informed by his ſteward that his corn-chandler's bill comes to 123l. 19s. His brewer's 41 l. 10s. His butcher's 212 l. 6 d. To his lordſhip's baker is owing 24 l. To his tallow chandler 13l. 8s. To his taylor 137 l. 9 s. 9 d. To his draper 74 l. 13 s. 6 d. His coachmaker's demand was 214 l. 16 s. 6d. His wine-merchant's 68 l. 12 s. His confectioner's 16 l. 2 s. His rent 86 l. 2 s. And his ſervant's wages for half a year came to 46 l. 5 s. What money muſt he ſend to his banker for, in caſe he would carry with him 50 l. to defray his expences on the road.

	l.	s.	d.
Corn-chandler - - - -	123	19	–
Brewer - - - - -	41	10	–
Butcher - - - -	212	–	6
Baker - - - -	24	–	–
Tallow-chandler - - -	13	8	–
Taylor - - - -	137	9	9
Draper - - - - -	74	13	6
Coach-maker - - -	214	16	6
Wine-merchant - - -	68	12	–
Confectioner - - -	16	2	–
Rent - - - - -	86	2	–
Servants wages - - -	46	5	–
For expences - - -	50	–	–
Anſwer £	1108	18	3

2. A collecter of caſh hath been out with bills, and gives an account that A paid him 13 l. and half a crown; B 2 l. 13 s. C 14 s. and a groat; D 1 l. 9 s. 8½ d. E 11 l. 6¼ d. F 17 s. and a teſter; G 12 s. 2 d. H a pound and half a guinea; I a moidore, and 13 s. K two broad pieces of 23 ſhillings each, a Jacobus of 25 s. and a ſhilling;

shilling; L nine pounds and a mark; M 12 l. 12 s. N a bank-note of 15 l. and O three crown-pieces and an angel. What cash had he in charge?

	l.	s.	d.
A - - - - -	13	2	6
B - - - -	2	13	0
C - - - - -	–	14	4
D - - - -	1	9	8 $\frac{1}{2}$
E - - -	11	0	6 $\frac{1}{4}$
F - - -	–	17	6
G - - -	–	12	2
H - - - - -	1	10	6
I - - -	2	–	–
K - - - - -	3	12	–
L - - -	9	13	4
M - - -	12	12	–
N - - - - -	15	–	–
O - - - - -	1	5	–
	£ 76	2	6 $\frac{3}{4}$

3. A corn-factor buys seventy quarters of oats for 46 l. 7 s. 6 d. thirty-eight quarters of beans for 100 l. twelve quarters of pease, which cost 16 l. 16 s. eighty-eight quarters of barley, for 73 l. 8 d. sixteen ditto of wheat for 56 l. 9 s. 10 d. and six quarters of rye for 4 l. 1 s. 6 d. the water carriage of all comes to 13 l. 2 s. 7 d. his riding charges to 1 l. 13 s. and if he clears eighteen guineas by the bargain, what do his bill of parcels amount to?

	l.	s.	d.
Oats - - - - -	46	7	6
Beans - - - -	100	–	–
Pease - - - -	16	16	–
Barley - - - -	73	–	8
Wheat - - - -	56	9	10
Rye - - - -	4	1	6
Water carriage - - - -	13	2	7
Riding charges - - -	1	13	–
Commission - - -	18	18	–
	£ 330	9	1

4. A

4. A of Amfterdam is debtor to B of Briftol, for mercery wares as per factory, 418 l. 2 s. 6 d. for forty Cwt. of Chefhire cheefe 52 l. 18 s. for Englifh broad-cloth fifteen pieces, 317 l. 12 s. 10 d. for 19 fodder of lead 320 l. for 12 tons of bar iron 173 l. 3 d. for eight tons of copper 1110 l. 10 s. 1 d. for his acceptance of a bill drawn 88 l. 14 s. for another paid for honour 50 l. 10 dozen of Morocco fkins 28 l. 15 s. 4. paid convoys, infurances, and port charges 43 l. warehoufe room, poftage, fledage, boatage, and incidental charges 5 l. 5 s. the factorage of all came to 112 l. 6 s. For what fum muft B draw to clear the account?

	l	s.	d.
Mercery wares - - -	418	2	6
Chefhire cheefes - - -	52	18	-
Broad-cloth - - -	317	12	10
Lead - - - -	320	-	-
Bar-iron - - -	173	-	3
Copper - - - -	1110	10	1
Accepted bill - - -	88	14	-
Ditto on honour - - -	50	-	-
Morocco fkins - - -	28	15	4
Convoys, infurances, &c. - -	43	-	-
Warehoufe room, &c. - -	5	5	-
Factorage - - -	112	6	-
	£ 2720	4	-

5. A rate or affeffment, for and towards the relief of the poor of the parifh of Gofberton, &c.

	l.	s.	d.
Francis Fane, Efq. - - -	11	3	7
John Robinfon, Gent. - - -	3	17	$5\frac{1}{4}$
Richard Calthorp, Gent. - -	4	13	6
Thomas Baley, Gent. - - -	5	7	$4\frac{1}{4}$
Mr. John Torry - - -	3	18	6
Mr. John Turver - - -	1	17	$4\frac{1}{2}$
Henry Worley - - -	-	3	2
Jonathan Cheavin - - - -	-	15	$10\frac{3}{4}$
William Trickett - - -	-	5	9
Anthony Birks - - -	-	6	$10\frac{1}{4}$
Carried over	32	9	$5\frac{1}{2}$

	l.	s.	d.
Brought over	32	9	5½
Mr. Robert Cole - - -	1	2	7
John Wright - - - -	–	10	3½
Mr. John Shaw - - -	–	19	11½
Thomas Ladd - - -	–	5	9
John Lambson - - - -	–	3	6¾
Thomas Hooper - - -	–	2	9¾
Francis Mastin - - -	–	10	3
William Crawforth - - -	–	15	10½
Thomas Oldgate - - -	–	17	5¼
William Wiseman - - -	–	3	11
Mrs. Margaret Parkinson - - -	–	11	7¾
Samuel Lane - - -	–	2	4¼
Mrs. Alice Sharp - - -	–	2	3
William Curtis - - -	–	9	7¾
John Linsey - - - -	–	–	11
Mrs. Silv. Flear - - - - -	–	18	9
John Pattison - - - -	–	2	4½
John Gennils - - - -	–	19	3
Mr. John Pacy - - -	–	18	11¾
William Bilton - - -	1	4	1½
Mr. Robert Allen - - -	2	12	3
Mrs. Eliz. Wilcox - - - -	2	9	6¾
John Mason - - - -	1	18	7½
Mr. John Thimbleby - - -	5	7	9¾
John Smith - - - -	3	18	6
Henry Ward - - -	5	17	3¾
Alexander Codling - - -	1	13	2
William Lambson - - -	1	5	4
John Gibbons - - -	–	17	10¾
£	69	12	7

6. The Right Hon. the Lord Bolsover Debtor
To Paul Purseproud, upholder.

1768.		l.	s.	d.
April 19.	A rich crimson damask bed, laced, complete - -	75	–	–
May 5.	A set of window curtains and valences, ditto - - -	16	11	8
7.	A fine carpet, counterpane, and an otter-down quilt -	12	10	–
	Carried over	104	1	8

			l.	s.	d.
		Brought over	164	1	8
June	6.	A crimſon velvet eaſy-chair, and two ſtools, ditto -	13	7	6
	13.	A wrought dimity bed and furniture complete - -	28	18	4
Aug.	10.	A down bed, bolſter, pillows, and quilt - -	15	-	-
		Chairs 10, with two armed ditto, walnut-tree framed -	34	12	6
Nov.	20.	A fire-ſcreen, bed, table, and dreſſing glaſs - - -	8	14	6
		The lady Wanton's picture, in a rich frame - -	21	-	-
			£ 225	14	6

7. A perſon ſaid he had 20 children, and that it happened there was a year and half between each of their ages; his eldeſt was born when he was 24 years old, and the age of his youngeſt is now one-and-twenty. What was the father's age?

When the eldeſt was born	-	24
Then $19 + 9\frac{1}{2}$ - -	=	$28\frac{1}{2}$
Youngeſt - -	-	21
Father's age -	-	$73\frac{1}{2}$

8. A ſheepfold was robbed three nights ſucceſſively; the firſt night 84 ſheep were ſtolen; the ſecond night 42 were loſt; the laſt night they took 21 and left 20: how many were there at firſt?

20 left
21 taken the 3d night.
42 taken the 2d night.
84 taken the 1ſt night.

167 ſheep in all.

SECT.

SECT. IV.

SUBTRACTION.

SUBTRACTION takes a lefs number from a greater, by which the excefs, difference, or remainder may be known.

In fetting down numbers for work, always place the greateft number or fum uppermoft, in fuch order that units may ftand under units, tens under tens, &c. alfo pounds under pounds, fhillings under fhillings, pence under pence, &c.

RULE.

Begin with the loweft or leaft denomination, (as in addition) and take or fubtract the figure, or figures, in that place of the fubtrahend, from the figure, or figures, that ftands over them of the fame denomination, fetting down the remainder. But if that cannot be done, increafe the upper figure or figures, with one of the next fuperior denomination; and from that fum make fubtraction; and fo proceed to the next fuperior denomination, where you muft pay the one borrowed, adding unity to the fubtrahend in that place.

1. If a perfon hath 105 miles to travel, and hath gone 99, how many miles hath he yet to go?

	Miles.
	105
	99
Anfwer	6

2. If a perfon be 49 years of age this prefent year 1772, what year was he born in?

	1772
	49
Anfw.	1723

3. In

3. In fifteen hundred ninety-two there died a noble prince; how many years is that ago?

	1772
	1592
Anſwer	180

A collector of exciſe has received 2479 l. 12 s. $6\frac{1}{4}$ d. and paid into the office, by ſeveral remittances, 1977 l. 17 s. $7\frac{1}{2}$ d. how much remains in his hands?

	l.	s.	d.
Received	2479	12	$6\frac{1}{4}$
Remitted	1977	17	$7\frac{1}{2}$
In hand £	501	14	$10\frac{3}{4}$

5. Having a piece of ground 127 feet in front, I let off to A 57 feet, to build on at one end; and to B at firſt $27\frac{1}{2}$ feet; which he afterwards, by conſent, extended to 42 feet; what ground was left in the center?

	Feet.
	127
$57 + 42 =$	99
Anſwer	28

6. Your grandfather, if living, is 119 years of age; your father actually 63; you are not ſo old as your grandſire by 83 years; what is the difference in years between your father and you.

	Years.		Years.
Grandfather's	119	Father's	63
	— 83		— 36
Your age	36	Anſwer	27

7. In the city of Peking in China, is a bell weighing, it is ſaid, 120000 pounds; at Nankin, in the ſame country, is another weighing 50000 pounds. The firſt exceeds the

the great bell at Erfurd, in Upper Saxony, by 94600 pounds; how much then is the German bell inferior in weight to the second?

Pekin bell	120000	Nankin bell	50000
	—94600		— 25400
Erfurd bell	25400	Anſwer	24600

8. Miſs Kitty told her ſiſter Charlotte, whoſe father had before left them twelve thouſand twelve hundred pounds a-piece, that their grandmother by will had raiſed her fortune to fifteen thouſand pounds, and had made her own twenty thouſand; pray what did the old lady leave between them?

Miſs Kitty	£ 20000	Miſs Charlotte	£ 15000	
By father	13200		13200	
Grandmother	6800	+	1800	
				= 8600 Anſ.

9. What is the difference between the ages of A, born in the year 1693, and B that will be born 13 years hence; the queſtion being put in the year 1758?

Anno 1758
— 1693

65 + 13 = 78 the Anſwer.

To prove ſubtraction, add the remainder to the leſs number, which ought to make up the greater, if the work be right.

10. A horſe in his furniture is worth 35 l. 10 s. out of it 12 l. 12 s. how much does the price of the furniture exceed that of the horſe?

	l.	s.
Horſe and furniture	35	10
Horſe - -	12	12
Furniture - -	22	18
	12	12
Anſwer £	10	6

11. A

11. A merchant at his outſetting in trade owed 280 l. He had in caſh, commodities, the ſtocks, and good debts, 11505 l. 10 s. He cleared the firſt year by commerce 393 l. 13 s. 1 d. What, at the year's end was his net balance?

	l.	s.	d.
To caſh, &c. - -	11505	10	–
Commerce - -	393	13	1
	11899	3	1
Debts - - -	280	–	–
Anſwer £	11619	3	1

12. A trader failing, was indebted to A 71 l. 12 s. 6 d; to B 34 l. 9 s. 9 d.; to C 16 l. 8 s. 8 d.; to D 44 l.; to E 19 l. 19 s.; to F 11 l. 2 s. 3 d; to G 66 l. 17 s. 6 d.; to H a fine of thirty marks. At the time of this diſaſter he had by him in caſh 3 l. 13 s. 6 d.; in commodities 23 l. 10 s.; in houſhold furniture 13 l. 8 s. 6d. in plate 7 l. 18 s. 5 d.; in a tenement 56 l. 15 s.; in recoverable book debts 87 l. 13 s. 10 d. Suppoſing theſe things faithfully ſurrendered to his creditors, what will they then loſe by him?

Debtor

	l.	s.	d.
To A	71	12	6
B	34	9	9
C	16	8	8
D	44	–	–
E	19	19	–
F	11	2	3
G	66	17	6
H	20	–	–
Debtor	284	9	8
Creditor	192	19	3
Anſwer £	91	10	5

Credior.

	l.	s.	d.
By Caſh - - -	3	13	6
Commodities -	23	10	–
Houſhold furniture -	13	8	6
Plate - - -	7	18	5
Tenement - -	56	15	–
Book debts -	87	13	10
£	192	19	3

13. You were born 34 years after me; how old ſhall I be when you are 17; and how old will you be, when I am 70 years of age?

$70 - 34 = 36$ You. $34 + 17 = 51$ I.

C

14. A

14. A made a bond for 114l. 10s. the intereſt came to 19l. He then paid off forty guineas, and gave a freſh bond for what was behind. By that time there was 13l. 4s. 8d. due on the ſecond for intereſt. He paid off 37l. 14s. 2d. more, took up the old bond, and ſigned a new one ſtill for the reſidue. The principal again ran on till there was 9l. 11s. 3d. more due, and then he determined to take it up: Pray what money had his creditor to receive?

		l.	s.	d.
Firſt bond - - -		114	10	–
Intereſt - - - -	+	19	–	–
	£	133	10	–
Paid - - - - -	–	42	–	–
Second bond - - -		91	10	–
Intereſt - - -	+	13	4	8
	£	104	14	8
Paid - - -	–	37	14	2
Third bond - - -		67	–	6
Intereſt - - -	+	9	11	3
Anſwer - - -	£	76	11	9

15. Received from my factor at Alicant, on account of ſales of tin, to the value of 197l. 12s. ſterling; of bees-wax 71l. 7s. 6d. of ſtockings 47l. 3s. 6d.; of tobacco, the net proceeds whereof were 943l. 15s. 10d.; of cotton 123l. 3s. 7d.; and of wheat to the amount of 116l. 5s. 6d. He at the ſame time adviſes, that he has per order ſhipped for my account, and riſk, Alicant wines, to the value of 226l. 16s. 6d.; figs 157l. 11s. 3d; fruit 90 cheſts, coſt 104l. 6s.; olives 136l. 10s.; oil 193l. 17s.; raiſins 143l. 4d.; and Spaniſh wool to the value of 73l. 13s. 8d. The commiſſion of the whole conſignment came to 71l. 18s. 11d. which of us is to draw for the difference, and how much?

Debror.

Debtor.		Factor.		Creditor.			
	l.	s.	d.		l.	s.	d.
To Tin - -	197	12	–	By Wines - -	226	16	6
Bees-wax -	71	7	6	Figs - -	157	11	3
Stockings -	47	3	6	Fruit - -	104	6	–
Tobacco -	943	15	10	Olives - -	136	10	–
Cotton - -	123	3	7	Oil - - -	193	17	–
Wheat - -	116	5	6	Raisins - -	143	–	4
				Wool - -	73	13	8
Debtor	£1499	7	11	Commission	71	18	11
Creditor	1107	13	8		£ 1107	13	8
Balance	£ 391	14	3				

16. A, B and C open an account with a banker, Jan. 11, 1739, and put into his hands, viz. A 17 l. 17 s. B 34 l. 11 s. 6 d. C 28 l. 18 s. 10 d. On the 21st A withdrew 9 l. 10 s. and C advanced 12 l. and a crown. The 24th B called for 6 l. 10 s. The 30th C wanted 19 l. 8 s. 4 d. On the 12th of February B deposited with him eleven broad pieces, and three moidores. On the 19th A sent for 5 l. and a noble more; but on the 24th returned him 42 l. On the 2d of March C paid in twenty guineas, and B drew for six. The 14th B sent in 17 l. 8 s. 8 d; and the 17th A had cash 12 l. 2 s. 6 d. On the 19th they sent for five guineas a man; and on the 24th they returned that sum, and ten marks a piece more. How much did their said banker owe them, jointly and separately, at Lady-day?

Debtor. Banker's account current. Creditor.

A

1739.	l.	s.	d.		l.	s.	d.
				21st Jan. - -	9	10	–
11th Jan. - -	17	17	–	19th Feb. - -	5	6	8
24th Feb. - -	42	–	–	17th Mar. - -	12	2	6
24th Mar. - -	11	18	4	19 - - - - -	5	5	–
Debtor	71	15	4		£ 32	4	2
Creditor	32	4	2				
To A -	£ 39	11	2				

Debtor. Banker's account current. Creditor.

B

	l.	s.	d.		l.	s.	d.
11th Jan. - -	34	11	6	24th Jan. - -	6	10	—
12th Feb. - -	16	14	—	2d Mar. - - -	6	6	—
14th Mar. - -	17	8	8	19th - - - -	5	5	—
24th - - - - -	11	18	4		£18	1	—
Debtor -	80	12	6				
Creditor -	18	1	—				
To B -	£62	11	6				

C

	l.	s.	d.		l.	s.	d.
11th Jan. - -	28	18	10	30th Jan. - -	19	8	4
21ſt - - -	12	5	—	19th Mar. - -	5	5	—
2d Mar - -	21	—	—		£24	13	4
24th - - -	11	18	4				
Debtor -	74	2	2				
Creditor -	24	13	4				
To C -	£49	8	10				

	l.	s.	d.
To A	39	11	2
B	62	11	6
C	49	8	10
	£151	11	6

17. B born 161 years ago, died when C was 47 years of age; who it ſeems came into the world 180 years ſince, and outlived B 43 years. The ſum of the ages of theſe two perſons is required?

$180 - 47 = 133$ years ſince B died.
$161 - 133 = 28$, B's age.
$47 + 43 = 90$, C's age.
$28 + 90 = 118$, the anſwer.

18. Sam. was born 28 years before Toby, who died at 12, and lived 19 years after him. Rachael came to light when Sam. was 16, and died 11 years before him. Joſhua (when Rachael was ſeven years old, being himſelf then 14) went abroad, where he continued nine years; and returning, ſurvived Rachael four years. How old was each of theſe, and what is the ſum of their ages?

28.

	28	
Toby -	12 years old.	
	19	
Sam. -	59 years old.	
	$-27=16+11$	Then $59+12+32+43=146$.
Rachael	32 years old.	
	$-11=7+4$	
Joſhua	43 years old.	

19. A chaiſe, horſe, and harneſs, were together valued at 50 l. The horſe in harneſs was worth 38 l. 16 s. 6 d.; the chaiſe and harneſs were eſtimated at 13 l. 13 s. Their ſeveral valuations are required?

	l.	s.	d.
Together - -	50	–	–
Horſe & harneſs —	38	16	6
Chaiſe - -	£ 11	3	6

	l.	s.	d.
Chaiſe and harneſs	13	13	–
	11	13	6
Harneſs alone -	£ 2	9	6

	l.	s.	d.
	50	–	–
	13	13	–
Horſe alone -	£ 36	7	–

20. From the creation to the flood was 1656 years; thence to the building of Solomon's temple 1336 years; thence to Mahomet, who lived 622 years after Chriſt, 1630 years. In what year of the world was Chriſt then born?

From the creation to the flood - -	1656
To the building of Solomon's temple	1336
To Mahomet - - - -	1630
	4622
Mahomet after Chriſt	— 622
Anſwer A. M.	4000

21. A is 13 years younger than B, and 17 years older than C, who in the year 1711 was known to be 24 years of age. How old was each of these persons in 1733?

$$\left.\begin{aligned}1733 - 1711 = 22 + 24 &= 46 \text{ C's}\\ 17 + 46 &= 63 \text{ A's}\\ 13 + 63 &= 76 \text{ B's}\end{aligned}\right\}\text{age.}$$

22. W, X, Y, Z send their money to the Bank, and draw upon it in the following manner, viz. June 4th, 1771, Z sends in 70 l. 8 s. Y had 116 l. 14 s. 10 d. remaining on balance; and the 14th sent in 120 l. more. W paid in 47 l. 18 s. 2 d. in cash, and delivered in a Bank-note for 200 l. X paid in a bill of exchange on a good man, for 33 l. 14 s. 9 d. and in cash made it up 100 l. Y on the 16th drew for 43 l. 12 s. 6 d. and on the 20th Z for eleven guineas. W on the 24th added 14 l. 12 s. 10 d. and X withdrew 47 l. 10 s. 8 d. Y on the 28th paid in 18 l. 5 s. and two days after drew for 88 l. 13 s. 4 d. W sent for 63 guineas on the 30th, and in five days after for 15 l. 10 s. 9 d. more. Z on the 7th of July demanded 12 l. 8 s. 3 d. and X 7 l. 3 s. 1 d. Z on the 15th remitted them 31 l. 12 s. 4 d. and per assignment they received for him, the same time, double that sum. Y received on the 12th 81 l. 19 s. 8 d. and W 10 l. 10 s. Y three days atfer sent in 42 l. and W 52 l. On the 19th X sent for 31 l. 18 s. 10 d. and on the 24th paid in 19 l. 19 s. The question is, how stood these gentlemen's cash severally, and what money can they jointly raise?

Debtor. The Bank. Creditor.

To cash received. W By cash paid.

1771.	l.	s.	d.		l.	s.	d.
4th June - -	47	18	2	30th June - -	66	3	–
	200	–	–	5th July - -	15	10	9
24th - - -	14	12	10	12th - - -	10	10	–
15th July - -	52	–	–				
					£ 92	3	9
Debtor -	314	11	–				
Creditor -	92	3	9				
To W -	£ 222	7	3				

Debtor.

Debtor. The Bank. Creditor.

To cash received. X By cash paid.

1771.	l.	s.	d.		l.	s.	d.
4th June - -	100	–	–	24th June - -	47	10	8
24th July - -	19	19	–	7th July - -	7	3	1
				19th - - -	31	18	10
Debtor -	119	19	–		£86	12	7
Creditor -	86	12	7				
To X -	£33	6	5				

Y

	l.	s.	d.		l.	s.	d.
4th June - -	116	14	10	16th June - -	43	12	6
14th - - -	120	–	–	30th - - -	88	13	4
28th - - -	18	5	–	12th July - - -	81	19	8
15th July - -	42	–	–		£214	5	6
Debtor -	296	19	10				
Creditor -	214	5	6				
To Y -	£82	14	4				

Z

	l.	s.	d.		l.	s.	d.
4th June - -	70	8	–	20th June - -	11	11	–
15th July - -	94	17	–	7th July - -	12	8	3
Debtor -	165	5	–		£23	19	3
Creditor -	23	19	3				
To Z -	£141	5	9				

To W - -	222	7	3
X - -	33	6	5
Y - -	82	14	4
Z - -	141	5	9
In all -	£479	13	9

23. Moses was born Anno Mundi 2433. Homer 832 years after him. Julius Cæsar lived 40 years before our Saviour; and Alexander 312 years before Cæsar. Now as Christ was incarnate 4000 years after the creation, the sum of the intervals between Homer and the three great personages last mentioned is required?

Moses born A. M. - - - - - -	2433
	+ 832
Homer born A. M. - - - -	3265
Christ born A. M. - - -	4000
	— 40
Cæsar born A. M. - - -	3960
	— 312
Alexander born A. M. - -	3648

3960 — 3265 = 695 from Homer to Cæsar.
4000 — 3265 = 735 from Homer to Christ.
3648 — 3265 = 383 from Homer to Alexander.

Answer 1813

24. A merchant, taking an inventory of his capital, finds in his vault 28 pieces of brandy, which cost him 874 l. 10 s. 6 d. Bourdeaux claret 40 tuns, which stood him in 754 l. 4 s.; 22 lasts, four bushels of corn in his granary, worth 675 l. 17 s. 3 d.; with two lasts of Canary seed, worth 113 l. In his warehouse were 10 casks of indigo, worth 632 l. 12 s. a parcel of saffron, worth 253 l. 5 s. W. P. of Stafford owed him 384 l. 10 s. In the hands of F. G. of Lynn, he had wines to the amount of 1011 l. 10 s. Pepper in the keeping of S. Q. of the Custom-house, value 1552 l. 16 s. 8 d. Besides which, R. O. owes him on bond 300 l.; and T. M. on note 260 l. 14 s. He has in India bonds to the value of 459 l. and the interest of those securities made 25 l. 14 s 6 d. He had Bank-stock to the value of 2134 l 4 s. 6 d. There lay in his banker's hands 1892 l. 17 s. 6 d. He was at this time indebted to D. E. 713 l. 13 s. To M. F. 352 l. 10 s. 8 d. To L. P. the foot of his account 172 guineas. To J. B. on balance 57 l. 12 s. 10 d. To an insurance 190 l. The present state of this person's fortune is required?

Stock

Stock debtor,

	l.	s.	d.
To D. E. -	713	13	—
M. F. -	352	10	8
L. P. -	180	12	—
J. B. -	57	12	10
Insurances -	190	—	—
	£1494	8	6

Contra creditor.

	l.	s.	d.
By Brandy - -	874	10	6
Claret - - -	754	4	—
Corn . - -	675	17	3
Canary seed -	113	—	—
Indigo - -	632	12	—
Saffron - -	253	5	—
W. P. - -	384	10	—
Wines per F. G.	1011	10	—
Pepper per S. Q.	1552	16	8
A bond on R. O.	300	—	—
A note on T. M.	260	14	—
India bonds -	459	—	—
Interest - -	25	14	9
Bank-stock -	2134	4	6
Banker - -	1892	17	6
	£11324	15	11
	—1494	8	6
The merchant's present worth,	£9830	7	5

25. Seth was born when Adam was 130 years of age, and 800 years before our said grandsire's death. Seth at the age of 105 years had Enos. He at 90 was father to Canaan, who at 70 had Mahalaleel. This man at 65 got Jared; who having lived 162 years, was father to Enoch. This patriarch at 65 years of age had Methuselah; and by the time he was 187 years of age, his son Lamech came into the world; who at 182 years old was father to Noah; and when Noah was 600 years old, the flood swept away the bulk of mankind. In what year of the world did this happen, and how long after the death of Adam?

Adam at -	130	years had Seth.
Seth at -	105	Enos.
Enos at -	90	Canaan.
Canaan at	70	Mahalaleel.
Mahalaleel at	65	Jared.
Jared at -	162	Enoch.
Enoch at -	65	Methuselah.
Methuselah	187	Lamech.
Lamech at	182	Noah.
Noah at -	600	entered the ark.
Year of the flood	1656	
Adam 130 + 800 =	930	
After his death -	726	

26. In

26. In a company S had 3l. 17s. 2d. more than T, who had six guineas less than R, who had within 16s. 8d. as much as W, who was known to have 100 guineas, wanting 10 marks of 13s. 4d. each. Pray what money had they among them?

l.	s.	d.		l.	s.	d.		l.	s.	d.	
104	–	–	—	6	13	4	=	98	6	8	W had.
98	6	8	– –		16	8	=	97	10	–	R had.
97	10	–	—	6	6	–	=	91	4	–	T had.
91	4	–	+	3	17	2	=	95	1	2	S had.
							£	382	1	10	the answer.

27. If the mean distance between the earth and sun be 81 millions of miles, and between the earth and moon 240 thousand miles, how far are these two luminaries asunder in an eclipse of the sun, when the moon is lineally between the earth and sun? And in another of the moon, when the earth is in a line between her and him?

Distance of the sun from the earth -	81000000
- - - of the moon - - - - - -	+ 240000
From each other in an eclipse of the moon	81240000
	81000000
	− 240000
- - - - - - in an eclipse of the sun -	80760000

28. Hipparchus and Archimedes, of Syracuse, about 200 years before Christ; Possidonius 50 years before the said grand period; and Ptolemy 140 years after it; all advanced the science of astronomy. How long did each of these persons flourish before the year of Christ 1758?

	200		50
	+ 1758		+ 2758
Hipparchus and Archimedes	1958	Possidonius.	1808
	1758		
	— 140		
Ptolemy -	1618		

29. A

29. A grant was made by the crown, anno 1239, which was forfeited 137 years before the Revolution in 1688; how long did the same subsist?

Revolution A. D.	1688			1551
	— 137		Granted A. D.	1239
Forfeited A. D.	1551		Answer	312

30 The building of Solomon's temple was in the year of the world 3000. Troy was, by computation, built 443 years before the temple, and 260 years before London. Now Carthage was built 113 years before Rome; founded 744 years before Christ, born anno mundi 4000. Is London or Carthage the antientest city, and how much?

Solomon's temple built A. D. - -		3000
Troy before - - - - - -		— 443
	A. M.	2557
		+ 260
London built - - - - - -	A. M.	2817
Christ born - - - - - - -	A M.	4000
Rome built before - - - - - -		— 744
	A. M.	— 3256
Carthage before - - - - - - - -		— 113
	A. M.	3143
		2817
London built before Carthage - -		326 years.

31. A public edifice was finished towards the close of the 10th of king John, who began his reign 134 years after the conquest in 1066; and it stood till within 70 years of the peace of Utrecht, in 1713. Of what duration was it?

Conquest - - - -	A. D.	1066
	134 + 10 =	+ 144
Edifice finished -	A. D.	1210
Peace of Utrecht -	A. D.	1713
		— 70
Demolition - - - -	A. D.	1643
		— 1210
Duration - - - - - -		433 years, the answer.

32. A,

32. A, born anno 1438, died at 48 years of age. B died anno 1502, aged threescore and seventeen. C, in the year 1577, was 22 years of age, and survived that time 54 years. D, anno 1616, had lived just half his time, and died in 1648. E was 13 years old at the death of D, and fourteen years after that was father to F, who was 31, when his son G was born; who, at his grandsire's death, was seven years of age. The years of Christ, wherein those men were born, and the year wherein the first five of them died, are severally quired?

A born 1438 + 48 = 1486 died.
B died 1502 — 77 = 1425 was born.
C in 1577 — 22 = 1555 was born.
And 1577 + 54 = 1631 died.
D died 1648 — 1616 = 32 half his age.
And 1648 — 64 = 1584 died.
E in 1648 — 13 = 1635 was born.
And 1635 + 13 + 14 + 31 + 7 = 1700 died.
F in 1635 + 27 = 1662 was born.
G in 1662 + 31 = 1693 was born.

33. The powder-plot was discovered 88 years after the Reformation in 1517. The murder of king Charles the First was committed 43 years after that discovery. The accession of the Brunswick family to the crown was in 1714, just 54 years after the return of king Charles the Second, who had lived in exile ever since the death of his father Charles the First. How long was that?

Reformation - - - - -	A. D.	1517
Powder-plot discovered - -	A. D.	+ 88
		1605
		+ 43
King Charles murdered - -	A. D.	1648

1714 — 54 = 1660 — 1648 = 12 years, the answer.

34. Arphaxad was born to Shem two years after the Deluge, and 500 years before his father's death; but at 35 years of age he had Seth, who at 30 was father to Eber; who at 34 had Peleg, and he lived 430 years after that. The question is, whether Shem or Eber died the first; and at

at ninescore and fourteen years after the death of the longer liver, what interval might be wanting to complete the term of 1000 years after the Flood?

Selah 35 + Eber 30 - - - - - - - -	= 65
Eber had Peleg at 34, lived after 430 - -	= 464
	529
Shem died after the birth of Arphaxad - - -	500
Eber was survivor - - - - - -	29

1000 — 502 + 29 + 194 = 275, interval required.

35. B was born 14 years after C, who came into the world 19 years before A, who was 23 years of age eight years ago. What then is the age of D, who is within 22 years of being as old as those three together?

$$\left.\begin{array}{l} 22 + 8 = 31 \text{ A} \\ 19 + 31 = 50 \text{ C} \\ 50 - 15 = 36 \text{ B} \end{array}\right\} \qquad \begin{array}{r} 117 \\ -22 \end{array}$$

$$117 \qquad 95 \text{ the answer.}$$

36. Of the noble family of Cornaro, the grandsire's age was 134 years; and he was 93 years older than the son, at the time when the son and father's age together made 112 years. Distinguish their ages?

Grandsire's age - -	= 134
	— 93
Sons - - - -	= 41

112 — 41 = 71, father's age.

37. K is 19 years older than L, who was 27 years of age in the south-sea year 1720. How old is M, in 1740, who in the year 1738 was within 24 years of being as old as both of them together.

$$19 + 27 = 46 \text{ K's age in } 1720$$
$$1738 - 1720 = 18.$$
$$\left.\begin{array}{l} 46 + 18 = 64 \text{ K's} \\ 27 + 18 = 45 \text{ L's} \end{array}\right\} \text{age in } 1738.$$

$$64 + 45 = 109 + 2 = 111.$$
$$111 - 24 = 87, \text{ the answer.}$$

38. If

38. If Sampson was born 17 years after Timothy, and Timothy 26 years before Jacob, who 28 years hence will be just 50. In what year of Christ were they severally born, the question being proposed anno 1758?

$1758 - 50 = 1708.$
$1708 + 28 = 1736$ Jacob.
$1736 - 26 = 1710$ Timothy.
$1710 + 17 = 1727$ Sampson.

39. A, born anno Christi 318, lived 207 years before B, who lived 104 years after C, who was successor to D 84 years. E was also 112 years after D, but predecessor to F, by 47 years. In what year of Christ did each of those gentlemen flourish?

$$\left.\begin{array}{l} 318 + 207 = 525 \text{ B} \\ 525 - 104 = 421 \text{ C} \\ 421 - 84 = 337 \text{ D} \\ 337 + 112 = 449 \text{ E} \\ 449 + 47 = 496 \text{ F} \end{array}\right\} \text{flourished.}$$

40. A was born when B was 18 years of age. How old shall A be, when B is 41; and what will be the age of B, when A is 72?

$41 - 18 = 23$ A. - - - $72 + 18 = 90$ B.

41. B, born anno 1108, lived 48 years before C, who was 113 years senior to D; and X was 114 years before Y, who was 74 years after Z, born anno 1527. In what years of Christ were these men severally born?

B born - - - - - - - - - - A.D. 1108
+ 48

C born - - - - - - - - - A.D. 1156
+ 113

D born - - - - - - - - - A.D. 1269

Z born - - - - - - - - - A.D. 1527
+ 74

Y born - - - - - - - - - A.D. 1601
− 114

X born - - - - - - - - - A.D. 1487

42. A, born 445 years before the year 1733, died anno [illegible]. B born 37 years ago, will die 18 years hence. C, [illegible] years ago, died 197 years since. D, born anno 1578,

1758, lived till within 75 years of the ſaid 1733. The length of thoſe people's lives is ſeverally required?

$1733 - 445 = 1288$ A born.
$1362 - 1288 = 74$ his age.
$37 + 18 = 55$ B's } age.
$256 - 197 = 59$ C's } age.
$1733 - 75 = 1658$ D died.
$1658 - 1578 = 80$ his age.

43. If I am now 42 years older than you are, what will be the difference of our ages 14 years after my deceaſe, in caſe you ſhall then ſurvive?

$42 - 14 = 28$, the anſwer.

44. A, born anno 1441, lived till B was ſeven years of age; which was 23 years before the Reformation, in 1517. B ſurvived this remarkable æra juſt 49 years. C, born nine years after the death of A, lived but till B was 36 years of age. The ſum of the ages of theſe three perſons is required?

Reformation - - -	A. D.	1517
		23
A died - - -	A. D.	1494
born - - -	A. D.	1441
A's age - - - - -		53
1517 + 49, B died - -	A. D.	1556
1494 — 7, B born - -	A. D.	1487
B's age - - - - - -		79

$36 - \overline{7 + 9} =$ C's age - - $= 20$
$53 + 79 + 20 = 152$, the anſwer.

45. A ſnail in getting up a May-pole, only 20 feet high, was obſerved to climb eight feet every day; but every night it came down again four feet. In what time by this method did he reach the top of the pole?

$20 - 8 + 4 = 16$ to go the 2d. morning.
$16 - 8 + 4 = 12$ to go the 3d.
$12 - 8 + 4 = 8$ to go the 4th, and at night got to the top.

46. The ſemi-diameter of the earth's orbit, or annual path round the ſun in the center of the ſyſtem, is about 81000000 miles; that of Venus 59000000. When they are

are both on the ſame ſide the ſun, they are in perigæo; when on different ſides, in apogæo. What is the difference of their diſtance in both theſe circumſtances?

81000000 — 59000000 = 22000000 miles in perigæo.
81000000 + 59000000 = 140000000 in apogæo.
Then 140000000 — 22000000 = 118000000, the anſwer.

47. B was 14 years old, when C was 25. How old ſhall C be, when B comes to be 25?

25 — 14 = 11
25 + 11 = 36, the anſwer.

48. A, born 17 years after C, and 13 before B, died 42 years before the late king's inauguration in 1727, aged 47 years. C died anno 1712, and B exactly eight years before him. D, born 23 years before C, died at 64. E, born 11 years after B's death, will die ſuppoſe 12 years after the year 1733. And F, born juſt in the midway of the interval between the births of A and D, is not to reach the time of E's death by 14 years. What is the ſum of all their ages, and which of them lived longeſt?

1727
— 42
A died A. D. 1685
Aged — 47 — 1704
Born 1638 — 13 = 1651 B born.
— 17 — Aged 53
C born - - - 1621
Died - - - 1712 — 1621 = 91 C's age.
1621 — 23 = 1598 D born.
B died - - - 1704 — + 64 age.
+ 11 — 1662 died.
1715
1733 + 12 = 1745 E died.
1745 — 1715 = 30 E's age.
1638 = 1598 = 40 its half = 20.
1745 — 14 = 1731 F died.
1638 — 20 = 1618 born.
Aged 113

A B C D E F
47 + 53 + 91 + 64 + 30 + 113 = 398 ſum.
And 113 — 91 = 22 F, oldeſt.

49. Three

49. Three and thirty years before the Restoration in 1660, the crown granted demesnes, to certain uses for 210 years then to come. The proprietor, in 1715, procured a reversionary grant of 99 years, to commence after the expiration of the first. In what year of Christ will the second term end?

Restoration - - - - - -	A. D.	1660
Grant before - - - - -		— 33
First Grant made - -	A. D.	1627
Duration - - - - -		+ 210
End of the first grant - -	A. D.	1837
Reversionary grant's continuance -		+ 99
Its expiration - - -	A. D.	1936

50. A young fellow owed his guardian 74 l. 18 s. 2 d. on balance. He paid off 41 l. 14 s. 8 d. and then declared his sister owed the gentleman half as much again as himself: on hearing this, she paid off in a pet 13 l. 12 s. 10 d. and gives out that her uncle William was not less in arrear than her brother and she together. The uncle hereupon pays 24 l. 7 s. 3 d. And then the uncle's brother, who, by the bye, was not the uncle of those children, for 150 l. undertakes to set them all clear, and has 35 l. 15 s. 5 d. he says, to spare. Can that be true?

		l.	s.	d.
Brother debtor to guardian the at first - - -		74	18	2
Paid - - - - - - - -	—	41	14	8
Remains debtor - - - -		33	3	6
	+	16	11	9
Sister debtor at first - - - - -		49	15	3
Paid - - - - - - - -	—	13	12	10
Sister remains debtor - - - - -		36	2	5
	+	33	3	6
Uncle William debtor at first - - -		69	5	11
Paid - - - - - - - -	—	24	7	3
Remains debtor - - - - - -		44	18	8

Then 33l. 3s. 6d. + 36l. 2s. 5d. + 44l. 18s. 8d. = 114l. 4s. 7d.
∴ 150l. — 114l. 4s. 7d. = 35l. 15s. 5d. as was proposed.

51. Five

51. Four notable discoveries preceded the Reformation in 1517, viz. 1st. The invention of the compass 215 years before that period. 2d. Gunpowder, found out 42 years after the use of the compass. 3d. Printing discovered 77 years before the Reformation: And, 4th. America became known 148 years after the invention of gunpowder. The question is, in what year of Christ did each of these happen to be found?

The Reformation - - -	A. D.	1517
		—215
Invention of the compass - -	A. D.	1302
		+ 42
Gunpowder - - ,	A. D.	1344
		+ 148
America discovered - -	A. D	1492
		1517
		— 77
Printing invented - - -	A. D.	1440

* *

SECT. V.

MULTIPLICATION.

MULTIPLICATION is a rule, by which the greater of two numbers may be speedily increased as often as there are units in the less, and in a concise manner performs the office of addition.

In every operation in multiplication, are two given numbers, called factors, viz.

First, the multiplicand, or number to be multiplied, which is generally the greater of the two.

Secondly, The multiplier or multiplicator, or number by which we multiply, which denotes the number of times the multiplicand is increased by, or added to itself; and from thence will arise a third number, called the product.

This

This in geometrical operations is called the RECTANGLE, or PLANE.

Let 7 Multiplicand } Factors.
 5 Multiplier }

35 product.

By addition 7 } add
7
7
7
7

35 sum.

MULTIPLICATION TABLE.

2 × 2 = 4	4 × 7 = 28	7 × 9 = 63
2 × 3 = 6	4 × 8 = 32	7 × 10 = 70
2 × 4 = 8	4 × 9 = 36	7 × 11 = 77
2 × 5 = 10	4 × 10 = 40	7 × 12 = 84
2 × 6 = 12	4 × 11 = 44	
2 × 7 = 14	4 × 12 = 48	8 × 8 = 64
2 × 8 = 16		8 × 9 = 72
2 × 9 = 18	5 × 5 = 25	8 × 10 = 80
2 × 10 = 20	5 × 6 = 30	8 × 11 = 88
2 × 11 = 22	5 × 7 = 35	8 × 12 = 96
2 × 12 = 24	5 × 8 = 40	
	5 × 9 = 45	9 × 9 = 81
3 × 3 = 9	5 × 10 = 50	9 × 10 = 90
3 × 4 = 12	5 × 11 = 55	9 × 11 = 99
3 × 5 = 15	5 × 12 = 60	9 × 12 = 108
3 × 6 = 18		
3 × 7 = 21	6 × 6 = 36	10 × 10 = 100
3 × 8 = 24	6 × 7 = 42	10 × 11 = 110
3 × 9 = 27	6 × 8 = 48	10 × 12 = 120
3 × 10 = 30	6 × 9 = 54	
3 × 11 = 33	6 × 10 = 60	11 × 11 = 121
3 × 12 = 36	6 × 11 = 66	11 × 12 = 132
	6 × 12 = 72	
4 × 4 = 16		12 × 12 = 144
4 × 5 = 20	7 × 7 = 49	
4 × 6 = 24	7 × 8 = 56	

N. B. This table is to be perfectly learned by heart, so as to be readily remembered without pausing.

Then multiplication may be easily performed, observing the following

RULE.

Always begin with that figure which ſtands in the units place of the multiplier, and with it multiply the figure which ſtands in the units place of the multiplicand; if their product be leſs than ten, ſet it down underneath its own place of units, and proceed to the next figure of the multiplicand. But if their product be above ten (or tens) then ſet down the overplus only (or odd figures, as in addition) and bear (or carry) the ſaid ten (or tens) in mind, until you have multiplied the next figure of the multiplicand with the ſame figure of the multiplier; then to their product add the ten or tens beared in mind, ſetting down the overplus of their ſum above the tens, as before; and ſo proceed in the very ſame manner, until all the figures of the multiplicand are multiplied with that figure of the multiplier.

394786	8643597	796534289
7	9	11
2763502	77792373	8761877179

2. When the multiplier is any number between 12 and 20; multiply by the figure in the units place; and as you multiply, add to the product of each ſingle figure, that of the multiplicand, which ſtands next on the right-hand.

4721217	4713176	94713761
15	16	18
70818255	75410816	1704847698

12345	72453	6729004
13	17	19
160485	1231701	127851076

2. But when the multiplier conſiſts of ſeveral figures, the multiplicand muſt be multiplied with every ſingle figure of the multiplier; always placing the firſt figure, or cypher, of every particular product, directly underneath that figure of the multiplier you then multiply with.

4739284

```
      4739284          6247386495
        94785               27356
      -------      --------------
     23696420        37484318970
    37914272        31236932475
   33174988        18742159485
  18957136        43731705465
 42653556        12494772990
 ------------     ---------------
 449213033940     170903504957220
```

4. If there be a cypher, or cyphers, intermixed with the figures, move for every figure, or cypher, one place toward the left hand, and take care that every first figure of the several products ſtand directly under its reſpective multiplier.

```
        630700325            50710984
          6072008             4050607
        ---------            --------
       5045602600           354976888
   1261400650            304265904
  4414902275          253554920
 3784201950         202843936
 ----------------    ---------------
 3829617419002600    205410266767288
```

5. Cyphers placed at the end of either or both factors, are to be omitted till the laſt product, and then the number of cyphers as are at the end of both muſt be annexed to it.

```
  42600          429000
   2200            5600
  -----          ------
   852            2574
  852            2145
 --------       ----------
 93720000       2402400000
```

6. Any number given, being multiplied by 1, undergoes no alteration; but if by 10, a cypher is to be annexed; if by 100, annex two cyphers; by 1000, annex three, &c.

7157 ×	1 =	7157	And thus for as many cyphers as you please.
	10 =	71570	
	100 =	715700	
	1000 =	7157000	
	10000 =	71570000	
	100000 =	715700000	

7. In geometrical progressions, converging series, &c. when multiplications have been very operose, I have frequently added, subtracted, or divided; or multiplied a product by a smaller, when the former happens to be a multiple of the latter; as I shall endeavour to explain in the example following:

8496427	
874359	
76467843	By subtracting the right-hand figure from a cypher, and each preceding figure from that following.
42482135	By dividing the multiplicand by 2.
25489281	By dividing the product of 9 by 3.
33985708	By add. the last prod. to the multiplicand.
59474989	By adding the two last products together.
67971416	By multiplying the product of 4 by two.
7428927415293	

But before the learner attempts to perform operations by this method, he ought to be acquainted with division.

8. If the multiplier be any number near 100, 1000, 10000, &c. increase the multiplicand by as many cyphers as there are figures in the multiplier; and subtract the multiplicand from itself thus increased, as often as the multiplier wants units of that by which the multiplicand was increased.

Let 7943628 × 999	And 4372845 × 9997
7943628000	43728450000
7943628	13118535 = multiplic. × 3.
7935684372	43715331465

9. If the multiplier be a repetend of the same figure, multiply by one of the repeating figures; and the figures of that

that product added, as if they had been wrote down in as many products as the multiplier repeated the same figure, give the product required.

547856789	54018
22222	3333
1095713578	162054
12174473565158	180041994

10. When the repeating figure is a high digit, collect the product of as many ones as there are digits in the multiplier, from the multiplicand, according to the rule in the last contraction; which product being multiplied into the repetend, will give the true product.

784325634 into 7777777.

871472839519374 Products collected for 1111111.
7

6100309876635618 Product of 7777777.

11. Find the product of the given multiplicand by the like number of nines, and divide that product by 9; the quotient multiplied by the digit which repeats in the given multiplier, will be the product required.

Ex. Let 4538769 be multiplied by 7777777.

45387690000000
4538769

9)45387685461231

5043076162359
× 7

35301533136513

N.B. Division must be learned before examples of this kind be attempted.

 12. When

12. When the multiplier can be parted into periods, which are multiples of one another, the operation may be contracted in the following manner.

8649347864	
1325769612	
103792174368	product of 12.
830337394944 -	foregoing product $\times$ 8.
4982024369664 - -	last product $\times$ 6.
1141713918048 - - - -	first product $\times$ 11.
11467042561708308768	

13. To multiply by a factor, consisting of as many cyphers between two digits as there are places in the multiplicand, multiply by a single digit; and the product by the second figure will fall directly to the left-hand of the product by the first figure; but if the product of the first figure be less than 10, then a cypher must be put down between the two products.

$$\begin{array}{r} 84629 \\ 7000003 \\ \hline 592403253887 \end{array}$$

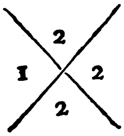
14. The proof of multiplication, is by making the multiplicand to be the multiplier; then, if the product comes out the same as before, your work is right.

15 Or by casting away the nines, which, though not infallible, serves to confirm the other. Thus, in the last example, make a cross, and add all the figures, or digits, of the multiplicand together, as units, thus, $8+4+6+2+9=29$; cast away the nines, and set the remainder 2 on one side the cross. Do the same with the multiplier $7+3=10$; set the remainder 1 on the other side the cross. Do the like by the product, and set the remainder at top. Lastly, multiply the figures on the sides, and set the remainder at the bottom, after the nines (if any) are cast away; which must be the same with the top, if the work is right.

QUESTIONS

QUESTIONS *to exerciſe* MULTIPLICATION.

1. A is 17, B 7; what will their ages ſeverally be, when the elder is double the age of the younger?



A 17
B — 7

10 A's age when B was born.
× 2

20 A's age, the anſwer.

2. Trajan's bridge over the Danube is ſaid to have had 20 piers to ſupport the arches, every pier being 60 feet thick, and ſome of them were 150 feet above the bed of the river; they were alſo 170 feet aſunder: pray what was the width of the river in that place, and how much did it exceed the length of Weſtminſter-bridge, which is about 1200 feet from ſhore to ſhore; and is ſupported by 11 piers, making the number of arches 12?

Arches -	21 × 170 =	3570
Piers -	20 × 60 =	1200
Width of the Danube -		4770
of the Thames -		1200
Difference -		3570 the anſwer.

3. By God's bleſſings upon a merchant's induſtry, in ten years time he found himſelf poſſeſſed of 13000 l. It appeared from his books, that the laſt three years he had cleared 873 l. a year; the three preceding, but 586 l. a year; and before that, but 364 l. a year. The queſtion is, what was the ſtate of his fortune at every year's end that he continued in trade, and conſequently what had he to begin with?

Merchant's whole ſtock - -		£ 13000
Gain per annum	364 × 4 = -	1456
Ditto - -	586 × 3 = -	1758
Ditto - -	873 × 3 = -	2619
Whole gain - - - -	=	£ 5833
Original ſtock - - - -	=	£ 7167

4. What

4. What difference is there between twice eight-and-twenty, and twice twenty-eight; as alſo between twice five-and-fifty, and twice fifty-five?

$$28 \times 2 = 56$$
$$2 \times 8 + 20 = 36$$

Anſwer 20 difference

Alſo $55 \times 2 = 110$
$$2 \times 5 + 50 = 60$$

Anſwer 50 difference.

5. What number taken from the ſquare of 54, will leave 19 times 46?

```
    54              56
  × 54            × 19
  ----            ----
   216             414
  270              46
  ----            ----
then 2916        — 874 = 2042, the anſwer.
```

6. The remainder of a diviſion ſum is 423; the quotient 423; the diviſor is the ſum of both, and 19 more. What then was the number to be divided?

```
   423
   423
    19
  ----
   865 diviſor
 × 423
  ----
  2595
 1730
3460
------
365895
 + 424
------
366318 the anſwer.
```

7. There

7. There are two numbers; the greater is 73 times 109; and their difference 17 times 28; I demand their ſum and product?

$109 \times 73 = 7957$ the greater number.
$28 \times 17 = 476$

7481 the leſs number.
$7957 + 7481 = 15438$ their ſum.
$7957 \times 7481 = 59526317$ their product.

8. There are two numbers, the leſs is 187, the difference 34; give the ſquare of their product, ditto of their ſum and difference, and the ſum of thoſe ſquares.

$187 + 34 = 221$ the greater; then $221 \times 187 = 41327$ prod.
$41327 \times 41327 = 1707920929$ ſquare of their product.
$221 + 187 = 408$; and $408 \times 408 = 166464$ ſquare of their ſum.
$221 - 187 = 34$; and $34 \times 34 = 1156$ ſquare of their differ.
Laſtly, $1707920929 + 166464 + 1156 = 1708088549$ anſw.

9. A perſon dying, left his widow the uſe of 5000 l. To a charity he bequeathed 846 l. 10 s. To each of his three nephews 1230 l. To each of his four nieces 1050 l. To twenty poor houſekeepers five guineas each; and 200 guineas to his executors. What muſt he have died poſſeſſed of?

	l.	s.	d.
To his widow - - -	5000	—	—
To a charity - - - -	846	10	—
To nephews 1230 × 3 -	3690	—	—
To nieces 1050 × 4 -	4200	—	—
To 20 poor houſekeepers	105	—	—
To executors - - -	210	—	—
	14051	10	—

10. In the partition of lands in an American ſettlement, A had 757 acres allotted to him, B had 2104 acres, C 16410, D 12881, E 11008, F 9813, H 13800, and I 8818 acres; now how many acres did the ſettlement contain, ſince the allotments made above want 416 acres of $\frac{1}{5}$th of the whole?

Firſt $757 + 2104 + 16410 + 12881 + 11008 + 9813 + 13800 + 8818 + 416 = 76007$.
Then $76007 \times 5 = 380035$, the anſwer.

SECT.

SECT. VI.

DIVISION.

DIVISION is a rule by which we speedily discover how often one *number* is contained, or may be found in another; or by which any number may be decreased; or divided into as many parts as there are units in the number you divide by.

To perform division, two numbers are always given.

I. The dividend, or number to be divided.

II. The divisor, or number by which the said dividend is to be divided.

And from thence will arise a third, called the quotient, which shews how often the divisor is contained in the dividend.

Lastly, If the divisor doth not exactly measure the dividend, a fourth number occurs, called the remainder; which is always less than the divisor, and consequently a fractional part of the quotient.

Division by a single figure, or not exceeding 12 in the divisor, is performed by the following

RULE.

First, observe how often the divisor is contained in the first figure of the dividend (or in case the first figure of the dividend be less than the divisor, in the two first figures) and set the quotient figure under it; and if any thing remains, carry it to the next figure in the dividend, where it must be reckoned as so many tens; and so on, bearing the remainder of each figure to the next in your mind, until you have finished your operation.

	Dividend,				
Divisor 2	5738473		11	8579475321079	
Quotient	2869236	1 rem.		779952301916	- 3
5	18647279		12	2157963058731	
	3729455	- 4		179830254894	- 3

2. But when the divisor consists of many places, distinguish by a point so many of the foremost places of the dividend

dividend towards the left-hand, as are either equal to the divisor, or else being greater, it comes neareſt to it; then conſider how often the diviſor is contained in this firſt period of the dividend, and aſſume that number for a quotient which multiply into the diviſor; and whenever it proves greater than the dividend, ſtrike that figure out, and put a leſs in the quotient: then ſubſcribe the product of the quotient figure into the diviſor, under the dividend, and draw a line under it; ſubtract it therefrom, ſubſcribing the remainder under the line; then dot and bring down another figure, proceeding as before, till your diviſion be finiſhed; always obſerving, that for every figure or cypher you bring down, you put a figure or cypher in the quotient.

EXAMPLE.

```
59157)2520705739155549(42610438 98
      236628·········
      ------
      ·154425
       118314
       ------
       ·361117
        354942
        ------
        ··61753
          59157
          -----
          ·259691
           236628
           ------
           ·230635
            177471
            ------
            ·531645
             473256
             ------
             ·583894
              532413
              ------
              ·514819
               473256
               ------
                41563
                -----
```

3. Many

3. Many figures may be saved, if you work by the short Italian method; that is, omit setting down your multiplications, and multiply and subtract together; always remembering to carry to the next figure as many as you borrowed.

873469) 432756284563574 (495445498
8336868
4756474
3891295
3974196
4803203
4358585
8647097
7858764
―――――
871012

4. When the divisor consists of several cyphers after a figure or figures, cut them all off by a dash of your pen underneath them; and also cut off as many cyphers, or figures, in the dividend; but when division is finished, bring down the cyphers, or figures, cut off from the dividend, to the remainder.

35000) 29628754963 (846535
162
228
187
125
204
―――――
29963

5. As unity, or 1, neither multiplies or divides, any number may be divided by 10, 100, 1000, by only cutting off by a comma so many figures to the right-hand of the dividend, as there are cyphers in the divisor; those to the left-hand being the quotient, and those to the right the remainder.

	Dividend.	Quotient.	Rem.
10	9056724417	905672441	- 7
100	9056724417	90567244	- 17
1000	9056724417	9056724	- 417
10000	9056724417	905672	4417

6. To

6. To divide by any number consisting of nines; viz. 9, 99, 999, &c. This may be performed by addition, as the multiplying by those numbers was by subtraction.

RULE.

Divide the given dividend into periods of as many places of figures as there are nines in the divisor, beginning from the left-hand; and annex as many cyphers to the right-hand of the number, as may be wanted to complete a period. Then write the figures of the left-hand period under those of the second period, which is next thereto, towards the right-hand; add these two together, and place their sum under the third period; observing if the sum of the two figures in the highest place exceed nine, to place the figure that would (in common addition be carried) under the lowest place of the second period. Add the third period to those figures which stand under it, including the carried figure, and place them under the fourth period; and so proceed till you have placed figures under the right-hand period; and under them place such a figure as would have been there placed, had the work proceeded a period farther. Then add the whole together; and beginning at the right-hand, cancel as many figures as there were cyphers annexed to the dividend; and from the figures that remain, cut off from the right-hand as many figures as the divisor contained nines; so shall the figures to the left be the quotient, and those cut off the remainder; only if the remainder be all nines, add one to the quotient.

Let 8765806137663 be divided by 9999.

The N° with three cyphers annexed	8765, 8061, 3766, 3000
	8765, 6826, 0592
	1. 2.
By addition ariseth this N° - -	8766682805943,592
From which the three last figures being left out for the three cyphers annexed to the dividend	876668280,5943
These properly separated - -	876668280 the quotient.
And - -	5943 remainder.

7. But if your divisor be 11, 111, 1111, &c. 22, 222, 2222, &c. or 33, 333, 3333, &c. divide the given dividend by the digit, which repeats in the divisor, and multiply the quotient

quotient by 9: then divide the product by 99,999,9999, and the result will be the quotient required.

Let 222671883 be divided by 777.

$$\begin{array}{r} 7\,)\,222671883 \\ \hline 31810269 \\ \times 9 \\ \hline 286,292421 \\ 286,578 \\ 1 \\ \hline 286579,\underline{000} \end{array}$$

Therefore 286579 is the quotient required.

8. If the divisor be large, and a quotient of many figures be required, as in resolving of high equations, and calculating astronomical tables, or those of interest, under the divisor set down its double; to this double add the divisor, setting down their sum against the figure 3; and proceed on by a continual addition, until there be ten times the divisor in the table; which, if true, will be the divisor itself, with a cypher to the right-hand of it.

Let it be required to divide 702518074O2 by 79863.

1	79863
2	159726
3	239589
4	319452
5	399315
6	479178
7	559041
8	638904
9	718767
	798630

$$\begin{array}{r} 79863\,)\,702518074O2\,(\,879654 \\ 638904\cdot\cdot\cdot\cdot\cdot \\ \hline \cdot 636140 \\ 559041 \\ \hline \cdot 770997 \\ 718767 \\ \hline \cdot 522304 \\ 479178 \\ \hline \cdot 431260 \\ 399315 \\ \hline 319452 \\ 319452 \\ \hline 0 \end{array}$$

9. Divi-

9. Division and multiplication interchangeably prove each other; for in division, if you multiply the divisor by the quotient, and to the product add the remander, (if any) the sum will be the dividend. So to prove multiplication, if the product be divided by the multiplier, the quotient will be the multiplicand; or, if the product be divided by the multiplicand, the quotient will be the multiplier.

10. Or cast away the nines in the divisor, and quotient, and set the remainders on the sides of a cross. Do the same with your dividend, and set the remainder at top. Multiply the figures on the sides, cast away the nines, and set the remainder at the bottom, which must be equal to the top. Note, If there be a remainder, it must be added to the product, on the sides of the cross, and the nines thrown out as before.

QUESTIONS *performed by* Division *in conjunction with the rest of the foregoing general rules.*

1. What is the difference, and what the sum of six dozen dozen, and half a dozen dozen?

$$12 \times 12 \times 6 = 864 = 6 \text{ dozen dozen.}$$
$$12 \times 12 = 144 \div 2 = 72 = \tfrac{1}{2} \text{ dozen dozen.}$$

936 sum.

792 difference.

2. Subtract 30079 out of fourscore and thirteen millions as often as it can be found, and say what the last remainder exceeds, or falls short of 21180?

```
30079 ) 93000000 ( 3091
         276300
          55890
       ---------
   Rem.   25811
        — 21180
       ---------
           4631, the answer.
```

3. What number added to the forty-third part of 4429, will make the sum 240?

43) 4429 (103

Then 240 − 103 = 137, the answer.

4. What number deducted from the 26th part of 2262, will leave the 87th part of the same?

26) 2262 (87
− 26

61, the answer.

5. What number multiplied by 72084, will produce 5190048 exactly?

72084) 5190048 (72, the answer.

6. What number divided by 419844, will quote 9494, and leave just a third part of the divisor remaining?

$$
\begin{array}{r}
3\,)\,419844 \\
\hline
139948 \\
\hline
419844 \\
9494 \\
\hline
1679376 \\
3778596 \\
1679376 \\
3778596 \\
\hline
3985998936 \\
139948 \\
\hline
3986138884
\end{array}
$$
the answer.

7. The sum of two numbers is 360; the less is 114; what is their difference, product, and larger quote?

$$
\begin{array}{r}
360 \\
-114 \\
\hline
246
\end{array}
$$

246 − 114 = 132 difference.
246 × 114 = 28044 product.
114) 246 = 2 $\frac{3}{19}$ quotient; viz. 6) $\frac{18}{114}$ ($\frac{3}{19}$.

8. I

8. I would plant 2072 elms in 14 rows, the trees in each row 25 feet asunder; how long will this grove be?

dist.

14) 2072 (148 in each row. $148 - 1 = 147$.

$147 \times 25 = 3675$ feet $= 1225$ yards.

9. A brigade of horse, consisting of 384 men, is to be formed into a square body, having 32 men in front; how many ranks will there be?

32) 384 (12, the answer.

10. What number is that, from which if you deduct the 25th part of 22525, and to the remainder add the 16th part of 9696, the sum will be 1440?

25) 22525 (901

16) 9696 (606

$1440 + 901 - 606 = 1735$, the answer.

11. There are two numbers, whose product is 1610; the greater is given 46: what is their sum, difference, and quotes; what the sum of their squares, and what the cube of their difference?

46) 1610 (35 lesser.

$46 + 35 = 81$ their sum.

$46 - 35 = 11$ difference.

35) 46 ($= 1\frac{11}{35}$ quotient.

$46 \times 46 = 2116$

$35 \times 35 = 1225$

3341, sum of their squares.

$11 \times 11 \times 11 = 1331$, cube of their difference.

12. What number multiplied by 57, will produce just what 134 multiplied by 71 will do?

$134 \times 71 = 9514$

57) 9514 ($166\frac{52}{57}$, the answer.

381

394

52

 13. There

13. There are other two numbers, the greater 7050, which divided by the leſs, quotes 94; what is the difference of their ſquares, and what the ſquare of the product of their ſum and difference?

94) 7050 (75 the leſs.

$7050 \times 7050 = 49702500$ ſquare of the greater.
$75 \times 75 = 5625$ ſquare of the leſs.

49696875 diff. of their ſquares.

$7050 + 75 = 7125$ ſum . . . $7050 - 75 = 6975$ diff.
$7125 \times 6975 = 49696875$ prod. of their ſum and diff.
$49696875 \times 49696875 = 2469779384765625$, anſwer.

14. Six of the female cricketers, that played lately in the Artillery-ground, fetched in company ſtrokes as follow; viz. ABCDE 207, ACDEF 213, ADEFB 189, AEBCF 234, ABCDF 222, BFDCE 250: how many did they fetch on the other ſide, ſince theſe ſix perſons wanted but fourſcore and thirteen notches to decide the game?

$207 + 213 + 189 + 234 + 222 + 250 = 1315.$
They being each mentioned five times - 5) 1315 (263.
Then $263 + 93 = 356$;
And $356 - 1 = 355$, the anſwer.

15. In order to raiſe a joint ſtock of 10000 l. L, M, and N together ſubſcribe 8500 l. and O the reſt. Now M and N are known together to have ſet their hands to 6050 l. and N has been heard to ſay, that he had undertaken for 420 l. more than M. What did each proprietor advance?

Firſt $6050 - 420 = 5630$

$$\left.\begin{array}{r} 8500 - 6050 = 2450 \text{ L's} \\ 2)\,5630 = 2815 \text{ M's} \\ 2815 + 420 = 3235 \text{ N's} \\ 10000 - 8500 = 1500 \text{ O's} \end{array}\right\} \text{ſubſcription.}$$

£ 10000

16. One of the ſmarts in the accomptant's office making his addreſſes in an old lady's family, who had five fine daughters;

daughters; ſhe told him their father had made a whimſical will, which might not ſoon be ſettled in Chancery, and till then he muſt refrain his viſits. The young gentleman undertook to unravel the will, which imported, That the firſt four of the girls fortunes were together to make 25000 l.; the four laſt 33000 l.; the three laſt, with the firſt, 30000 l.; the three firſt, with the laſt, were to make 280000 l.; and the two laſt, and two firſt, 32000 l. Now, ſir, if you can make appear what each is to have, and as you like, ſeemingly, my third daughter, Charlotte, who I am ſure will make you a good wife, and you are welcome; what was Miſs Charlotte's fortune?

$25000 + 33000 + 30000 + 28000 + 32000 = 148000$; each be mentioned times 4) 148000 (37000.

Then $37000 - 25000 = 12000$ youngeſt.
$37000 - 33000 = 4000$ eldeſt.
$37000 - 30000 = 7000$ ſecond.
$37000 - 28000 = 9000$ fourth.
$37000 - 32000 = 5000$ Miſs Charlotte.

17. A father dying worth 5460 l. left his wife with child, to whom he bequeathed, if ſhe had a ſon, $\frac{1}{3}$d of his eſtate, and $\frac{2}{3}$ds to the ſon; but if ſhe had a daughter, $\frac{1}{3}$d to her, and $\frac{2}{3}$ds to her mother. It happened that ſhe had both a ſon and a daughter; how ſhall the eſtate be divided to anſwer the father's intention?

It is plain that the father deſigned the ſon's fortune to be double the mother's, and that the mother ſhould have double the daughter's fortune.

For every pound the daughter had, the mother muſt have two, and the ſon four.

Then $1 + 2 + 4 = 7$ diviſor for the daughter's portion,

$$\left.\begin{array}{ll} 7\,)\,5460\,(\,780\text{ l.} & \text{daughter's} \\ \text{Alſo } 780 \times 2 = 1560 & \text{mother's} \\ \text{And } 1560 \times 2 = 3120 & \text{ſon's} \end{array}\right\}\text{ part.}$$

18. Fair ladies, of you I muſt yet enquire,
How the poll ſtood for the knights of our ſhire:
The number of votes, as I have ſeen,
Were five thouſand, two hundred, and nineteen;
Which among four was juſt ſo divided,
As one the ſecond, and the third exceeded,
By twenty-two, and fourſcore bating ſeven;
The fourth by no more than ſixſcore and ten;

Then how many votes had each candidate?
You need not in finding much trouble your pate.

L. Diary.

$$\begin{array}{r} 5219 \\ 22 \\ 73 \\ 130 \\ \hline 4\,)\,5444\,(\end{array} \quad - \; - \quad 1361 \text{ first}$$

$$\left.\begin{array}{l} 1361 \text{ first} \\ 1361 - 22 = 1339 \text{ second} \\ 1361 - 73 = 1288 \text{ third} \\ 1361 - 130 = 1231 \text{ fourth} \end{array}\right\} \text{candidate.}$$

$$\overline{5219} \text{ proof.}$$

19. A general disposing his army into a square battle, finds he has 284 soldiers over and above; but increasing each side with one soldier, he wants 25 soldiers to fill up the square: How many soldiers had he?

Since the number of soldiers exceeds the less square by 284, and wants 25 to fill up the greater,

$284 + 25 = 309$, and $309 + 1 = 310$.

2) 310 (155 side of the greater square.

$155 \times 155 = 24025$.

Answer $24025 - 25 = 24000$ the number of soldiers required.

20. What number is that, which multiplied by 20, and divided by 6, gives 140 in the quotient?

First $140 \times 6 = 840$. And $840 \div 20 = 42$, the answer.

21. A man being 100 years of age upon his birth-day, had his three sons with him at dinner, viz. William, James, and Thomas; the Father saying to them, Well, sons, I am this day just 100 years old; the youngest, William, said, Father, my brother Thomas is four times as old as I am, and my brother James is three times as old as I am, and all our ages together are just 100 years: How old was each of the three sons?

4 Thomas.
3 James.
1 William.

$$\left.\begin{array}{l} 8\,)\,100\,(\,12\tfrac{1}{2} \text{ William's} \\ \text{Also } 12\tfrac{1}{2} \times 3 = 37\tfrac{1}{2} \text{ James's} \\ \text{And } 12\tfrac{1}{2} \times 4 = 50 \text{ Thomas's} \end{array}\right\} \text{age, the answer.}$$

22. A

22. A man dies and leaves a legacy of 900 l. to be disposed of among four of his relations, viz. A, B, C, D, in this manner; B is to have twice as much as A; C twice as much as A and B; and D to have as much and half as much as C. What must each person have?

A 1
B 2
C 6
D 9

$$\left.\begin{array}{rcrcl} & & \text{l.} & & \\ 18 &)\ 900\ (& 50 & = & \text{A's} \\ 50 \times 2 & = & 100 & = & \text{B's} \\ 100 \times 3 & = & 300 & = & \text{C's} \\ 300 \times 1\frac{1}{2} & = & 450 & = & \text{D's} \end{array}\right\}\text{part, the answer.}$$

$$900$$

23. A labourer, after 40 weeks working, lays up 28 crowns — three weeks wages, and finds that he has expended 36 crowns + 11 weeks wages. What was his weekly pay?

First $11 + 3 = 14$ weeks wages wanting.
Also $40 - 14 = 32$ weeks. And $28 + 36 = 64$ crowns.
$\therefore$ 32) 64 (2 crowns, his weekly pay. Q. E. F.

CHAPTER II.

Containing Tables of weights, measures, and time; with Addition, Subtraction, *and* Reduction *thereof from one denomination to another.*

SECT. I. TABLES.

TROY WEIGHT.

BY this weight are weighed jewels, gold, silver, and all liquors.

Grains.			
24 =	1 pennyweight,		
480 =	20 =	1 ounce.	
5760 =	240 =	12 =	1 pound.

The

The moneyers also at the mint subdivide a grain.

Thus { 24 blanks = 1 periot.
20 periots = 1 droite.
24 droites = 1 mite.
20 mites = 1 grain. }

The carat is a weight which goldsmiths and jewellers use to weigh precious stones and pearls; it weighs four grains, each of which is subdivided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, &c.

Carat, or caract, is also the name which represents what degree of fineness gold is of; as fine gold, in its purity or perfection, is 24 carats; and standard gold, of which our coin is made, is 22 carats of fine gold, and two of alloy, (or a baser metal, as copper or silver.)

Whence we may observe, that this carat is $\frac{1}{24}$th part of any quantity or weight.

APOTHECARIES WEIGHT.

Apothecaries compound their medicines according to the following division of an ounce Troy; but buy and sell their drugs by Averdupoise weight.

Grains			
20 =	1 scruple.		
60 =	3 =	1 dram.	
480 =	24 =	8 =	1 ounce.

AVERDUPOISE WEIGHT.

By Averdupoise weight are weighed such commodities as are either very coarse and drossy, or subject to waste; as all kind of grocery wares; and pitch, tar, rosin, wax, tallow, soap, flax, hemp, &c. copper, tin, steel, iron, lead, &c.; also flesh, butter, cheese, salt, and most other common necessaries of life.

N. B. 681 grains of barley hath been found to weigh exactly one ounce Averdupoise weight; therefore a pound containeth 10896 grains.

The pound Averdupoise is greater, but the ounce less, than those of Troy weight: the pound Averdupoise being equal to 14 oz. 11 pwt. 15 $\frac{1}{2}$ grains; and one ounce equal to 18 pwt. 5 $\frac{1}{2}$ grains Troy.

Drams.

Drams.					
16 =	1 ounce.				
256 =	16 =	1 pound.			
7168 =	448 =	21 =	1 quarter.		
28672 =	1792 =	112 =	4 =	1 hundred.	
573440 =	35840 =	2240 =	80 =	20 =	1 ton.

N.B. A ſtone of fleſh meat in London is 8 lb. Averdupoiſe, but in moſt other places 14 lb.

Alſo 28 lb. of wool makes a tod in Norfolk, and in the ſouthern counties; but 30 lb. in Yorkſhire, and other northern ones.

A ſtone, horſeman's weight, is 14 lb.
A fother, or fodder, of lead, $19\frac{1}{2}$ cwt.

LIQUID MEASURE.

As the original of Troy weight was a corn of wheat taken out of the middle of the ear, and being well dried, 32 were to make a pennyweight; ſo eight pound Troy weight of wheat (or 61440 grains) were enacted, by ſeveral ſtatutes, to make one gallon wine meaſure. This gallon, by which all wines, brandies, ſpirits, ſtrong-waters, mead, perry, cyder, vinegar, oil, &c. are meaſured and ſold, containeth 231 cubic inches.

WINE MEASURE.

Cub. In.							
$28\frac{7}{8}$ =	1 pint.						
231 =	8 =	1 gallon.					
9702 =	336 =	42 =	1 tierce.				
14553 =	504 =	63 =	$1\frac{1}{2}$ =	1 hogſhead.			
19404 =	672 =	84 =	2 =	$1\frac{1}{3}$ =	1 puncheon.		
29106 =	1008 =	126 =	3 =	2 =	$1\frac{1}{2}$ =	1 butt or pipe.	
58212 =	2016 =	252 =	6 =	4 =	3 =	2 =	1 ton.

Note, $31\frac{1}{2}$ gallons is a wine or vinegar barrel, and 236 gallons a ton of ſweet oil.

A ton of 252 gallons, at $7\frac{1}{2}$ lb. to the gallon, weighs 1890 lb. = 16 cwt. 3 qrs. 14 lb.

The beer or ale gallon (which are both one) is much larger than the wine gallon; it being probably made at firſt to correſpond with Averdupoiſe weight, as the wine gallon did with Troy weight. For one pound Averdupoiſe being nearly equal to 14 oz. 12 pwts. Troy; and as one pound Troy is in proportion to the cubic inches in a wine gallon, ſo

so is one pound Averdupoise to the cubic inches in an ale gallon, viz. $12 : 14\frac{12}{20} :: 231 : 282$ nearly.

ALE MEASURE.

Cub. In.						
$35\frac{1}{4}$	= 1 pint.					
282	= 8	= 1 gallon.				
2256	= 64	= 8	= 1 firkin.			
4512	= 128	= 16	= 2	= 1 kilderkin.		
9024	= 256	= 32	= 4	= 2	= 1 barrel.	
13536	= 384	= 48	= 6	= 3	= $1\frac{1}{2}$	= 1 hogshead.

BEER MEASURE.

Cub. In.							
$35\frac{1}{4}$	= 1 pint.						
282	= 8	= 1 gallon.					
2538	= 72	= 9	= 1 firkin.				
5076	= 144	= 18	= 2	= 1 kilderkin.			
10152	= 288	= 36	= 4	= 2	= 1 barrel.		
15228	= 432	= 54	= 6	= 3	= $1\frac{1}{2}$	= 1 hogshead.	
30456	= 864	= 108	= 12	= 6	= 3	= 2	= 1 butt.

N. B. This distinction, or difference, between and beer ale measure, is only used in London; but in all other places of England the following table of beer or ale, whether it be strong or small, is to be observed according to a statute of excise made in the year 1689.

BEER *and* ALE *in the Country*.

Cub. In.						
$35\frac{1}{4}$	= 1 pint.					
282	= 8	= 1 gallon.				
2397	= 68	= $8\frac{1}{2}$	= 1 firkin.			
4794	= 136	= 17	= 2	= 1 kilderkin.		
9588	= 272	= 34	= 4	= 2	= 1 barrel.	
14382	= 408	= 51	= 6	= 3	= $1\frac{1}{2}$	= 1 hogshead.

DRY MEASURE.

By an act of parliament, made in 1697, it was decreed, That every round bushel with a plane and even bottom, being made eighteen inches and a half wide throughout, and eight inches deep, should be esteemed a legal Winchester bushel,

bushel, according to the standard in his majesty's Exchequer. Now a vessel thus made, will contain 2150,42 cubic inches; consequently the corn gallon doth contain 268⅘ cubic inches.

Cub. In.							
33.6 =	1 pint.						
268.8 =	8 =	1 gallon.					
537.6 =	16 =	2 =	1 peck.				
2150.4 =	64 =	8 =	4 =	1 bushel.			
8601.6 =	256 =	32 =	16 =	4 coom.			
17203.2 =	512 =	64 =	32 =	8 =	2 =	1 quarter.	
172032 =	5120 =	640 =	320 =	80 =	20 =	10 =	1 last.

But the farmer generally delivers to the merchant 10½ quarters of oats, colesed, and some other grain, for a last, in consideration of waste, &c. by exportation.

The miners in Derbyshire have a vessel called an ore-dish, by which they buy and sell their lead-ore.

Its length 21.3, breadth 6, depth 8.4 } inches;

consequently its contents 1073.52 cubic inches, very nearly equal to two pecks, or four corn gallons.

Nine of those dishes they call a load of ore; which, if pretty good, will produce about three hundred weight of lead.

LONG MEASURE.

Inches.						
12 =	1 foot.					
36 =	3 =	1 yard.				
72 =	6 =	2 =	1 fathom.			
198 =	16½ =	5½ =	2¾ =	1 pole.		
7920 =	660 =	220 =	110 =	40 =	1 furlong.	
63360 =	5280 =	1760 =	880 =	320 =	8 =	1 mile.

LONG MEASURE.

The navigators, or seamen, reckon 60 English miles to a degree; so that the circumference of the earth, according to them, is 360 degrees × 60 = 21600 miles.

But

But Mr. Norwood, by an experiment made between London and York, in the year 1635, found, that 367196 feet = 69 miles and 958 yards make a degree; according to whom the circumference of a great circle = 25035 miles.

And according to the Tranſactions of the Royal Academy of Sciences at Paris, anno 1687, 57060 toiſes = 365184 Engliſh feet = 69 miles 288 yards make a degree on this terraqueous globe.

SQUARE MEASURE.

Sq. Inches.						
144 =	1 ſqr. foot.					
1296 =	9 =	1 ſqr. yard.				
39204 =	$272\frac{1}{4}$ =	$30\frac{1}{4}$ =	1 perch.			
1568160 =	10890 =	1210 =	40 =	1 rood.		
62722640 =	43560 =	4840 =	160 =	4 =	1 acre.	
4014489600 =	27878400 =	3097600 =	102400 =	2560 =	640 =	1 ſq. m.

This table will be uſeful in menſuration of ſuperficies.

N. B. The leaſt part of long meaſure was at firſt a barley-corn taken out of the middle of the ear; and being well dried, three of them in length were to make one inch.

CLOTH MEASURE.

Inches.			
$2\frac{1}{4}$ =	1 nail.		
9 =	4 =	1 quarter.	
36 =	16 =	4 =	1 yard.
45 =	20 =	5 =	1 ell Engliſh.
27 =	12 =	3 =	1 ell Flemiſh.
54 =	24 =	6 =	1 ell French.

Note, all Scotch and Iriſh linens are bought and ſold by the yard; but all Dutch linens are bought by the ell Flemiſh, and ſold by the ell Engliſh.

TIME.

Time only ſhews the duration or mutation of things, a year being the ſtandard, or integer, by which ſuch continuation or change is computed. And a year is that ſpace of time in which the ſun (apparently) completes its revolution from any one point in the ecliptic (an imaginary circle in the heavens) to the ſame point again.

Seconds,

Seconds.				
60 =	1 minute.			
3600 =	60 =	1 hour.		
86400 =	1440 =	24 =	1 day.	
31556937 =	525949 =	8766 =	365.5° 48′ 57″ =	1 year.

SECT. II.

ADDITION of WEIGHTS, MEASURES, &c.

RULE.

ALWAYS begin with thofe figures of the loweft or leaft denomination, and add them altogether into one fum; then confider how many of the next fuperior denomination are contained in that fum, fo many units you muft carry to the faid next fuperior denomination, to be added together with thofe figures that ftand there; and if any thing remain over, that overplus muft be fet down underneath its own denomination; but if you cannot otherways difcover how many of the next fuperior denomination are contained in that fum, divide it by the number of units contained in one of the next denomination fuperior thereto, and fet down the remainder, and carry the quotient. And fo proceed on from one denomination to another, until all are finifhed.

1. A merchant buys fix bags of Canterbury hops, Nº 1. of which weighed 2 cwt. 2 qrs. 10 lb.; Nº 2. 2 cwt. 1 qr. 16 lb.; Nº 3. 2 cwt. 0 qrs. 24 lb.; Nº 4. 2 cwt. 3 qrs. only; Nº 5. 2 cwt. 1 qr. 12 lb.; Nº 6. 2 cwt. 1 qr. 16 lb.; befides a couple of pockets, ditto, that weighed $58\frac{1}{2}$ lb. each. How many hundreds weight has he to pay carriage for, on bringing them to town?

Bags,

	C.	qrs.	lb.
Bags, Nº 1 - - - - - - - - -	2	2	10
2 - - - - - - - -	2	1	16
3 - - - - - - - -	2	–	24
4 - - - - - - - -	2	3	–
5 - - - - - - - -	2	1	12
6 - - - - - - - -	2	1	16
Pockets - 1 - - - - - - - -	–	2	$2\frac{1}{2}$
2 - - - - - - - -	–	2	$2\frac{1}{2}$
Cwt.	15	2	27, the anſwer.

2. In a gentleman's ſervice of plate there are fourteen diſhes, weighing 193 oz. 13 dwt.; plates thirty-ſix, weighing 421 oz. 11 dwt.; four dozen of ſpoons, weighing 104 oz. 6 dwt; ſix ſalts, chaſed, weighing 32 oz.; knives and forks, weighing 83 oz. 9 dwt.; fourteen preſenters, weighing 113 oz. 4 dwt.; in mugs, tumblers, beakers, and other odd pieces, weighing 264 oz. 18 dwt.; ſilver tea-kettle and lamp, weighing 126 oz. 9 dwt.; and the reſt of that equipage 93 oz. 2 dwt. What quantity of plate had the butler under his care?

		oz.	dwt.
Fourteen diſhes - - - - - -	wt.	193	13
Thirty-ſix plates - - - - - -	wt.	421	11
Four dozen of ſpoons - - -	wt.	104	6
Six ſalts - - - - - - - - -	wt.	32	–
Knives and forks - - - - - -	wt.	83	9
Four preſenters - - - - - -	wt.	113	4
Mugs, tumblers, beakers, &c. -	wt.	264	18
A ſilver tea-kettle, &c. - - -	wt.	126	9
And the reſt of that equipage -	wt.	93	2
	oz.	1432	12, the anſwer.

The diſtance betwixt two places is ſuch, that if three miles and five furlongs is taken from it, what remains is equal to eight miles, four furlongs, and 100 yards; what is the diſtance of theſe two places?

M.	F.	Yds.
3	5	—
8	4	100
12	1	100, the anſwer.

4. In

4. In my survey of the river Glen, from Baston-hedges to the Outfall, I found the several distances as follow:

	M.	F.	P.
To Clarke's house	1	6	4
To the toll-bridge	–	6	24
To the falling in of Bourn-Eau	–	1	32
To Gutram-cot	2	5	28
To Pinchbeck-bar	2	2	12
To Money-bridge	1	–	4
To Herring-bridge	1	2	4
To New-bridge	–	5	12
To Bondman-bridge	–	–	36
To Stone-goat	–	4	–
To Surfleet bridge	–	7	2
To the Half-penny-toll	1	1	28
To the Outfall	1	1	32
From Baston-hedges to the Outfall	14	5	18

5. A father was 18 years four months old (reckoning 13 months to one year, and 28 days to one month) when his eldest child was born. Betwixt the eldest and second, were 11 months, 10 days. Betwixt the second and third, were three years, eight months. When the third is 12 years, six months, 20 days, how old is the father?

	Y.	M.	D.
	18	4	–
	–	11	10
	3	8	–
	12	6	20
Years	35	4	2, the answer.

SECT. III.

SUBTRACTION *of* WEIGHTS, MEASURES, &c.

I refer the young student to the general rule for subtraction, Chap. I. Sect. III.

1. Having

1. Having bought two hundred weight and three quarters of ſugar, and ſold thereof one hundred, two quarters, and 14 pounds; what is yet unſold?

	cwt.	qr.	lb.	
	2	3	–	
	1	2	14	
Cwt.	1	–	14,	the anſwer.

2. A father was 24 years, 9 months, 10 days old, when his eldeſt ſon was born; and is now 56 years, 3 months, and 22 days. How old is the ſon?

Y.	M.	D.	
56	3	22	
24	9	10	
31	7	12;	anſwer.

3. Received in lieu of two gold repeaters, ſent to Jamaica in 1756, the five cheſts of indigo following; and on a like adventure in 1758, the ſubſequent five cheſts. The queſtion is, how much indigo I had leſs the ſecond time than the firſt?

Anno 1756.

	cwt.	qr.	lb.		cwt.	qr.	lb.
Nº 1	2	1	16	Tare		1	15
2	2	2	11			1	19
3	2	–	12			1	13
4	2	–	19			1	14
5	2	3	17			1	21
Groſs	12	–	19		1	3	26
—	1	3	26	Tare			
Net first	10	–	21				

Anno 1758.

	cwt.	qr.	lb.		cwt.	qr.	lb.
Nº 1	1	3	7	Tare		1	4
2	1	3	17			1	4
3	1	2	10			1	2
4	1	–	13			–	27
5	2	–	11		cwt.	1	6
Groſs	8	2	2		1	1	15
	1	1	15	Tare			
	7	–	15				

Net	10	–	21	firſt venture.
	7	–	15	ſecond venture.
	3	–	6	difference.

4. Jacob, by contract, was to ſerve Laban for his two daughters 14 years; and when he had accompliſhed 11 years, 11 months, 11 weeks, 11 days, 11 hours, and 11 minutes; pray how long had he yet to ſerve?

11 Y.

							Y.	M.	W.	D.	H.	M.
Y.	M.	W.	D.	H,	M.		14	-	-	-	-	-
11	11	11	11	11	11	=	12	2	-	4	11	11
						Answer	1	9	3	2	12	49

5. When the air presseth with its full weight in very fair weather it may be demonstrated, that there presseth upon a human body about 302 cwt. 2 qrs. 25 lb. of that fluid matter; and in very foul weather, when the air is most light, 273 cwt. 1 qr. 20 lb. What difference of weight lies on such a body in the two greatest alterations of the weather?

	cwt.	qrs.	lb.
	302	2	25
	273	1	20
Cwt.	29	1	5, the answer.

* *

SECT. IV.

REDUCTION.

REDUCTION alters or changes any superior denomination proposed, into any inferior or lesser denomination required; still keeping them equivalent in value. And the contrary.

RULE.

Consider how many units of the denomination required, make one of that denomination proposed to be reduced (which is easily known by its respective table) and with that number of units multiply or divide the denomination proposed, and their product, or quotient, will be the number required.

1. Reduce 753 l. into pence.

```
  753 l.
   20
 -----
 15060 shillings.
    12
 ------
 180720 pence.
```

Or, 753 l. may be reduced into pence at one operation.
Thus

```
  753 l.
  240
 -----
  3012
 1506
 ------
 180720 = pence in 753 l. as before.
```

But when the numbers proposed to be reduced are of several denominations, and it is required to bring them all to the lowest, you must reduce the highest or greatest denomination to the next less, adding the numbers that are of that less denomination thereto; then reduce their sum to the next lower denomination, adding thereto all the numbers that are of that denomination, and so proceed gradually on until all be finished.

2. Reduce 375 l. 17 s. $10\frac{3}{4}$ d. into farthings.

```
   l.  s.  d.
  375  17  10¾
   20
 ------
  7517 = the shillings in 375 l. 17 s.
    12
 ------
 90214 = the pence in 375 l. 17 s. 10 d.
     4
 ------
 360859 = farthings in 375 l. 17 s. 10¾ d.
```

3. In 384627 farthings, how many pounds sterling?

```
 4 | 384627
12 |  96156¾ pence.
20 |    8013 shillings 0¾ d.
   | £. 400 13 0¾ d.
```

Note,

Note, the remainder is always of the fame denomination with the dividend.

4. In 648385 grains, how many penny-weights, ounces, and pounds?

```
                20)
24)648385) 27016 pennyweights.
   168     -----
      38 12)1350 oz. 16 pwts. 1 grain.
      145 -------
      ----    112 lb. 6 oz. 16 pwts. 1 grain.
        1 gr. -----
      ----
```

5. In 17 cwt. 3 qrs. 14 lb. how many ounces?

```
   4                     cwt. qr. lb.
  ---                     17   3  14
  71 quarters.           112   84 + 14 — 98.
  28                    -----
 ----                   2002 lb.
 568                      16
142                    -------
----                    32032, anfwer, as before.
2002 pounds.           -------
  16
------
32032 ounces.
------
```

6. Reduce 93 tuns, 15 cwt. 2 qrs. 12 lb. 13 oz. 14 dr. into drams.

```
 Tuns. cwt. qrs. lb. oz. dr.
   93   15   2   12  13  14
   20
 ----
 1875 hundreds.
  22568          1875 × 100 = 187500 }
 -------         1875 ×  12 =  22500 } fum 210068.
  21068 lb.        58 +  12 =     68 }
     16
 --------
 3361101 ounces.
      16
 ---------
53777630 drams.
---------
```


In 7.

7. In 17 tankards, each weighing 27 oz. 14 dwt. 16 gr. how many grains?

oz.	dwt.	gr.
27	14	16
20		

554 penny-weights.
24

2232
1108

13312 grains in one tankard.
17

226304 grains, the answer.

8. In 53777630 drams, how many tuns?

drams.	16) ozs.	28) lb.		4)
16) 53777630 (	3361101)	210068		7502
57	16	140		
97	110	68	20	1875 cwt. 2 qrs.
17	141			
16		12		93 tun, 15 cwt.
30	13			
14				

	Tuns.	cwt.	qrs.	lb.	oz.	dr.
Answer	93	15	2	12	13	14

9. The silk mill at Derby contains 26586 wheels, and 97746 movements, which wind off, or throw 73726 yards of silk every time the great water-wheel, which gives motion to all the rest, goes about, which is three times in a minute. The question is, how many yards of silk may be thrown by this machine in a day, reckoning ten hours a day's work; and how many in the compass of a year, deducting for Sundays and great holidays, 63 days, provided no part of it stand still?

73726

73726 yards at 1 circumvolution of the wheel.
×3

221178 yards in a minute.
× 60

13270680 yards in an hour.
× 10

132706800 yards in a day.
302 = 365 — 63

2654136
39812040

40077453600 yards, the answer.

10. In 4712 nails of Holland, how many yards, ells English, and ells Flemish?

4 | 4712
4 | 1178 quarters
294½ yards.

5 | 1178
235 ells English ¾ yards.

3 | 1178
392 ells Flemish ½ yard.

Sometimes multiplication and division are both required to answer the question; as in the following.

11. In 491 barrels of beer (London measure) how many hogsheads, gallons, and pints?

461 barrels
2

3) 982 (327 hogsheads, and 1 barrel = 36 gallons.
54

1308
1635

17694 gallons.

17694 gallons.
×8

141552 pints.

12. I desire to know how many days, hours, minutes, and seconds, since our Saviour's nativity, it being accounted 1772 years?

```
          31556937 seconds in a year.
              1772 years.
          --------
          63113874
         220898559
        220898559
        31556937
        -----------
60 | 55918892364 seconds in 1772 years.
   |-----------
60 |  931981539 minutes 24″.
   |-----------
24 | 15199692 hours 19′ 24″
   |-----------
     633320 days 12 hours 19′ 24″ = 1772 years.
```

13. In how long time would a million of million of money be in counting, supposing 100 l. to be counted every minute, without intermission, and the year to consist of 365 days, five hours, 45 minutes?

```
100 ) 1000000000000 = 10000000000 minutes.
                                          Years  days  h. m.
In 1 year minutes 525945 ) 10000000000 ( 19013  144   5  55
                            4740550    the answer required.
                              704500
                              1785550
                             ---------
                      1440 ) 207715 Rem. 100 pounds.
                              637
                               611
                              -----
                         60 ) 355
                              -----
                                55
```

A Geographical QUESTION.

14. There is a city in a certain island 708 miles more distant from the tropic of Cancer, than another under the same meridian is from the artic polar circle. What cities are those, what are the distances of those cities from the equator, and what from each other; remembering the polar circle is about $23\frac{1}{2}$ degrees from the pole, as is the tropic from

from the equator; and in this please to consider 60 geographical miles as a degree?

Tropic of Cancer 23° 30′ from the equator.
21 30 half the temperate zone.
60) 708 (11 48½ 2 = 5 54 half 708 miles.

50 54 latitude of the first city.
23° 30′ polar circle from the pole.
21 30 half the temperate zone, as before.

45 —
— 5 54 the half of 708 miles.

90 — 39 6 = 50 54 latitude of the second city.

Consequently they have each the lat. north, and answer to Chichester in Sussex, Tongeren in Germany, and Upres in Flanders.

15. I would put 60 hogsheads of London beer into 30 wine-pipes, and would know what the cask must hold that receives the difference; 231 solid inches being the gallon of wine, and 282 that of beer.

60 × 54 = 3240 gal. of beer. 30 × 126 = 3780 gal. wine meas.

282	231
648	378
2592	1134
648	756
913680 inches of beer.	873180

913680 — 873180 = 40500 inches difference.
282) 40500 (143 gallons, two quarts, and almost a pint.
Remains 33 inches, 35¼th, being a pint.

Chronological QUESTIONS.

16. England was conquered by William I. Oct. 4, 1066; his son William II. came to the crown, Sept. 9, 1087; and left it Aug. 2, 1100. William III. received it Feb. 3, 1689; and died March 8, 1701. How many days did each of these princes govern, respect being had to the intercalary days (added to February every leap-year) as they arose in the course of time?

Note; Every fourth year is leap-year, or bissextile; to find which are such, divide the year of our Lord by 4, and when nothing remains, those are the leap-years, and to such you add one day more than 365.

$\frac{1066}{4}$ remains 2, ſo that 1068 was leap-year.

And in the reign of William I. were five intercalary days.
Between September 9, and October 4, are 25 days.
1087 — 1066 = 21 years all but 25 days, William I. reigned.
365 x 21 = 7665 days.
Therefore 7665 + 5 — 25 = 1645 days, William I. reigned.

$\frac{1087}{4}$ remains 3, therefore 1088 was leap-year.

And in the reign of William Rufus 4 intercalary days.
Betwixt Aug. 2, and Sept. 9. are 38 days.
1100 — 1087 = 13 years all but 38 days, William II. reigned.
365 x 13 = 4745 days.
Therefore 4745 + 4 — 38 = 4711 days, William II. reigned.

$\frac{1689}{4}$ remains 1, ſo that 1692 was leap-year.

And in the reign of William III. 3 intercalary days.
From Feb. 3, to March 8, are 33 days.
1731 — 1689 = 12 years and 33 days, William III. reigned.
365 x 12 = 4380 days.
Therefore 4380 + 33 + 3 = 4416 days, William III. reigned.

17. Richard I. ſucceeded his father Henry II. July 7, 1189; John, his brother, ſucceeded him April 6, 1199; Richard II. ſucceeded Edward III. on the 21ſt of June, 1377, and was depoſed by Henry IV. on the 30th of Sept. 1399. The third Richard cauſed his nephew, Edward V. and his brother, to be murdered on the 18th of June, 1483; and was ſlain himſelf on the 22d of Aug. 1485; how many days was the realm governed by the three Richards, reſpect being ſtill had to the intercalary days as they happened?

$\frac{1189}{4}$ remains 1, ſo that 1192 was leap-year.

And in the reign of Richard I. were 2 intercalary days.
Betwixt April 6, and July 7, are 92 days.
1199 — 1189 = 10 years all but 92 days, Richard I. reigned.
365 x 10 = 3650 days.
And 3650 + 2 — 92 = 3560 days, Richard I. reigned.

$\frac{1377}{4}$ remains 1, ſo that 1380 was leap-year.

And in Richard II's reign were 5 intercalary days.
From June 21, till September 30, are 101 days.
1399 — 1377 = 22 years, 101 days, Richard II. reigned.
365 x 22 = 8030 days.

And

And $8030 + 101 + 5 = 8136$ days, Richard II. reigned.

$\frac{1483}{4}$ remains 3, so that 1484 was leap-year.

And in Richard III's reign was 1 intercalary day.

From June 18, till August 22, are 65 days.

$1485 - 1483 = 2$ years, 65 days, Richard III. reigned

$365 \times 2 = 730$ days.

And $730 + 65 + 1 = 796$ days, Richard III. reigned.

Lastly, $3560 + 8136 + 796 = 12492$ days, the answer.

18 The first queen Mary came to the crown, July 8, 1553; she reigned five years, four months, and nine days. Her sister Elizabeth succeeded; and James I. came to her throne the 14th of March, 1602; he left it to his son Charles I. on the 27th of March, 1625, who was forced from it Jan. 30, 1648. The question is, how many days did those princes reign; and at the death of Charles I. how long had England been under an uninterrupted succession of Protestant princes (Mary I. being the last professed Papist that enjoyed the crown) not neglecting the intercalary days in February, as before?

$\frac{1533}{4}$ remains 1, so that 1556 was leap-year.

Therefore from 1553, till 1602, were 12 intercalary days, and only one leap-year in queen Mary's reign; so that in queen Elizabeth's reign were 11 intercalary days.

From July 8, to Nov. 17, being four months and nine days, are 132 days. Then $365 \times 5 = 1825$ days.

And $1825 + 132 + 1 = 1958$ days, Mary reigned.

$1602 - 1553 = 49$ years.

Betwixt March 14, and July 8, are 115 days.

Then $365 - 115 = 250$. Also $365 \times 49 = 17885$ days.

And $17885 + 250 + 12 = 18147$ days to the beginning of James's reign.

Then $18147 - 1958 = 16189$ days. Elizabeth reigned.

$\frac{1602}{4}$ remains 2, so that 1604 was leap-year.

Consequently from 1602 till 1625, were six intercalary days.

$1625 - 1602 = 23$; but as the date altered at our Lady-day, the interval was no more than 22 years, and 13 days.

Also $365 \times 22 = 8030$ days.

Therefore $8030 + 13 + 6 = 1049$ days. James I. reigned.

$\frac{1625}{4}$ remains 1, so that 1628 was leap-year.

There-

Therefore from 1625, till 1648, were six intercalary days.
1748 — 1625 = 23 whole years, Charles reigned.
From March 27, till Jan. 30, are 309 days.
Also 365 × 13 = 8395 days.
Therefore 8395 + 309 + 6 = 8710 days, Charles I. reigned.
Then 16189 + 8049 + 8710 = 32948 days, the answer.

19. A grant was made Dec. 14, in the 10th of Henry I. who began his reign Aug. 2, 1100; it was resumed Nov. 19, in the 4th of Henry III. who came to the crown Oct. 19, 1216; it was revived the 16th day of July, in the 13th of Henry VII. who ascended the throne Aug. 22, 1485: but it was a second time revoked, and finally suppressed in the 16th of his successor Henry VIII on the 10th of May. Now as this man's father died July 21, 1509, the question is, how many days was this grant in force, and how many did it lie dormant?

Henry I. began his reign Aug. 2, 1100.
Then 1100 + 9 = 1109, when the grant began, Dec. 14.
Henry III. began Oct. 19. 1216 + 3 = 1219, Nov. 19.

The first continuance of this grant 110 years wanting 26 days; and in that period are 27 intercalary days.

∵ 365 × 110 = 40150, and 40150 + 27 — 26 = 40151 days, first continuance.

Henry VII. began his reign Aug. 22, 1485.
1485 + 13 = 1498, July 16, grant reassumed.
Henry VII. died, and Henry VIII. succeeded, July 21, 1509.
1509 + 15 = 1524, May 20, grant ended.
1524 — 1497 = 27 years, wanting 57 days.
And in those 27 years, are 7 intercalary days.
365 × 27 = 9855 + 7 — 57 = 9805 days, last in force.
Then 40151 + 9805 = 49956 days, the grant was in force.

Q. E. F.

Again, Henry III. began his reign. Oct. 19, 1216.
1216 + 3 = 1219, Nov. 19, grant reassumed,
Henry VII. began his reign, Aug. 22, 1485.
1485 + 13 = 1498. July 16.
1498 — 1219 = 279 years, bating 126 days.
In which period are 69 intercalary days.

∵ 279 × 365 = 101835, and 101835 + 69 — 126 = 101778 days, superseded. Q. E. F.

N B. This question was taken from Clare's Introduction to Trade, &c. who makes the time of the continuance of the grant nine days less than found by the solution above; so would advise the young accomptant to try which is right.

CHAP.

CHAPTER III.

RULES of PRATICE.

THE rules of Practice, from their great and frequent use, derive their name, and are contrived speedily and compendiously, to cast up any sort of goods or merchandize.

SECT. I.

PRACTICE by MULTIPLICATION.

CASE I.

To multiply by a mixed number; that is, a fraction joined with a whole number.

RULE.

When you have multiplied by the whole number, take $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{7}{8}$, or whatever part it may be of the multiplicand, which, added to the product, will give the true answer.

1. In 57 fodder of lead, each $19\frac{1}{2}$ cwt. how many cwt.?

$$\begin{array}{r} 57 \\ 19\frac{1}{2} \\ \hline 1083 \\ 28\frac{1}{2} \\ \hline 1111\frac{1}{2} \end{array}$$ cwt. in all.

3. In 27 hogsheads of sugar, each containing $7\frac{3}{4}$ cwt. how many hundred weight?

$$\begin{array}{rl} 27 & \\ 7\frac{3}{4} & \\ \hline 189 & \\ 13\frac{1}{2} & = \frac{1}{2} \\ 6\frac{3}{4} & = \frac{1}{4} \\ \hline 209\frac{1}{4} & \end{array}$$

2. In 359 pieces of Norwich stuffs, each $23\frac{1}{4}$ yards, how many yards.

$$\begin{array}{r} 359 \\ 23\frac{1}{4} \\ \hline 1077 \\ 718 \\ 89\frac{3}{4} \\ \hline 8346\frac{3}{4} \end{array}$$ yards.

4. In 354 pieces of Kersey, each $27\frac{7}{8}$ yards, how many yards?

$$\begin{array}{rl} 354 & \\ 27\frac{7}{8} & \\ \hline 2478 & \\ 708 & \\ 177 & = \frac{4}{8} \\ 88\frac{1}{2} & = \frac{2}{8} \\ 44\frac{1}{4} & = \frac{1}{8} \\ \hline 9867\frac{3}{4} & \end{array}$$

CASE

CASE II.

To cast up any number of things, not exceeding 12, at any given price.

RULE.

Multiply the price by the quantity or number of things, always observing to carry from one denomination to another, as in addition of money.

1. What cost seven stone of beef, at 2 s. 7 d. per stone?

s.	d.	
2	7	
	7	
18	1,	the answer.

2. What cost 9 cwt. of treacle, at 1 l. 17 s. $4\frac{1}{4}$ d. per cwt?

l.	s.	d.	
1	17	$4\frac{1}{4}$	per cwt.
		9	
16	16	$2\frac{1}{4}$,	the answer.

3. What cost five sheep, at 1 l. 17 s. 6 d. each?

l.	s.	d.
1	17	6
		5
£9	7	6

4. What cost 10 yards of broad-cloth, at 17 s. $10\frac{1}{2}$ d.?

	s.	d.
	17	$10\frac{1}{2}$
		10
£8	18	9

5. What cost 11 geese, at 1 s. $7\frac{3}{4}$ d. each?

s.	d.
1	$7\frac{3}{4}$
	11
18	$1\frac{1}{4}$

6. What cost 12 cwt. of sugar, at 3 l. 17 s. $7\frac{3}{4}$ d.

l.	s.	d.
3	17	$7\frac{3}{4}$
		12
£46	11	9

Note, If the given quantity is 13, multiply the price by 12; and as you multiply, add to it the price of one, and the result will be the answer.

	l.	s.	d.
What cost 13 cwt. at	4	13	$7\frac{3}{4}$?
			13
	£60	17	$4\frac{3}{4}$

Thus

Thus performed, $3 \times 12 = 36 + 3 = 39$ farthings.
3 and carry 9.

Then	$7 \times 12 = 84 + 9 + 7 = 100$ d.	4 and carry 8.
Also	$3 \times 12 = 36 + 8 + 3 = 47$ s.	7 and carry 4.
And	$1 \times 12 = 12 + 4 + 1 = 17$ angels,	1 and carry 8.
Lastly	$4 \times 12 = 48 + 8 + 4 = 60$.	

CASE III.

When the quantity exceeds twelve.

RULE.

Find two numbers in the multiplication table, which being multiplied together, will make the quantity; then multiply the price by one of those numbers (it matters not which you multiply first by) and that product by the other number, and the last product will be the answer.

1. What cost 15 cwt. of treacle, at 1l. 7s. 9d. per cwt.?

			3
	4	3	3
			5
£	20	16	3

2. What cost 27 ounces of silver, at 5s. 9$\frac{3}{4}$d. per oz.

			3
		17	5$\frac{1}{4}$
			9
£	7	16	11$\frac{1}{4}$

3. What cost 56 lb. of Hyson tea, at 15s. 9$\frac{1}{2}$d. per lb.?

			8
	6	6	4
			7
£	44	4	4

4. What cost 77 cwt. of madder, at 3l. 15s. 6d. p. cwt.?

			7
	26	8	6
			11
£	290	13	6

5. What cost 108 lb. of nutmegs, at 12s. 3$\frac{3}{4}$d. per lb.

			12
	7	7	9
			9
£	66	9	9

6. What cost 132 gallons of wine, at 5s. 4d. per gallon?

			12
	3	4	–
			11
£	35	4	–

To

To find the price of a grofs; firft find the price of a dozen, by multiplying by 12; which product multiplied alfo by 12, gives the price of a grofs.

What coft feven grofs of buckles, at 1s. 11¼d. per pair?

	1	11¼	
		12	
1	3	3,	price of 1 dozen pair.
		12	
13	19	–	price of a grofs,
		7	
97	13	–,	the anfwer.

And to find the price of 1 cwt. at fo much per pound, multiply by four; which product multiplied by feven, gives the price of a quarter; then four times the laft product will be the anfwer.

What coft 3 cwt. of tea, at 4s. 5¼d. per lb.

	s.	d.	
	4	5¼	
		4	
	17	9	
		7	
6	4	3,	price of a quarter.
		4	
24	17	–,	price of a cwt.
		3	
£74	11	–,	the anfwer.

CASE IV.

When the quantity is a prime number, viz. fuch an one as no two numbers in the multiplication table can be found to anfwer it.

RULE.

Multiply by fuch numbers as come neareft the quantity; and for what is wanting, multiply the price by that number, and

and add to it the other product, and the total will be the answer.

1. What cost 29 lb. of Bohea tea, at 6s. 9d. per lb.?

	s.	d.
	6	9
		4
1	7	0
		7
9	9	–
	6	9
£9	15	9

3. What cost 68 yards of Holland, at 9 s. $4\frac{1}{2}$d. per yard?

		6	
1	12	3	
		11	
17	14	9	
	10	9	= price x 2.
£18	5	6	

5. What cost 117 sheep, at 1 l. 7 s. 6 d.?

l.	s.	d.	
1	7	6	
		10	
13	15	–	
		11	
151	4	–	
9	12	6	= price x 7.
£160	17	6	

2. What cost 38 lb. of leather, at $11\frac{3}{4}$ d. per pound?

		d.	
		$11\frac{3}{4}$	
		4	
	3	11	
		9	
1	15	3	
	1	$11\frac{1}{2}$	= price x 2.
£1	17	$2\frac{1}{2}$	

4. What cost 76 quarter of wheat, at 1l. 17s. 9d. per qr.?

		12	
22	13	–	
		6	
135	18	–	
7	11	–	= pr. X 4.
£143	9	–	

6. What cost 135 yards of broad-cloth, at 17s. $7\frac{3}{4}$d.?

	s.	d.	
	17	$7\frac{3}{4}$	
		12	
10	11	9	
		11	
116	9	3	
2	12	$11\frac{1}{4}$	= price x 3.
£119	2	$2\frac{1}{4}$	

To

To find the amount per annum of officers ſalaries at ſo much per diem, multiply the ſalary or wages per day by 10, and that product by 9: this laſt product multiplied by 4 gives the ſalary for 360 days; to this add one day's ſalary multiplied by 5, which gives the anſwer.

If an officer's ſalary be 17 s. $10\frac{3}{4}$ d. a day, what is that a year?

	s.	d.
	17	$10\frac{3}{4}$
		10
8	18	$11\frac{1}{2}$
		9
80	10	$7\frac{1}{2}$
		4
322	2	6
4	9	$5\frac{3}{4}$
£326	11	$11\frac{3}{4}$, the anſwer.

CASE V.

When the quantity is 1, 2, 3, 4, 5, 6, 7, or more hundreds.

RULE.

Multiply the price by 10, and that product by 10 alſo, which gives the value of one hundred; then multiply the product by the number of hundreds; then multiply that product, which gave the price of 10, by 2, 3, 4, or 5, as the tens happen, which place under the laſt product without drawing a line; and for the units always multiply the price by them, and ſet that down under the former products; ſo you will have three lines, the ſum of which will be the anſwer.

1. What

1. What cost 795 yards of brocade, at 1 l. 7 s. $10\frac{1}{4}$d. per yard?

l.	s.	d.
1	7	10
		10
13	18	$6\frac{1}{2}$
		10
139	5	5
		7
974	17	11
125	6	$10\frac{1}{2}$
6	19	$3\frac{1}{4}$
£ 1107	4	$-\frac{3}{4}$

2. What cost 368 lamb-hogs, at 17 s. 9 d. each?

	s.	d.
	17	9
		10
8	17	6
		10
88	15	–
		3
266	5	–
53	5	–
7	2	–
£ 326	12	–

CASE VI.

To multiply weights and measures.

RULE.

Place the multiplier under the lowest denomination of the multiplicand; then multiply the lowest denomination by the multiplier, and divide the product by as many of the lowest denomination as make one of the next superior; setting down underneath the remainder, if any, and carry the quotient to the product of the multiplier and the next superior denomination, and so proceed till all is finished.

1. In 11 pieces of kersey, each 17 yards, three quarters, three nails, how many yards in all?

Y.	qrs.	n.
17	3	3
		11
197	1	1, answer.

3. In 38 pieces of tapestry, each 37 ells flemish, 2 qrs. 3 nails, how many ells Flemish?

E.F.	qrs.	n.	
37	2	3	
		6	
227	1	2	
		6	
1365	–	–	
75	2	2	= 1 piece × 2.
1440	2	2,	answer.

2. In 42 pieces of Holland, each 27 ells English, two quarters, three nails, how many ells in all?

El. E.	qrs.	n.
27	2	3
		6
165	1	2
		7
1157	–	2

As the next superior denomination contains only 3, 4, or 5 of the inferior ones, questions of this kind may be performed without division.

4. What

4. What is the weight of ſeven tankards, each weighing 11 oz. 16 dwt. 21 grs.?

oz. dts. gr.
11 16 21
7

82 18 3

24) 147 (6

3

5. What is the net weight of 39 hogſheads of ſugar, each weighing 7 cwt. 3 qrs. 17 lb.?

Cwt. qr. lb.
7 3 17
6

47 1 18
6

284 1 24
23 2 23

308 – 19, anſwer.

28) 102 (3
18

28) 108 (3
24

28) 91 (1
23

7. What is the weight of 37 ſmall parcels of tea, each weighing 13 ounces 12 dr.?

oz. dr.
13 12
12

10 5 –

1 2 15 –
13 12

1 3 12 12

16) 144 (9

0

As 20 pennyweights make one ounce, we carry as in multiplication of ſhillings; and if pounds troy be uſed, we carry the ſame as in pence.

6. What is the weight of 29 parcels of tea, each 25 lb. 7 oz. 13 dr.

lb. oz. dr.
25 7 13
7

Cwt.
1 2 10 6 11
4

6 1 13 10 12
25 7 13

6 2 11 2 9

16) 91 (5
11

16) 54 (3
6

28) 178 (6
10

16) 44 (2
12

All the needful diviſions are here put down.

8. What is the weight of 105 ingots of ſilver, each weighing 21 oz. 17 dwt. 19 gr.?

oz. dwt. gr.
21 17 19
10 }

218 17 22
10

2188 19 4
109 8 23

Ozs. 2298 8 3

24) 190 (7
22

24) 95 (3
23

24) 220 (9
4

QUES-

QUESTIONS *to exercise the foregoing* RULES.

1. A person dying, left his widow 1780l. and 1250l. to each of his four children; 30 guineas a-piece to 15 of his poor relations, and 150l. to charities; he had been $25\frac{1}{2}$ years in trade, and at an average had cleared 126l. a year: what had he to begin with?

	To widow - - -	£ 1780	–
£ 1250 × 4	children - - -	5000	–
£ 31 10 × 15	poor relations -	472	10
	To charity - -	150	–
	Worth - - -	£ 7402	10
£ 126 cleared yearly × $25\frac{1}{2}$	-	3213	–
		£ 4182	10, answer.

2. Supposing that for a quarter's rent I paid in money 7 l. 7 s. 6 d. and was allowed for a small repair 18 s. 9 d. and for the king's tax 8 s. 9 d. what did my tenement go at a year?

£ 7	7	6	quarter's rent.
	18	9	repairs.
	8	9	tax.
8	15	–	
	4		
35	–	–,	the answer.

3. At Leicester, and other places, they weigh their coals by a machine, in the nature of a steelyard, waggon and all: three of these draughts together amount to 117 cwt. 2 qrs. 10 lb.; and the tare of the waggon was $13\frac{1}{4}$ cwt. How many coals had the customer to pay for?

		Cwt.	qr.	lb.
Cwt. qr.		117	2	10
13 1 × 3	=	39	3	–
		77	3	10

4. A gentleman hath 536l. per annum, and his expences are, one day with another, 18 s. $10\frac{3}{4}$ d. I desire to know how much he layeth up at the year's end?

	18	$10\frac{3}{4}$
		4
3	15	7
		13
49	2	7
		7
343	18	1
	18	$10\frac{3}{4}$
344	16	$11\frac{3}{4}$, expended in one year.

$4 \times 13 \times 7 = 364.$

	l.	s.	d.
Year's rent	536	–	–
Expended -	344	16	$11\frac{3}{4}$
Laid up -	191	3	–$\frac{1}{4}$, the answer.

5. A gentleman expendeth daily 1 l. 7 s. $10\frac{1}{2}$ d. and at the year's end layeth up 340 l. I demand how much is his yearly income?

l.	s.	d.	
1	7	$10\frac{1}{2}$	expences yer day.
		$\times 4$	
5	11	6	
		$\times 13$	
72	9	6	
		$\times 7$	
507	6	6	
1	7	$10\frac{1}{2}$	
508	14	$4\frac{1}{2}$,	expended in one year.

	l.	s.	d.
Year's expences	508	14	$4\frac{1}{2}$
To lay up - -	340	–	–
Answer £	848	14	$4\frac{1}{2}$

Her

Her Grace the Dutchefs of PLINTIMON, Debtor

To CREW, Cabinet-maker.

1758.			l.	s.	d.
October	3.	A chimney-glafs, and a pair of fconces	5	18	–
		A pair of pier-glaffes, 72 inches, in gilt frames	60	–	–
	10.	A pair of Indian cabinets, at 43 l. 10 s. each	87	–	–
		A fine Indian four-leaved fkreen, and a fire-fkreen	17	10	–
November	18.	A book-cafe with glafs doors, and a corner-cupboard, ditto	21	–	–
	30.	A walnut-tree table, and a fet of dreffing-boxes, japanned	3	4	10
December	7.	A tea-table and ftand, plated, weight 103 ounces, at 8 s. 4 d. per ounce	42	18	4
	30.	A dozen and half of fine matted chairs, at 18 s. 6 d. each	16	13	–
January	24.	Twelve elbow-chairs, ftuffed with hair, at 1 l. 15 s. 6 d. each	21	6	–
		A dreffing-table, with implements for writing	6	16	6
			£ 282	6	8

l.	s.
43	10
	× 2
£ 87	–

	l.	s.	d.
		8	4
			9
	3	15	–
	× 11		–
	41	5	–
Price × 2 =	1	13	4
	£ 42	18	4

l.	s.	d.
	18	6
		× 6
5	11	–
	× 3	
16	13	–

l.	s.	d.
1	15	6
		× 12
21	6	–

The Reverend Mr. EUCLID PEACHY, Debtor

To JOHN PERCIVAL and Partners.

1772.					l.	s.	d.
March	27.	Oats	5 quarters	at 2s. 3d. per bushel	4	10	–
April	9.	Beans	9 bushels	at 4 10	2	3	6
May	16.	Bran	7 quarters	at 1 10	5	2	8
June	19.	Tares	19 bushels	at 1 11	1	16	5
	24	Pease	16 bushels	at 3 11½	3	3	4
July	2.	Pale malt	28 bushels	at 3 2	4	8	8
	17.	Hops	17 pounds	at 1 4 per pound	1	2	8
					22	7	3

OPERATIONS.

s.	d.
2	3
	8
18	–
	× 5
£4 10	–

s.	d.
4	10
	× 9
£2 3	6

s.	d.
1	10
	× 8
14	8
	× 7
£5 2	8

	s.	d.
	1	11
		× 4
	7	8
		× 4
l.		
1	10	8
Price × 3 =	5	9
£1	16	5

s.	d.
3	11½
	× 4
15	10
	× 4
£3 3	4

s.	d.
3	2
	× 4
12	8
	× 7
£4 8	8

	s.	d.
	1	4
		× 2
	2	8
		× 8
l.		
1	1	4
	1	4
£1	2	8

Messieurs

Meſſieurs DRAKE and COMPTON, Debtors

To RICHARD and JACOB BROUGHTON, Dyers.

					l.	s.	d.
1762.	For dying the goods following, delivered per order, to Edward Evelin, Packer.						
May 7.	Exeter ſtuffs, yellow	70 pieces	at 14 s. —d. per piece		49	—	—
13.	Norwich ditto, blue	30	at 11 6		17	5	—
20.	Tammies, black	42	at 12 8		26	12	—
July 1.	Colcheſter bays, green	28	at 9 —		12	12	—
28.	Camblets, orange	21	at 15 —		15	15	—
					121	4	—

	s.	d.
	14	—
	× 10	
l.		
7	—	—
× 7		
£49	—	—

	s.	d.
	11	6
		× 5
l.		
2	17	6
		× 6
£17	5	—

	s.	d.
	12	8
		× 6
l.		
3	16	—
	× 7	
£26	12	—

	s.	d.
	9	—
	× 4	
l.		
1	16	—
	× 7	
£12	12	—

	s.	d.
	15	—
	× 3	—
l.		
2	5	—
	× 7	
£15	15	—

G 4

Mr.

Mr. ROWLAND, Upholder, Debtor

To WILLIAM WAREHOUSE and Company.

1762.						l.	s.	d.
October	7.	Superfine blankets	17 pair	at 1l. 3s. 10d.		20	5	2
	9.	Medium ditto	13	at – 13	6	8	15	6
		Harateen blue	31 pieces	at 2 5	8	70	15	8
November	6.	Cheney ditto	20	at 1 18	–	38	–	–
	18.	Green Linseys	10	at 1 8	–	14	–	–
December	9.	Printed quilts ¾ Callico	15	at – 16	8	12	10	–
						£ 164	6	4

l.	s.	d.
1	3	10
	×	2
2	7	8
	×	8
19	1	4
1	3	10
£ 20	5	2

	s.	d.
	13	6
	×	13
£ 8	15	6

l.	s.	d.
2	5	8
	×	5
11	8	4
	×	6
68	10	–
2	5	8
£ 70	15	8

l	s.	d.
1	18	–
× 4		
7	12	–
× 5		
£ 38	–	–

l.	s.	d.
1	8	
× 10		
£ 14	–	–

l.	s.	d.
16	8	–
	× 3	
2	10	–
	× 5	
£ 12	10	–

Sir

Sir EDWARD CORNWALL Debtor

To BERNARD, Bricklayer,

For work and materials in his houfe on Tower-hill, London,

1772						l.	s.	d.
March	28.	Bricks - -	25 thoufand - -	at 15s. 7d. per thoufand	-	19	9	7
	30.	Tiles - -	11 ditto - -	at 19 5 - -	-	10	13	7
April	1.	Lime - -	28 hundred - -	at 15 11 per hundred -	-	22	5	8
	12.	Sand - -	19 load - -	at 3 10 per load -	-	3	12	10
June	28.	Work for myfelf	90 days - -	at 3 – per diem -	-	13	10	–
		Ditto for a labourer	90 - - -	at 1 8 - -	-	7	10	–
		Ditto for my man	90 - - -	at 2 6 - -	-	11	5	–
						£88	6	8

s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
15 7	19 5	15 11	3 10	3 –	1 8	2 6
× 5	× 11	× 4	× 2	× 10	× 9	× 9
l. 3 17 11	£10 13 7	l. 3 3 8	7 8	l. 1 10 –	15 –	l. 1 2 6
× 5		× 7	× 9	× 9	× 10 –	× 10
£19 9 7		£22 5 8	l. 3 9 –	£13 10 –	£7 10 –	£11 5 –
			3 10			
			£3 12 10			

SECT.

SECT. II.

PRACTICE *by* DIVISION.

CASE I.

HAVING the price of any number of things not exceeding 12, to find the price of one.

RULE.

Place the divisor under the highest denomination (or to the left-hand) of the dividend. Then divide the highest denomination of the dividend by the divisor, and bring the remainder, if any, in your mind only, into the next inferior denomination, adding thereto what is in the dividend; divide this number as above, and so proceed till the whole is finished.

1. If 3 yards of velvet cost 5 l. 3 s. $11\frac{1}{4}$ d. what cost 1 yard?

	l.	s.	d.
	5	3	$11\frac{1}{4}$
÷3			
£	1	14	$7\frac{3}{4}$, answer.

3. If 7 cwt. of lead cost 6 l. 5 s. $10\frac{1}{4}$ d. what cost 1 cwt.?

	l.	s.	d.
	6	5	$10\frac{1}{4}$
÷7			
£	–	17	$11\frac{3}{4}$, answer.

5. 11 geese cost 18 s. $1\frac{1}{2}$ d. what cost one?

	s.	d.
	18	$1\frac{1}{2}$
÷11		
	1	$7\frac{3}{4}$, answer.

2. If 4 yards of silk cost 3 l. 11 s. What cost 1 yard?

	l.	s.	d.
	3	11	–
÷4			
£	–	17	9, answer.

4. If 9 ells of linen cost 1 l. 8 s. $8\frac{1}{4}$ d. what cost 1 ell?

	l.	s.	d.
	1	8	$8\frac{1}{4}$
÷9			
£	–	3	$2\frac{1}{4}$, answer.

6. If 12 cwt. of loaf-sugar cost 46 l. 12 s. 9 d. what cost one?

	l.	s.	d.
	46	12	9
÷12			
£	3	17	$8\frac{3}{4}$, answer.

CASE II.

When the divisor exceeds 12.

RULE.

Find two or more numbers, whose product is equal to the given quantity; then divide the price by one of them, and that quotient by the other; the last quotient will be the answer.

1. If 21 fat heifers cost 164 l. 11 s. $3\frac{1}{4}$ d. what cost one?

	l.	s.	d.
	164	11	$3\frac{3}{4}$
÷ 7			
	23	13	$2\frac{1}{4}$
÷ 3			
£	7	16	$8\frac{3}{4}$, answer.

2. If 33 lb. of butter cost 15 s. $9\frac{3}{4}$ d. what cost one pound?

	s.	d.
	15	$9\frac{3}{4}$
÷ 11		
	1	$5\frac{1}{4}$
÷ 3		
	–	$\frac{3}{4}$, answer.

3. If 35 dollars are worth 7 l. 17 s. 6 d. what is the worth of one?

	l.	s.	d.
	7	17	6
÷ 5			
	1	11	6
÷ 7			
£	–	4	6, answer.

4. If 54 cwt. of treacle cost 101 l. 14 s. what per cwt.?

	l.	s.	d.
	101	14	
÷ 6			
	16	19	
÷ 9			
£	1	17	8, answer.

5. If 72 sheep cost 82 l. 10 s. what cost one?

	l.	s.	d.
	82	10	
÷ 8			
	10	6	3
÷ 9			
£	1	2	11, answer.

6. If 132 lb. of tobacco cost 5 l. 18 s. 3 d. what per lb.?

	l.	s.	d.
	5	18	3
÷ 12			
	–	9	$10\frac{1}{4}$
÷ 11			
£	–	–	$10\frac{3}{4}$, answer.

I. If

I. If there be a remainder in the first operation, and none in the last, place it over the whole divisor for a fractional part; but if there be two remainders, multiply the last remainder by the first divisor, and to the product add the first remainder, which will be the true remainder or numerator of the fraction, under which place the whole divisor for a denominator.

7. If 28 tod of wool cost 25 l. what cost one tod?

l.	s.	d.
25	–	–
$\div$ 7		
3	11	$5\frac{1}{7}$
$\div$ 4		
£ –	17	$10\frac{1}{4} + \frac{1}{28}$d. or $\frac{1}{7}$ far.

8. If 77 lb. of tea cost 26 l. what per pound?

l.	s.	d.
26	–	–
$\div$ 7		
3	15	$2\frac{3}{4}$, $\frac{3}{7}$ farth.
$\div$ 11		
–	6	$8\frac{3}{4}$, $\frac{10}{11}$ farth.

$7 \times 10 + 3 = 73.$

Answer 6 s. $8\frac{3}{4}$d. + $\frac{73}{77}$ farth.

II. If you have the price of any number of gross, to find the price of one, divide the whole price by the number of gross, which gives the price of one gross; this quotient divided by 12 gives the value of 1 dozen; which last quotient, divided by 12 also, gives the value of one.

9. If 7 gross pair of buckles cost 59 l. 17 s. what cost 1 pair?

l.	s.	d.	
59	17	–	
$\div$ 7			
8	11	–	per gross.
$\div$ 12			
–	14	3	per dozen.
$\div$ 12			
–	2	$2\frac{1}{4}$	per pair.

10. If 9 gross pair of gloves cost 91 l. 16 s. what cost 1 pair?

l.	s.	d.	
91	16	–	
$\div$ 9			
10	4	–	per gross.
$\div$ 12			
–	17	–	per dozen.
$\div$ 12			
–	1	5	per pair.

III. Also having the value of a last of oats, cole-seed, or other grain, customary measure, to find the value of one bushel; divide by 3, and that quotient by 7, which gives the value of a coom; this last quotient divided by 4, gives the value of a bushel.

11. If

11. If a laſt of cole-ſeed coſt 16l. what will one buſhel coſt at that rate?

	l.	s.	d.	
	16	–	–	
÷	3			
	5	6	8	
÷	7			
	–	15	$2\frac{1}{4}$, $\frac{8}{7}$	per coom.
÷		4		
	–	3	$9\frac{1}{2}$, $\frac{3}{4}$	

$7 \times 3 + 3 = 24$ remainder.
Anſ. 3s. $9\frac{1}{2}$d. + $\frac{24}{28}$, or $\frac{6}{7}$ far.
or 3 $9\frac{3}{4}$, very near.

12. If a laſt of oats coſt 5 guineas, what coſt one buſhel at that rate?

	l.	s.	d.	
	5	5		
÷	3			
	1	15	–	
÷	7			
	–	5	–	per coom.
÷		4		
£	–	1	3,	the anſwer.

IV. Having the price of an hundred weight, to find the price of one pound, divide the price of the hundred by 4, which gives the value of $\frac{1}{4}$ of a cwt, which divide again by 4, gives the value of 7 l.; this laſt quotient divided by 7, gives the anſwer.

13. If 1 cwt. of hops coſt 5l. 9s. 8d. what coſt 1 lb. at that rate?

	l.	s.	d.	
	5	9	8	
÷	4			
	1	7	5	
÷	4			
	–	6	$10\frac{1}{4}$	
÷		7		
	–	–	$11\frac{3}{4}$,	the anſwer.

14. Sugar at 3l. 13s. 6d. per cwt. what per lb.?

	l.	s.	d.	
	3	17	–	
÷	4			
	–	19	3	
÷		4		
	–	4	$9\frac{3}{4}$	
÷		7		
	–	–	$8\frac{1}{4}$,	the anſwer.

V. Having the price of 1 c. great weight, viz. 120 lb. per cwt. to find the price of one ſtone, or of one pound, divide by 8, which gives the price per ſtone; which quotient divided by 3, and that quotient again by 5, will give the price of one pound.

15. Cheeſe

15. Cheese at 1 l. 15s. per cwt. what per stone, and per pound?

	l.	s.	d.	
8	1	15	–	
3		4	$4\frac{1}{2}$	per stone.
5		1	$5\frac{1}{2}$	
			$3\frac{1}{2}$	per pound.

16. Ditto at 1 l. 5 s. per cwt. what per stone, and per pound?

	l.	s.	d.	
8	1	5	–	
3		3	$1\frac{1}{2}$	per stone.
5		1	$-\frac{1}{2}$	
			$2\frac{1}{2}$	per pound.

VI. Or by considering, that as 120 pence make 10 s. every 10 s. per cwt. gives 1 penny, and every 2 s. 6 d. gives 1 farthing a pound.

So that 1 l. 10 s. gives 3d. and 5 s. gives $\frac{1}{2}$ d. viz. by inspection, $3\frac{1}{2}$ d. per pound.

In the question above, 1 l. gives 2 d. and 5 s. gives $\frac{1}{2}$ d. = $2\frac{1}{2}$ d. per pound.

17. Ditto at 1 l. 7 s. 6 d. per cwt. what per pound?
Here 1 l. gives 2 d. and 7 s. 6 d. gives $\frac{3}{4}$ d.
Or $2\frac{3}{4}$ d. per pound.

18. Ditto at 1 l. 12 s. 6 d. per cwt. what per pound?
Here 1 l. 10 s. gives 3 d. and 2 s. 6 d. = $\frac{1}{4}$ d.
Or $3\frac{1}{4}$ d. per pound.

7. Having the price of a ton, to find the price of a hundred, a quarter, or a pound, divide by 5 and by 4; which last quotient is the price of an hundred; then proceed as per remark 4th.

19. Carriage at 7 l. per ton, what per pound?

	l.	s.	d.	
4	7	–	–	
5	1	15	–	
4	–	7	–	per cwt.
7	–	1	9	per qr.
4	–	–	3	
			$\frac{3}{4}$ d.	per lb.

20 Carriage at 2 l. 6 s. 8 d. per ton, what per pound?

	l.	s.	d.	
4	2	6	8	
5	–	11	8	
4	–	2	4	per cwt.
7	–	–	7	per qr.
4	–	–	1	
			$\frac{1}{4}$ d.	per lb.

VIII. As 252 gallons of wine, &c. make a tun, to find the price of a hogshead or gallon, divide the price of the tun by 4, which gives the price of an hogshead; then divide the

the price of a hogshed by 7, and that quotient divided by 9, gives the price of a gallon?

21. Port wine at 60 l. per ton, what per gallon?

	l.	s.	d.	
4	60	–	–	per ton.
9	15	–	–	per hogshead.
7	1	13	4	
	–	4	$9\frac{1}{7}$	per gallon.

22. Madeira wine at 95 l. per ton, what per gallon?

	l.	s.	d.	
4	95	–	–	per ton.
9	23	15	–	per hogshead.
7	2	12	$9\frac{1}{3}$	
	–	7	$6\frac{1}{2}$	nearly p. gal.

IX. Having the value of a wey (viz. 256 lb.) of cheese, &c. to find the value of 1 lb. or of an hundred weight; divide the value of a wey by 8, and that quotient by 4, which gives the value of 8 lb.; which divided by 8, gives the value of 1 lb.; or the value of 8 lb., multiplied by 7, gives the value of $\frac{1}{2}$ cwt.: which multiplied by 2, gives the value of an hundred weight.

Cheese at 2l. 18s. 8d. per wey, what per lb. and per cwt?

	l.	s.	d.	
8	2	18	8	per wey.
4	–	7	4	
8	–	1	10	price of 8 lb.
	–	–	$2\frac{3}{4}$	per lb.

	s.	d.	
Or	1	10	price of 8 lb.
	× 7		
	12	10	per $\frac{1}{2}$ cwt.
	× 2		
£ 1	5	8	per cwt.

24. Ditto at 2l. 8s. per wey, what per lb. and per cwt.?

	l.	s.	d.	
8	2	8	–	per wey.
4	–	6	–	
8	–	1	6	price of 8 lb.
	–	–	$2\frac{1}{4}$	per lb.

	s.	d.	
Or	1	6	price of 8 lb.
	× 7		
	10	6	per $\frac{1}{2}$ cwt.
	× 2		
£ 1	1	–	

X. A stone of wool or locks, in several manufacturing counties is 15 lb. and 16 such stones, 8 tods, or 240 lb. make one pack;

pack; therefore having the price of a pack of wool, &c. to find the price of a tod, ſtone, or pound, divide the price of a pack by 8, which gives the price of a tod, which divided by 2, gives the price per ſtone; which divided by 3, and that quotient again by 5, gives the price of one pound.

25. Wool at 7 l. 5 s. per pack, what per lb.?

	l.	s.	d.	
8	7	5	–	
2	–	18	$1\frac{1}{2}$	per tod.
3	–	9	–$\frac{3}{4}$	per ſtone.
5	–	3	–$\frac{1}{4}$	
	–	–	$7\frac{1}{4}$	per lb.

Or as 240 l. = 20s., by inſpection, 7 l. 5 s. per pack, gives $7\frac{1}{4}$d. per lb.

26. Locks at 4 l. 15 s. per pack, what per lb.

	l.	s.	d.	
8	4	15	–	
2	–	11	$10\frac{1}{2}$	per tod.
3	–	5	$11\frac{1}{4}$	per ſtone.
5	–	1	$11\frac{3}{4}$	
	–	–	$4\frac{3}{4}$	per lb.

Or by inſpection, 4 l. 15 s. per pack, gives $4\frac{3}{4}$d. per lb.

CASE III.

When the diviſor is a prime number, or not compoſed of two or more numbers.

RULE.

Take the whole diviſor, and divide as in diviſion of integers; multiplying the remainder by that number of the next inferior denomination which makes one of its ſuperior, adding to the product what there is in the dividend of the ſame denomination you are then reducing the remainder to: divide this ſum as above, and ſo proceed in this manner till all be finiſhed.

1. It

1. If 53 fat Lincolnshire sheep be sold for 99l. 16s. 4d. what was each sold for on an average?

l. s. d. l. s. d.
53) 99 16 4 (1 17 8
46
20
936
406
35
12
424

2. If 675 l. 12 s. 6 d. be equally divided amongst 138 men, what is each man's share?

l. s. d. l. s. d.
138) 675 12 6 (4 17 11
123
20
2472
1092
126
12
1518
138

3. At 315 l. 3 s. $10\frac{1}{4}$ d. per year, what per day?

l. s. d. l. s. d.
365) 315 3 $10\frac{1}{4}$ (– 17 $3\frac{1}{4}$
20
6303
2653
98
12
1186
91
4
365

4. A prize of 9475l. 16s. 8d. being divided equally amongst 747 sailors, what is each man's share, after deducting $\frac{1}{5}$ for the captain?

l. s. d.
5) 9475 16 8
1895 3 4 captain.
l. s. d. f.
747) 7580 13 4 (10 2 11 2 $\frac{166}{747}$ an.
110
20
2213
719
12
8632
415
4
1660 remainder 166.

CASE IV.

To divide weights and meaſures

RULE.

Weights and meaſures are divided exactly in the ſame manner as money, due regard being only had to the number of thoſe of an inferior denomination contained in a ſuperior one of the ſame ſpecies.

1. If 6 pieces of tapeſtry contain 227 ells Flemiſh, 1 quarter, 2 nails, what is the length of one piece?

El. F.	qr.	n.
227	1	2
÷ 6		
Ells Fle. 37	2	3, anſw.

3. What is the length of 1 piece of linen, if 21 pieces are 754 ells Engliſh, 4 quarters, 3 nails?

El. E.	qr.	n.
754	4	3
÷ 7		
107	4	1
÷ 3		
35	4	3, anſwer.

5. If 28 parcels of tea weigh 6 cwt. 1 qr. 13 lb. 10 oz. 12 dr. what the weight of 1 parcel?

Cwt.	q.	lb.	oz.	dr.
6	1	13	10	12
÷ 4				
1	2	10	6	11
÷ 7				
–	–	25	7	13, anſwer.

2. If 20 pieces of cloth contain 438 yards, 3 quarters, what does 1 piece contain?

Yds.	qr.	
438	3	
÷ 4		
109	2	3
÷ 5		
Yards 21	3	3, anſwer.

4. If 322 cwt. 2 qrs. 5 lb. is the weight of 25 hogſheads, what is the weight of 1 hogſhead?

Cwt.	qr.	lb.
322	2	5
÷ 5		
64	2	1
÷ 5		
Cwt. 12	3	17, anſwer.

6. If 10 ſilver punch-bowls of an equal weight, weigh 478 oz. 19 dwt. 14 gr. what is the weight of one?

Oz.	dwt.	gr.
478	19	14
÷ 10		
Oz. 47	17	23, anſwer.

7. If

7. If 103 ingots of silver, of an equal weight, weigh 1225 oz. 12 dwt. 13 gr. what is the weight of one ingot?

```
      oz. dwt. gr.  oz. dwt. dr.
103)1225  12  13(11  17  23 55/103
     195
     ---
      92
      20
     ---
    1852
     822
     ---
     101
    × 24
     ---
    2424
     364
     ---
      55
```

8. If 59 parcels of tea of an equal weight weigh 2 qr. 1 lb. 9 oz. 14 dr. what is the weight of one parcel?

```
   Qr. lb. oz. dr.  oz. dr.
59) 2   1   9  14 ( 15  10
       57
     × 16
     ---
      921
      331
     ---
       36
     × 16
     ---
      590
     ---
        0
```

9. If 335 cwt. 1 qr. 22 lb. is the weight of 26 hogsheads of sugar, what is the weight of one hogshead?

```
   Cwt. qr. lb.  cwt. qr. lb.
26) 335  1  22 ( 12   3  17
     75
     --
     23
    × 4
     --
     93
     --
     15
   × 28
    ---
    442
    182
    ---
      0
```

10. A common pasture contains 53 acres, 1 rood, 27 perches; another 65 acres, 2 roods, 19 perches; a third 47 acres, 3 roods; these being inclosed, are to be divided amongst 59 parishioners; how much is each man's share?

```
    A.  R. P.
    53  1  27
    65  2  19
    47  3  -
   ---  -----    A. R. P.
59)166  3  6 (2  3  12 18/59
   ---
    48
     4
    ---
   195
    ---
    18
  × 40
   ---
   726
   136
   ---
    18
```

11. If 117 pieces of Holland contain 4440 ells English, and 3 nails, what doth each piece contain?

El. E. qr. n. el. E. qr. n.

117)4440 – 3 (37 4 3
930
111
× 5
555
87
× 4
351
0

12. The Spectator's club of fat people, though it consisted but of 15 persons, is said (No 9.) to weigh no less than three tons; how much at an equality was that per man?

Tons.
15) 3 –(4 cwt. ans.
× 20
60

CASE V.

To reduce great hundreds (and quarters) 120 lb. to the hund. weight; to hundreds, quarters, and pounds, 112 lb. to the hundred weight.

RULE.

Consider them as small hundreds, and quarters, dividing by 2, and that quotient by 7; which last quotient added to the great weight, gives the small weight.

In 5 cwt. 2 qr. 2 lb. great weight, how much small weight?

	Cwt.	qr.	lb.
2	5	2	2
7	2	3	–
Add	–	1	16
Ct.	5	3	18, small weight.

In 17 cwt. 1 qr. 21 lb. great weight, how much small weight?

	Cwt.	qr.	lb.
2	17	1	21
7	8	2	14
	1	–	26 add.
	18	2	19 small weight.

CASE VI.

To reduce small weight to great weight.

RULE.

RULE.

Divide the hundreds and quarters, considered as great weight by 3, and that quotient by 5; which last quotient subtracted from the small weight, leaves the great weight.

	Cwt.	qr.	lb.	
3	18	2	10	sm. weight.
5	6	–	20	
Subtr.	1	–	28	
	17	1	12	gr. weight.

	Cwt.	qr.	lb.	
3	5	3	18	sm. weight.
5	1	3	20	
Subt.	–	1	16	
	5	2	2	gr. weight.

N. B. The pounds in the first division, are taken no notice of, as being the same both in great and small weight.

A wey being 256 lb. of cheese, wool, &c. in some counties of England, is composed of $8 \times 8 \times 4 = 256$ lb. $=$ 2 cwt. 1 qr. 4 lb.

Cheese at $2\frac{3}{4}$d. per lb. what per wey?

l.	s.	d.
		$2\frac{3}{4}$
		8
	1	10
		8
	14	8
		4
Answer, 2	18	8 per wey.

Or as 256 farthings $=$ 64d. $=$ 5s. 4d. $= \frac{1}{4}$ per lb.

	s.	d.	
	5	4	
		4	
1	1	4	$=$ 1d. per lb.
		2	
2	2	8	
	16	–	$=$ 5s. 4d. $\times$ 3.
2	18	8	$= 2\frac{3}{4}$ d. per lb.

		lb.	
120		256	
$\frac{1}{4}$	2	2	8
$\frac{1}{2}$		10	8
		5	4
£	2	18	8, as before.

Cheese at 2l. 8s. per wey, what per lb.?

	l.	s.	d.
8	2	8	–
8	–	6	–
4	–	–	9
	–	–	$2\frac{1}{2}$

In $51\frac{3}{4}$ weys of cheese, how many hundreds?

C.	qr.	lb.	
2	1	4	= 1 wey.
		8	
18	1	4	
		6	
109	2	24	
$\frac{1}{4}$) 6	3	12	
1	2	24	
118	1	4	= $51\frac{3}{4}$ weys.

$51\frac{3}{4}$ weys of cheese, at $2\frac{1}{4}$d. per lb.?

	l.	s.	d.	
$\frac{1}{112}$	256			
$\frac{1}{8}$		2	2 8	
			5 4	
		2	8	– per wey.
			8	
		19	4	
			6	
	115	4		
$\frac{1}{4}$)	7	4		
	1	16		
	£124	4,		the answer.

QUESTIONS *to exercise* PRACTICE *by* DIVISION.

1. A draper bought 420 yards of broad-cloth, at the rate of 14s. $10\frac{3}{4}$d. per ell English, how much did he pay for the whole?

	s.	d.	
5	14	$10\frac{3}{4}$	per ell English.
	2	$11\frac{3}{4}$	
	11	11	per yard.
		10	
£5	19	2	
		7	
41	14	2	
		6	
250	5	–	the answer required.

2. A draper bought of a merchant 8 packs of broad-cloth; every pack had 4 parcels in it, and each parcel contained 10 pieces; every piece being 26 yards: he gave after the rate of four pounds, seventeen shillings, and sixpence, for six yards; what came the whole to, and what did it cost per yard?

6) 4l. 17s. 6d. (16s. 3d. per yard.

	×	2	
1	12	6	
		13	
21	2	6	per piece.
	×	10	
211	5	–	per parcel.
	×	4	
845	–	–	per pack.
× 8			
£ 6760	–	–,	the answer.

3. An

3. An oilman bought 3 tuns of oil, which cost him 151 l. 14 s. which happened to leak out 85 gallons; this he is willing to sell again so as to be no loser: I demand how he must sell it a gallon?

```
  252 gallons in a tun.
    3 tun.
  ---
  756 gallons.
   85 leaked.
  ---  l.  s.
671)151 14(4s. 6¼d. 25/671, the answer.
    × 20
    ----
    3034
     350
    × 12
    ----
    4200
     174
     × 4
    ----
     696
    ----
      25
```

4. A draper bought 242 yards of broad-cloth, which cost him in all 256l. 10s; for 86 yards of which he gave 1l. 1s. per yard; what did he give a yard for the remainder?

```
 l.  s  d        l.  s.  d.
 1   1  4       256  10  -         242
      × 12       91  14  8        — 86
 ---------      -----------      ----
 12  16  -      164  15  4 price of - 156 yards.
    × 7
 ---------                l.  s. d.
 89  12  -          156) 164 15 4(1l. 1s. 1¼d. 37/39 ans.
  2   2  8                 8
 ---------               × 20
£ 91 14  8 price of 86 yds. ----
 ---------                175
                           19
                         × 12
                         ----
                          232
                           76
                          × 4
                         ----
                          304
                         ----
                          148
```

5. A gentleman, at his death, left his eldeſt ſon once and a half what he allotted his daughter; and to the young lady 1383 l. leſs than her mother, to whom he bequeathed four times what he left towards the endowment of Hertford college, Oxon, viz. 1640 guineas: I require what he intended for his youngeſt ſon, who claimed, under the will, half as much as his mother and ſiſter; how much leſs than 30000 l. did the teſtator die worth, his debts and funeral expences being 988 l. 10 s?

20)1640 guineas.
+ 82

£ 1722 Hertford college.
× 4

6888 Wife.
− 1383

2)5505 daughter.
+ 2752 10 s.

£ 8275 10 s. eldeſt ſon.

6888
+ 5505

2)12393

£ 6196 10 s. youngeſt ſon.

	l	s.
The mother	6888	–
Eldeſt ſon	8257	10
Youngeſt	6196	10
Daughter	5505	–
College	1722	–
Funeral	988	10

30000 — 29557 10 = 442 l. 10 s. the anſwer.

6. My purſe and money, quoth Dick, are worth 12 s. 8 d. but the money is worth ſeven of the purſe; pray what was there in it?

s.	d.	
8)12	8	purſe and money.
1	7	purſe.
11	1,	money, the anſwer

7.

7. A dealer bought two lots of snuff, that together weighed 9 cwt. 3 qrs. 16 lb. for 97 l. 17s. 6d. Their difference in point of weight was 1 cwt. 2 qrs. 16 lb.; and of price 8 l. 13 s. 3d. Their respective weights and values are required?

	Cwt.	qr.	lb.		l.	s.	d.
Weight -	9	3	16	Cost -	97	17	6
Difference -	1	2	16		8	13	3
	2)8	1	–		2)89	4	3
Lesser lot -	4	–	14		44	12	$1\frac{1}{2}$
Greater -	5	3	2		53	5	$4\frac{1}{2}$

8. A tradesman increased his estate annually a third part, abating 100 l., which he usually spent in his family; and at the end of $3\frac{1}{4}$ years, found that his net estate amounted to 3179 l. 11 s. 8 d. Pray what had he at out-setting?

		l.	s.	d.
Worth at the end of $3\frac{1}{4}$ years -		3179	11	8
	4)100 =	25	–	–
$3\frac{1}{4}$ years = 13 quarters	∴ 13)	3204	11	8
	—	246	10	$1\frac{1}{2}$
Worth at the end of 3 years -		2958	1	$6\frac{1}{2}$
	+	100	–	–
	4)	3058	1	$6\frac{1}{2}$
		764	10	$4\frac{1}{2}$
Worth at the end of 2 years -		2293	11	2
	+	100	–	–
	4)	2393	11	2
		598	7	$9\frac{1}{2}$
Worth at 1 year's end - -		1795	3	$4\frac{1}{2}$
	+	100	–	–
		1895	3	$4\frac{1}{2}$
	—	473	15	10
	Answer, £	1421	7	$6\frac{1}{2}$

9. A

9. A certain person bought two horses, with the trappings, which cost 100 l.; which trappings, if laid on the first horse A, both will be of equal value; but if the said trappings be laid on the other horse, he will be double the value of the first; how much did the said horses cost?

	l.	l.	s.	d.	
2)	100 (	50	—	—	price of the best horse.
3)	50 (	16	13	4	price of the trappings.
Difference		£ 33	6	8	price of the other horse.

* *

SECT. III.

PRACTICE *by* ALIQUOT PARTS.

THIS rule is only a contraction of the Golden rule; for when the value or price of one yard, ell, hundred, &c. is given, and the price or value of any other quantity of yards, ells, hundreds, &c. required; the first number or term being unity, the question may be performed by aliquot, or even parts of numbers. An aliquot part of any number is such, that if the said part be taken certain times, it shall just make the number whereof it is a part.

TABLES *of* ALIQUOT PARTS.

Of a pound.

s.	d.	
10	-	$= \frac{1}{2}$
6	8	$= \frac{1}{3}$
5	-	$= \frac{1}{4}$
4	-	$= \frac{1}{5}$
3	4	$= \frac{1}{6}$
2	6	$= \frac{1}{8}$
2	-	$= \frac{1}{10}$
1	8	$= \frac{1}{12}$
1	-	$= \frac{1}{20}$
Tenths of a l.		
2	-	$= \frac{1}{10}$
4	-	$= \frac{2}{10}$
6	-	$= \frac{3}{10}$
8	-	$= \frac{4}{10}$
12	-	$= \frac{6}{10}$
14	-	$= \frac{7}{10}$
16	-	$= \frac{8}{10}$
18	-	$= \frac{9}{10}$

s.	d.	
1	3	$= \frac{1}{4}$ of $\frac{1}{4}$
—	10	$= \frac{1}{4}$ of $\frac{1}{6}$
—	8	$= \frac{1}{5}$ of $\frac{1}{6} = \frac{1}{30}$
—	$7\frac{1}{2}$	$= \frac{3}{4}$ of $\frac{1}{8}$
—	6	$= \frac{1}{4}$ of $\frac{1}{10} = \frac{1}{40}$
—	5	$= \frac{1}{6}$ of $\frac{1}{8}$
—	4	$= \frac{1}{5}$ of $\frac{1}{12} = \frac{1}{60}$
—	$3\frac{3}{4}$	$= \frac{1}{8}$ of $\frac{1}{8}$
—	3	$= \frac{1}{4}$ of $\frac{1}{20} = \frac{1}{80}$
—	2	$= \frac{1}{6}$ of $\frac{1}{20} = \frac{1}{120}$
—	1	$= \frac{1}{12}$ of $\frac{1}{20}$
—	$-\frac{3}{4}$	$= \frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{20}$
—	$-\frac{1}{2}$	$= \frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$
—	$-\frac{1}{4}$	$= \frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$

s.	d.		s.	d.	
7	6	$= \frac{3}{8}$	13	4	$= \frac{2}{3}$
8	-	$= \frac{2}{5}$	15	-	$= \frac{3}{4}$
8	4	$= \frac{5}{12}$	16	-	$= \frac{4}{5}$
11	8	$= \frac{7}{12}$	16	8	$= \frac{5}{6}$
12	-	$= \frac{3}{5}$	17	6	$= \frac{7}{8}$
12	6	$= \frac{5}{8}$	18	4	$= \frac{11}{12}$

s.	d.	
-	9	$= \frac{3}{80}$
-	10	$= \frac{5}{120}$
1	2	$= \frac{7}{120}$
1	3	$= \frac{5}{80}$
1	4	$= \frac{2}{30}$
1	6	$= \frac{3}{40}$
1	9	$= \frac{7}{80}$
1	10	$= \frac{11}{120}$
2	3	$= \frac{9}{80}$
2	4	$= \frac{7}{60}$
2	8	$= \frac{4}{30}$
2	9	$= \frac{11}{80}$
3	-	$= \frac{3}{20}$
3	6	$= \frac{7}{40}$
3	8	$= \frac{11}{60}$
4	6	$= \frac{9}{40}$
4	8	$= \frac{7}{30}$
5	4	$= \frac{8}{30}$
5	6	$= \frac{11}{40}$
7	4	$= \frac{11}{30}$

Of a shilling.

d.	
6	$= \frac{1}{2}$
4	$= \frac{1}{3}$
3	$= \frac{1}{4}$
2	$= \frac{1}{6}$
$1\frac{1}{2}$	$= \frac{1}{8}$
1	$= \frac{1}{12}$
$-\frac{3}{4}$	$= \frac{1}{2}$ of $\frac{1}{8}$
$-\frac{1}{2}$	$= \frac{1}{2}$ of $\frac{1}{12}$
$-\frac{1}{4}$	$= \frac{1}{4}$ of $\frac{1}{12}$
And	
$10\frac{1}{2}$	$= \frac{7}{8}$
10	$= \frac{5}{6}$
9	$= \frac{3}{4}$
8	$= \frac{2}{3}$
$7\frac{1}{2}$	$= \frac{5}{8}$
$4\frac{1}{2}$	$= \frac{3}{8}$
$3\frac{3}{4}$	$= \frac{1}{8}$ of $\frac{5}{8}$
$5\frac{1}{4}$	$= \frac{1}{2}$ of $\frac{7}{8}$
$2\frac{1}{4}$	$= \frac{1}{2}$ of $\frac{3}{8}$

CASE I.

When the price of the integer is an aliquot part of a pound, or of a ſhilling.

RULE.

For the aliquot part of a pound, divide the given quantity by that part; the quotient will be pounds, and the remainder ſo many times that part. But for the aliquot parts of a ſhilling, divide as before, and that quotient by 20; the laſt quotient will be pounds, and the remainder ſhillings.

But if it be a compound aliquot part, take the firſt aliquot part, and then the aliquot part of that part, which laſt quotient will anſwer the queſtion.

1. What coſt 737 yards of Holland, at 10s. per yard?

$\frac{1}{2}$|737

£ 368 10 – s. d., anſwer.

2. 873 yards ditto, at 6 s. 8 d.?

$\frac{1}{3}$|873

£ 291 – – s. d., anſwer.

3. 3711 yards ditto, at 5s.?

$\frac{1}{4}$|3711

£ 927 15 – s. d., anſwer.

4. 1171 yards ditto, at 4s.?

$\frac{1}{5}$|1171

£ 234 4 – s. d. anſwer.

5. 743 yards of Iriſh cloth, at 3s. 4d.?

$\frac{1}{6}$|743

£ 123 16 8 s. d., anſwer.

6. 275 yds. ditto at 2s. 6d.?

$\frac{1}{8}$|275

£ 34 7 6 s. d., anſwer.

7. 765 yds. of ditto at 2s.?

|765

£ 76 10 s., anſwer.

8. 1761 yards ditto, at 1s. 8d.?

$\frac{1}{12}$|1761

£ 146 15 s., anſwer.

9. What coſt 757 yards of dowlas, at 1s.?

$\frac{1}{20}$|757

£ 37 17 s., anſwer.

10. What

10. What cost 957 yards of dowlas, at 1s. 3d.?

$\frac{1}{4}$ | 957
s. d.
$\frac{1}{4}$ | 239 1 3

£ 59 16 3, answer.

15s. + 1s. 3d. = 16s. 3d.

11. 1713 yards of ribbon, at $7\frac{1}{2}$d.?

$\frac{1}{4}$ | 1713
s. d.
$\frac{1}{8}$ | 428 – $7\frac{1}{2}$

£ 53 10 $7\frac{1}{2}$, answer.

10s. + $7\frac{1}{2}$d. = 10s. $7\frac{1}{2}$d.

Here, and in several questions below, the remainder is looked upon to be of the same denomination with the dividend; and the second remainder is always added to the first.

12. 737 yards ditto, at 10d.?

$\frac{1}{4}$ | 737
s. d.
$\frac{1}{6}$ | 184 – 10

£ 30 14 2, answer.

13s. 4d. + 10d. = 14s. 2d.

13. 757 yds. ditto, at $3\frac{3}{4}$d.?

$\frac{1}{8}$ | 757
s. d.
$\frac{1}{8}$ | 94 1 $6\frac{3}{4}$

£ 11 16 $6\frac{3}{4}$, ans.

15s. + 1s. $6\frac{3}{4}$d. = 16s. $6\frac{3}{4}$d.

14. 1511 yards ditto, at 5d.?

$\frac{1}{6}$ | 1511
s. d.
$\frac{1}{8}$ | 251 2 1

£ 31 0 7, answer.

7s. 6d. + 2s. 1d. = 9s. 7d.

Questions in practice admit of various ways of working, equally short, which serve as a proof to each other; and indeed practice is best proved by practice, though it may be proved by several other methods.

15. What cost 1511 yards of ribbon, at 6d. per yard?

$\frac{1}{4}$ | 1511
s. d.
$\frac{1}{10}$ | 377 1 6

£ 37 15 6. answer.

Or $\frac{1}{2}$ | 1511
s. d.
$\frac{1}{20}$ | 755 6 –

£ 37 15 6, answer.

$\frac{1}{40}$ | 1511
s. d.
£ 37 15 6

Remains 31 sixpences = 15s. 6d.

16. What

16. What cost 1109 yards of small ribbon, at 4d. per yard?

$\frac{1}{3}$ | 1109
 s. d.
$\frac{1}{12}$ | 221 1 4

£ 18 9 8, answer.

$\frac{1}{2}$ | 1109
 d.
$\frac{1}{20}$ | 369 8

£ 18 9 8, answer.

$\frac{1}{60}$ | 1109
 s. d.
£ 18 9 8, answer.

Remains 29 groats, or 4 times 2s. 5d.

17. What cost 751 yards ditto, at 3d. per yard?

$\frac{1}{4}$ | 751
 d.
$\frac{1}{20}$ | 187 9

£ 9 7 9, answer.

$\frac{1}{80}$ | 751
 d.
£ 9 9, answer.

Remains 31 threepences = 7s. 9d.

18. What cost 1511 yards of ditto, at 2d. per yard?

$\frac{1}{6}$ | 1511
 d.
$\frac{1}{20}$ | 251 10

£ 12 11 10

$\frac{1}{120}$ | 1511
 s. d.
12 11 10

Remains 31 twopences = 11s. 10d.

19. 1173 yards of ferreting, at $1\frac{1}{2}$d.?

$\frac{1}{8}$ | 1173
 d.
$\frac{1}{20}$ | 146 $7\frac{1}{2}$

£ 7 6 $7\frac{1}{2}$

21. 1071 yards of filleting, at $\frac{3}{4}$ f.?

$\frac{1}{4}$ | 1071
 d.
$\frac{1}{4}$ | 267 $2\frac{1}{4}$

$\frac{1}{20}$ | 66 $11\frac{1}{4}$

£ 3 6 $11\frac{1}{4}$, answer

20. 713 yards ditto, at 1d?

$\frac{1}{12}$ | 713
s. d.
$\frac{1}{20}$ | 59 5 1

£ 2 19 5, answer.

22. 1075 yards ditto, at $\frac{1}{2}$?

$\frac{1}{2}$ | 1075
$\frac{1}{12}$ | 537$\frac{1}{2}$
s. d.
$\frac{1}{20}$ | 44 - 9$\frac{1}{2}$

£ 2 4 9$\frac{1}{2}$

23. 717 yards, at $\frac{1}{4}$ d. (
12)
4)717)179$\frac{1}{4}$

14s. 11 $\frac{1}{4}$d. answer.

CASE II.

When the numerator of the fraction is more than unity, and the denominator 10.

RULE.

Multiply the given quantity by the numerator, and double the figure in the units place of the product for shillings; the figures to the left-hand will be pounds.

1. What cost 757 ells of fine Holland, at 18s. per ell?

757
9
s.
£ 681 6, answer.

2. 617 ells ditto, at 16s.?

617
8
s.
£ 493 12

3. 577 ells ditto, 14 s.?

577
7
s.
£ 403 18

717 ells of long-lawn, at 8s. per ell?

717
4
s.
£ 286 16, answer.

933 ells ditto, at 6s.?

933
3
s.
£ 279 18, answer.

714 ells ditto, at 4s.?

714
2
s.
£ 142 16, answer.

4. 1187 ells ditto, at 12s.?

```
 |1187
 |   6
 |----- s.
£ 712  4, answer.
```

933 ells ditto, at 2 s.?

£ 93 6s. answer.

CASE III.

When the numerator is any number under 12, and the denominator 12, or under, with or without a cypher annexed.

RULE.

Multiply the given quantity by the numerator, and divide the product by the denominator; the quotient will be pounds or shillings, according to the nature of the question; and the remainder so many times the aliquot part expressed by the denominator.

1. What cost 737 yards of broad cloth, at 18 s. 4 d.?

```
  |737
  |  11
  |----
12|8107
  |----  s.  d.
£  675  11   8, answer.
```

Rem. 7 times 1 s. 8 d. = 11 s. 8 d.

2. 371 yards ditto, at 17 s. 6 d.?

```
 | 371
 |   7
 |----
8|2597
 |----  s.  d.
£ 324  12   6, answer.
```

Rem. 5 times 2 s. = 6 d. 12 s. 6 d.

3. 931 yards ditto, at 16 s. 8 d.?

```
 | 931
 |   5
 |----
6|4655
 |----  s.  d.
£ 775  16   8, answer.
```

Rem. 5 times 3 s. 4 d. = 16 s. 8 d.

4. 573 yards ditto, at 16 s.?

```
 | 573
 |   4
 |----
5|2292
 |----  s.
£ 458   8, answer.
```

Rem. 2 times 4 s. = 8 s.

5. 746 yards ditto at 15 s.?

```
  | 746
  |   3
  |----
 4|2238
  |----  s.  d.
£ 559  10  –     answer.
```

6. 865 yards ditto, at 13 s. 4 d.?

```
  | 865
  |   2
  |----
 3|1730
  |----  s.  d.
£ 576  13  4, answer.
```

Rem. 2 times 6 s. 8 d. = 13 s. 4 d.

7. 713 yards ditto, at 12 s. 6 d.?

```
  | 713
  |   5
  |----
 8|3565
  |----  s.  d.
£ 445  12  6  answer.
```

Rem. 5 times 2 s. 6 d. = 12 s. 6 d.

8. 783 yards ditto, at 12 s.?

```
  | 783
  |   3
  |----
 5|2349
  |----  s.
£ 469  16, answer.
```

Rem. 4 times 4 s. = 16 s.

9. What cost 137 ells of Holland, at 11 s. 8 d.

```
   | 137
   |   7
   |----
 12| 959
   |----  s.  d.
£  79  18  4, answer.
```

Rem. 11 times 1 s. 8 d. = 18 s. 4 d.

10. 537 ells ditto, at 8 s. 4 d.?

```
   | 537
   |   5
   |----
 12|2685
   |----  s.
£ 223  15, answer.
```

Rem. 9 times 1 s. 8 d. = 15 s.

11. 537 ells ditto, at 8 s.?

```
  | 537
  |   2
  |----
 5|1074
  |----  s.  d.
£ 214  16  –, answer.
```

Rem. 4 times 4 s. = 16 s.

12. 719 ells ditto, at 7 s. 6 d.?

```
  | 719
  |   3
  |----
 8|2157
  |----  s.  d.
£ 269  12  6, answer.
```

Rem. 5 times 2 s. 6 d. = 12 s. 6 d.

13. 157

13. What cost 157 ells of Holland, at 7s. 4d.?

```
   | 157
   |  11
   |-----
 30|1727
   |-----  s.  d.
 £   57  11   4, answer.
```

Rem. 17 times 8d. = 11s. 4d.

14. 737lb. of bohea tea, at 5s. 6d.?

```
   | 737
   |  11
   |-----
 40|8107
   |-----  s.  d.
 £  202  13   6, answer.
```

Rem. 27 times 6d. = 13s. 6d.

15. What cost 871 ounces of plate, at 5s. 4d.?

```
   | 871
   |   8
   |-----
 30|6968
   |-----  s.  d.
 £  232   5   4, answer.
```

Rem. 8 times 8d. = 5s. 4d.

16. 837 yards of kersey, at 4s. 8d.?

```
   | 837
   |   7
   |-----
 30|5859
   |-----  s.
 £  195   6, answer.
```

17. 713 yards ditto, at 4s. 6d.?

```
   | 713
   |   9
   |-----
 40|6417
   |-----  s.  d.
 £  160   8   6, answer.
```

Rem. 17 times 6d. = 8s. 6d.

18. 379 yards ditto, at 3s. 8d.?

```
   | 379
   |  11
   |-----
 60|4169
   |-----  s.  d.
 £   69   9   8, answer.
```

Rem. 29 times 4d. = 9s. 8d.

19. 719 yards ditto, at 3s.?

```
   | 719
   |   3
   |-----
 20|2157
   |-----  s.  d.
 £  107  17   –, answer.
```

20. 173 yards ditto, at 2s. 9d.?

```
   | 173
   |  11
   |-----
 80|1903
   |-----  s.  d.
 £   23  15   9, answer.
```

Rem. 63 times 3d. = 15s. 9d.?

I 21. What

21. What cost 1735 yards of Kersey, at 3s. 6d.?

	1735
	7
40	12145
	s. d.

£ 303 12 6, answer.

Rem. 25 times 6d. = 12s. 6d.

22. 931 yards ditto, at 2s. 8d.?

	931
	4
30	3724
	s. d.

£ 124 2 8, answer,

23. 107 yards ditto, at 2s. 4d.?

	107
	7
60	749
	s. d.

£ 12 9 8, answer.

Rem. 29 times 4d. = 9s. 8d.

24. 713 yards ditto, at 2s. 3d.?

	713
	9
80	6417
	s. d.

£ 80 4 3, answer.

Rem. 17 times 3d. = 4s. 3d.

25. 795 yards ditto, at 1s. 10d.?

	795
	11
120	8745
	s. d.

£ 72 17 6, answer.

Rem. 105 times 2d. = 17s. 6d.

26 What cost 173 yards of dowlas, at 1s. 9d.?

	173
	7
80	1211
	s. d.

£ 15 2 9, answer.

Rem. 11 times 3d. = 2s. 9d.

27. 713 yards ditto, at 1s. 6d.?

	713
	3
40	2139
	s. d.

£ 53 9 6, answer.

28. 913 yards ditto, at 1s. 4d.?

	913
	4
60	3652
	s. d.

£ 60 17 4, answer.

Rem. 52 times 4d. = 17s. 4d.

29. What

29. What cost 783 yards of dowlas at 1s. 3d.?

| 783
| 5
80 | 3915
s. d.
£ 48 18 9, answer.

Rem. 75 times 3d. = 18s. 9d.

30. 713 yards ditto, at 1 s. 2 d.?

| 713
| 7
120 | 4991
s. d.
£ 41 11 10, answer.

Rem. 71 times 2d. = 11s. 10d.

31. What cost 737 lb. of tobacco, at $10\frac{1}{2}$d.?

| 737
| 7
8 | 5159
d.
20 | 644 $10\frac{1}{2}$
s. d.
£ 32 4 $10\frac{1}{2}$, answer.

32. 673 lb. ditto, at 10 d.?

| 673
| 5
120 | 3365
s. d.
£ 28 – 10, answer.

Or 673
| 5
6 | 3365
s. d.
20 | 560 – 10
£ 28 – 10, answer.

33. 971 lb. ditto, at 9d.?

| 971
| 3
80 | 2913
s. d.
£ 36 8 3, answer.

Or 971
| 3
4 | 2913
d.
20 | 728 3
£ 36 8 3

34. 1713 yards of ribbon, at $7\frac{1}{2}$d.?

| 1713
| 5
8 | 8565
d.
20 | 1070 $7\frac{1}{2}$
s. d.
£ 53 10 $7\frac{1}{2}$, answer.

35. What cost 1735 yards of ribbon, at 8d.?

| 1735
| 2
3 | 3470
d.
20 | 1156 8
s. d.
£ 57 16 8, answer.

Or 30 | 1735
s. d.
£ 57 16 8, answer.

Rem. 25 times 8d. = 16s. 8d.

 36. What

36. What cost 753 yards of ribbon, at $4\frac{1}{2}$d.?

	753
	3
8	2259
20	282 $4\frac{1}{2}$ d.
£	14 2 s. $4\frac{1}{2}$ d., answer.

37. 783 yards ditto, at $5\frac{1}{4}$d.?

	783
	7
8	5481
$\frac{1}{2}$	685 $1\frac{1}{2}$ d.
20	342 $6\frac{3}{4}$ d.
£	17 2 s. $6\frac{3}{4}$ d., answer.

38. 575 yards ditto, at $3\frac{3}{4}$d.?

	575
	5
8	2875
$\frac{1}{2}$	359 $4\frac{1}{2}$ d.
20	179 $8\frac{1}{4}$ d.
£	8 19 s. $8\frac{1}{4}$ d., answer.

CASE IV.

When the price is less than a pound, or a shilling, by a single aliquot part of either,

RULE,

Take that aliquot part of the quantity, which subtract from the quantity; the remainder will be the price of the whole in pounds or shillings.

1. What cost 787 yards of velvet, at 19s. per yard.

$\frac{1}{20}$	787
	39 7
£	747 13, answer.

2. What cost 1135 yards ditto, at 18s. 4d.?

$\frac{1}{12}$	1135
	94 11 8
£	1040 8 4, answer.

3. What

3. What cost 785 yards of velvet, at 17 s. 6 d.

$\frac{1}{8}$ | 785
| 98 2 6

£ 686 17 6, answer.

4. 937 yds. ditto, at 18s. ?

$\frac{1}{10}$ | 937
| 93 14

£ 843 6, answer:

5. 731 yards ditto, at 16 s. 8 d. ?

$\frac{1}{6}$ | 731
| 121 16 8

£ 609 3 4, answer.

6. 573 yds. ditto, at 16 s. ?

$\frac{1}{5}$ | 573
| 114 12

£ 458 8, answer,

7. 1537 yds. ditto, at 15s.?

$\frac{1}{4}$ | 1537
| 384 5

£ 1152 15, answer.

8. 536 yards ditto, at 13 s. 4 d.?

$\frac{1}{3}$ | 536
| 178 13 4

£ 357 6 8, answer.

9. 1511 yards dito, at 19s. 10 d.?

$\frac{1}{120}$ | 1511
| 12 11 10

£ 1488 8 2, answer.

10. 1109 yards ditto, at 19 s. 9 d. ?

$\frac{1}{80}$ | 1109
| 13 17 3

£ 1095 2 9, answer.

11. 1109 yards ditto, at 19 s. 8 d.?

$\frac{1}{60}$ | 1109
| 18 9 8

£ 1090 10 4, answer.

12. 1511 yards ditto, at 19s. 6d.?

$\frac{1}{40}$ | 1511
| 37 15 6

£ 1473 4 6, answer.

13. 737 yards ditto, at 19 s. 4 d.?

$\frac{1}{30}$ | 737
| 24 11 4

£ 712 8 8, answer.

14. What cost 1736 yards of ribbon, at 8 d.?

$\frac{1}{3}$	1736	
	578	8
$\frac{1}{20}$	1157	4
£	57 17	4, answer.

15. 737 lb. of tobacco, at 11 d.?

$\frac{1}{12}$	737	
	61	5
$\frac{1}{20}$	675	7
£	33 15	7, answer.

16. 1371 lb. ditto, at $10\frac{1}{2}$d.?

$\frac{1}{8}$	1371	
	171	$4\frac{1}{2}$
$\frac{1}{20}$	1199	$7\frac{1}{2}$
£	59 19	$7\frac{1}{2}$, answer.

17. 783 lb. ditto, at 10 d.?

$\frac{1}{6}$	783	
	130	6
$\frac{1}{20}$	652	6
£	32 12	6, answer.

18. 1173 lb. ditto, at 9 d.?

$\frac{1}{4}$	1173	
	293	3
$\frac{1}{20}$	879	9
£	43 19	9, answer.

CASE V.

When the price is not an aliquot part of a pound, or of a shilling, but may be divided into such.

RULE.

Find two or more numbers, which are aliquot parts, whose sum makes the given price; proceed with them as before directed; then add the quotients together, which sum will be the answer.

1. What

1. What cost 731 yards of broad-cloth, at 18s. 4d.?

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$ | 731
365 10
182 15
121 16 8
£ 670 1 8, ans.

Or,

$\frac{9}{10}+\frac{1}{60}$ | 731
657 18
12 3 8
£ 670 1 8, answer.

2. 957 yards ditto, 17s. at 8d.?

$\frac{1}{2}+\frac{1}{3}+\frac{1}{20}$ | 957
478 10
319 –
47 17
£ 845 7, answer.

Or,

$\frac{8}{10}+\frac{1}{12}$ | 957
765 12
79 15
£ 845 7, answer.

3. 107 yards ditto, at 16s. 4d.?

$\frac{8}{10}+\frac{1}{60}$ | 107
85 12
1 15 8
£ 87 7 8, answer.

4. 377 yards ditto, at 15s. 4d.?

$\frac{6}{10}+\frac{1}{6}$ | 377
226 4
62 16 8
£ 289 – 8, answer.

5. 317 yards ditto, at 14s. 3d.?

$\frac{7}{10}+\frac{1}{80}$ | 317
221 18
3 19 3
£ 225 17 3, answer.

6. 101 yards ditto, at 12s. 2d.?

$\frac{6}{10}+\frac{1}{120}$ | 101
60 12
– 16 10
£ 61 8 10, answ.

7. What cost 713 gallons of rum, at 11s. 4d. per gallon?

$\frac{4}{10}+\frac{1}{6}$ | 713
285 4
118 16 8
£ 404 –– 8, answer.

8. 571 gallons ditto, at 10s. 8d.?

$\frac{1}{2}+\frac{1}{30}$ | 571
285 10
19 – 8
£ 304 10 8, answer.

9. What cost 109 gallons of rum, at 9s. 8d. per gallon?

$\frac{4}{10}+\frac{1}{12}$	109		
	43	12	
	9	1	8
£	52	13	8, answer.

10. 137 gallons ditto, at 9s. 2d.?

$\frac{1}{3}+\frac{1}{8}$	137		
	45	13	4
	17	2	6
£	62	15	10, answer.

11. 719 gallons ditto, at 8s 3d.?

$\frac{4}{10}+\frac{1}{80}$	719		
	287	12	
	8	19	9
£	296	11	9, answer.

12. 473 yards of Holland, at 8s. 4d.?

$\frac{1}{4}+\frac{1}{6}$	473		
	118	5	
	78	16	8
£	197	1	8, answer.

13. 157 yards ditto, at 7s. 4d.?

$\frac{1}{3}+\frac{1}{30}$	157		
	52	6	8
	5	4	8
£	57	11	4, answer.

14. 703 ells ditto, at 6s. 11d. per ell.?

$\frac{1}{3}+\frac{1}{80}$	703		
	234	6	8
	8	15	9
£	243	2	5, answer.

15. 959 ells ditto, at 5s, 10d.?

$\frac{1}{6}+\frac{1}{8}$	959		
	159	16	8
	119	17	6
£	279	14	2, answer.

16. 371 ells ditto, at 4s. 2d.?

$\frac{1}{5}+\frac{1}{120}$	371		
	74	4	
	3	1	10
£	77	5	10, answer.

17. 873 yards of kersey, at 3s. 10d. per yard?

$\frac{1}{6}+\frac{1}{40}$	873		
	145	10	
	21	16	6
£	167	6	6, answer.

18. 379 yards ditto, at 3s. 8d.?

$\frac{1}{6}+\frac{1}{60}$	379		
	63	3	4
	6	6	4
£	69	9	8, answer.

19. What

19. What cost 891 yards of kersey, at 3s. 7d. per yard?

$\frac{1}{6} + \frac{1}{80}$	891		
	148	10	
	11	2	9
£	159	12	9, answer.

20. 1735 yards ditto, at 3s. 6d.?

$\frac{1}{8} + \frac{1}{20}$	1735		
	216	17	6
	86	15	-
£	303	12	6, answer.

21. 907 yards ditto, at 3s. 2d.?

$\frac{1}{8} + \frac{1}{30}$	907		
	113	7	6
	30	4	8
£	143	12	2, answer.

22. 719 yards ditto, at 3s.?

$\frac{1}{10} + \frac{1}{20}$	719	
	71	18
	35	19
£	107	17, answer.

23. 873 yards ditto, at 2s. 10d.?

$\frac{1}{8} + \frac{1}{60}$	873		
	109	2	6
	14	11	-
£	123	13	6, answer.

24. 173 yards ditto, at 2s. 9d.?

$\frac{1}{8} + \frac{1}{80}$	173		
	21	12	6
	2	3	3
£	23	15	9, answer.

25. 931 yards ditto, at 2s. 8d.?

$\frac{1}{8} + \frac{1}{120}$	931		
	116	7	6
	7	15	2
£	124	2	8, answer.

26. 107 yards ditto, at 2s. 4d.?

$\frac{1}{10} + \frac{1}{60}$	107		
	10	14	
	1	15	8
£	12	9	8, answer.

27. 735 yards of Irish cloth, at 2s. 3d.?

$\frac{1}{10} + \frac{1}{80}$	735		
	73	10	
	9	3	9
£	82	13	9, answer.

28. 317 yards ditto, at 2s. 2d.?

$\frac{1}{10} + \frac{1}{120}$	317		
	31	14	
	2	12	10
£	34	6	10, answ.

29. What

29. What cost 137 yards of Irish cloth, at 1s. 11d.?

$\frac{1}{12} + \frac{1}{80}$ | 137

11	8	4
1	14	3
£ 13	2	7, answ.

30. 795 yards ditto, at 1s. 10d.

$\frac{1}{12} + \frac{1}{120}$ | 795

66	5	
6	12	6
£ 72	17	6, answ.

31. 713 yards ditto, at 1 s. 6d.?

$\frac{1}{20} + \frac{1}{40}$ | 713

35	13	
17	16	6
£ 53	9	6, answ.

32. 913 yards ditto, at 1 s. 4d.?

$\frac{1}{20} + \frac{1}{60}$ | 913

45	13	
15	4	4
£ 60	17	4, answ.

33. 873 yards of dowlas, at 1 s. 3d.?

$\frac{1}{20} + \frac{1}{80}$ | 873

43	13	
10	18	3
£ 54	11	3, answer.

34. 713 yards ditto, at 1 s. 2 d.?

$\frac{1}{20} + \frac{1}{120}$ | 713

35	13	
5	18	10
£ 41	11	10, answ.

35. 757 yards ditto, at $11\frac{1}{2}$ d.?

$\frac{1}{2} + \frac{1}{3} + \frac{1}{8}$ | 757

	378	6
	252	4
	94	$7\frac{1}{2}$
$\frac{1}{20}$	725	$5\frac{1}{2}$
	£ 36 –	$5\frac{1}{2}$, answ.

36. 371 yards ditto, at $9\frac{1}{2}$ d.?

$\frac{1}{2} + \frac{1}{6} + \frac{1}{8}$ | 371

	185	6
	61	10
	46	$4\frac{1}{2}$
$\frac{1}{20}$	293	$8\frac{1}{2}$
	£ 14 13	$8\frac{1}{2}$, ans.

37. 1713 yards of ribbon, at $8\frac{1}{2}$ d. per yard?

$\frac{1}{2} + \frac{1}{8} + \frac{1}{12}$ | 1713

	856	6
	214	$1\frac{1}{2}$
	142	9
$\frac{1}{20}$	1213	$4\frac{1}{2}$
	£ 60 13	$4\frac{1}{2}$

38. What

38. What coſt 587 yards of ribbon, at 7d. per yard?

$\frac{1}{3}+\frac{1}{4}$	587	
	195	8
	146	9
$\frac{1}{20}$	342	5

£ 17 2 5, anſw.

39. 713 yards ditto, at $6\frac{1}{2}$d?

$\frac{1}{4}+\frac{1}{6}+\frac{1}{8}$	713	
	178	3
	118	10
	89	$1\frac{1}{2}$
$\frac{1}{20}$	386	$2\frac{1}{2}$

£ 19 6 $2\frac{1}{2}$, anſ.

40. 731 yards ditto, at $5\frac{1}{2}$d.?

$\frac{1}{3}+\frac{1}{8}$	731	
	243	8
	91	$4\frac{1}{2}$
$\frac{1}{20}$	335	$-\frac{1}{2}$

£ 16 15 $-\frac{1}{2}$, anſwer.

41. 373 yards ditto, at 5d.?

$\frac{1}{4}+\frac{1}{6}$	373	
	93	3
	62	2
$\frac{1}{20}$	155	5

£ 7 15 5, anſwer.

42. 715 yards ditto, at $3\frac{1}{2}$d.?

$\frac{1}{6}+\frac{1}{8}$	715	
	119	2
	89	$4\frac{1}{2}$
$\frac{1}{20}$	208	$6\frac{1}{2}$

£ 10 8 $6\frac{1}{2}$, anſwer.

43. 757 yards ditto, at $2\frac{1}{2}$d.?

$\frac{1}{8}+\frac{1}{12}$	757	
	94	$7\frac{1}{2}$
	63	1
$\frac{1}{20}$	157	$8\frac{1}{2}$

£ 7 17 $8\frac{1}{2}$, anſwer.

CASE VI.

When the price of the integer is a farthing, or farthings joined with pence, or with ſhillings and pence,

RULE,

Work for the ſhillings and pence, as before directed; then obſerve what part of any of the foregoing lines the farthing or farthings are, which take, and then add all together.

1. What

1. What cost 715 yards of tape, at 1¼d. per yard?

1/12	715	
1/4	59	7
	14	10¾
1/20	74	5¾

£ 3 14 5¾, answer.

2. 495 yards ditto, at 1¾d.?

1/8	495	
1/6	61	10½
	10	3¾
1/20	72	2¼

£ 3 12 2¼, answer.

3. 351 yards of small ribbon, at 2¼d.?

1/6	351	
1/8	58	6
	7	3¾
1/20	65	9¾

£ 3 5 9¾, answer.

4. 741 yards ditto, at 2¾d.?

1/4	741	
1/12	185	3
Subt.	15	5¼
1/20	169	9¾

£ 8 9 9¾, answer.

5. 243 yards ditto, at 3¼d.?

1/4	243	
1/12	60	9
	5	-¾
1/20	65	9¾

£ 3 5 9¾, answer.

6. 747 yards ditto, at 3¾d.?

1/4	747	
1/4	186	9
	46	8¼
1/20	233	5¼

£ 11 13 5¼, answer.

7. 714 yards ditto, at 4¼d.?

1/4	714	
1/3	178	6
1/4	59	6
	14	10½
1/20	252	10½

£ 12 12 10½, answer.

8. 291 yards ditto, at 4¾d.?

1/4	291	
1/2	72	9
1/6	36	4½
	6	-¾
1/20	115	2¼

£ 5 15 2¼, answer.

9. What

9. What cost 747 yards of ribbon, at $5\frac{1}{4}$d. per yard?

$\frac{1}{3}$	747	
$\frac{1}{4}$	249	
$\frac{1}{4}$	62	3
	15	$6\frac{3}{4}$
$\frac{1}{20}$	326	$9\frac{3}{4}$

£ 16 16 $9\frac{3}{4}$ answer.

10. 210 yds. ditto, at $5\frac{3}{4}$d.?

$\frac{1}{4}+\frac{1}{8}$	210	
$\frac{1}{4}$	52	6
	35	–
	13	$1\frac{1}{2}$
$\frac{1}{20}$	100	$7\frac{1}{2}$

£ 5 – $7\frac{1}{2}$ answer.

11. 737 yds. ditto, at $6\frac{1}{4}$d.?

$\frac{1}{3}$	737	
$\frac{1}{2}$	245	8
$\frac{1}{8}$	122	10
	15	$4\frac{1}{4}$
$\frac{1}{20}$	383	$10\frac{1}{4}$

£ 19 3 $10\frac{1}{4}$ answer.

12. 1173 yards ditto, at $6\frac{3}{4}$d.?

$\frac{1}{2}$	1173	
$\frac{1}{8}$	586	6
	73	$3\frac{3}{4}$
$\frac{1}{20}$	659	$9\frac{3}{4}$

£ 32 19 $9\frac{3}{4}$, answer.

13. 1131 yards ditto, at $7\frac{1}{4}$d.?

$\frac{1}{2}$	1131	
$\frac{1}{6}$	565	6
$\frac{1}{4}$	94	3
	23	$6\frac{3}{4}$
$\frac{1}{20}$	683	$3\frac{3}{4}$

£ 34 3 $3\frac{3}{4}$, answer.

14. 711 yds. ditto, at $7\frac{3}{4}$d.?

$\frac{1}{3}+\frac{1}{4}$	711	
	237	
$\frac{1}{4}$	177	9
	44	$5\frac{1}{4}$
$\frac{1}{20}$	459	$2\frac{1}{4}$

£ 22 19 $2\frac{1}{4}$, answer.

15. 495 yds. ditto, at $8\frac{1}{4}$d.?

$\frac{1}{2}$	495	
$\frac{1}{3}$	247	6
$\frac{1}{8}$	82	6
	10	$3\frac{3}{4}$
$\frac{1}{20}$	340	$3\frac{3}{4}$

£ 17 – $3\frac{3}{4}$, answer.

16. 1157 yards ditto, at $8\frac{3}{4}$d.?

$\frac{1}{2}$	1157	
$\frac{1}{3}+\frac{1}{8}$	578	6
	192	10
	72	$3\frac{3}{4}$
$\frac{1}{20}$	843	$7\frac{1}{4}$

£ 42 3 $7\frac{1}{4}$, answ.

17. What

17. What cost 527 yards of ribbon, at $9\frac{1}{4}$d.?

	s.	d.
$\frac{1}{2}$	527	
$\frac{1}{2}$	263	6
$\frac{1}{12}$	131	9
	10	$11\frac{1}{4}$
$\frac{1}{20}$	406	$2\frac{3}{4}$

£ 20 6 $2\frac{3}{4}$ answer.

18. 715 yds. ditto, at $9\frac{3}{4}$d.?

	s.	d.
$\frac{1}{2}$	715	
$\frac{1}{2}$	357	6
$\frac{1}{4}$	178	9
	44	$8\frac{1}{4}$
$\frac{1}{20}$	580	$11\frac{1}{4}$

£ 29 – $11\frac{1}{4}$, answer.

19. 785 yds. ditto, at $10\frac{1}{4}$d.?

	s.	d.
$\frac{1}{2}$	785	
$\frac{1}{2}$	392	6
$\frac{1}{3}$	196	3
$\frac{1}{4}$	65	5
	16	$4\frac{1}{4}$
$\frac{1}{20}$	670	$6\frac{1}{4}$

£ 33 10 $6\frac{1}{4}$, answer.

20. 9113 yards ditto, at $10\frac{3}{4}$d.?

	s.	d.
$\frac{1}{2}+\frac{1}{3}$	9113	
$\frac{1}{8}$	4556	6
	3037	8
	569	$6\frac{3}{4}$
$\frac{1}{20}$	8163	$8\frac{3}{4}$

£ 408 3 $8\frac{3}{4}$, answ.

21. 415 yds. ditto, at $11\frac{1}{4}$d.?

	s.	d.
$\frac{1}{2}+\frac{1}{3}$	415	
	207	6
$\frac{1}{4}$	138	4
$\frac{1}{4}$	34	7
	8	$7\frac{3}{4}$
$\frac{1}{20}$	389	$-\frac{3}{4}$

£ 19 9 $-\frac{3}{4}$, answ.

22. 797 yds. ditto, at $11\frac{3}{4}$d.?

	s.	d.
$\frac{1}{3}+\frac{1}{2}+\frac{1}{8}$	797	
	398	6
	265	8
$\frac{1}{6}$	99	$7\frac{1}{2}$
	16	$7\frac{1}{4}$
$\frac{1}{20}$	780	$4\frac{3}{4}$

£ 39 – $4\frac{3}{4}$, ans.

23. 371 lb. of tobacco, at 1s. $-\frac{1}{4}$d.?

	£	s.	d.
$\frac{1}{40}$	371		
	9	5	6
$\frac{1}{2}$	4	12	9
$\frac{1}{12}$	4	12	9
		7	$8\frac{3}{4}$

£ 18 18 $8\frac{3}{4}$, answer.

24. 171 lb. ditto, at 1s. $-\frac{3}{4}$d.?

	£	s.	d.
$\frac{1}{40}$	171		
	4	5	6
$\frac{1}{8}$	4	5	6
		10	$8\frac{1}{4}$

£ 9 1 $8\frac{1}{4}$, answer.

25. What

25. What cost 907 lb. of tobacco, at 1 s. $1\frac{1}{4}$d.?

$\frac{1}{20}$	907		
$\frac{1}{12}$	45	7	
$\frac{1}{4}$	3	15	7
		18	$10\frac{3}{4}$
£	50	1	$5\frac{3}{4}$, answer.

26. 175 lb. ditto, at 1 s. $1\frac{3}{4}$d.?

$\frac{1}{20}$	175		
$\frac{1}{8}$	8	15	
$\frac{1}{6}$	1	1	$10\frac{1}{2}$
		3	$7\frac{3}{4}$
£	10	—	$6\frac{1}{4}$, answer.

27. 137 lb. ditto, at 1 s. $2\frac{1}{4}$d.?

$\frac{1}{20}$	137		
$\frac{1}{6}$	6	17	
$\frac{1}{8}$	1	2	10
		2	10
£	8	2	$8\frac{1}{4}$, answer.

28. 713 lb. ditto, at 1 s. $2\frac{3}{4}$d.?

$\frac{1}{30}+\frac{1}{40}$	713		
	23	15	4
$\frac{1}{8}$	17	16	6
	2	4	$6\frac{3}{4}$
£	43	16	$4\frac{3}{4}$, answ.

29. 9081 lb. ditto, at 1 s. $3\frac{1}{4}$d.?

$\frac{1}{20}$	9081		
$\frac{1}{4}$	454	1	
$\frac{1}{12}$	113	10	3
	9	9	$2\frac{1}{4}$
£	577	—	$5\frac{1}{4}$ answ.

30. 173 lb. ditto, at 1 s. $3\frac{3}{4}$ d.?

$\frac{1}{20}$	173		
$\frac{1}{4}$	8	13	
$\frac{1}{4}$	2	3	3
		10	$9\frac{3}{4}$
£	11	7	—$\frac{3}{4}$, answer.

31. 957 lb. ditto, at 1 s. $4\frac{1}{4}$d.?

$\frac{1}{20}$	957		
$\frac{1}{4}$	47	17	
$\frac{1}{3}$	11	19	3
$\frac{1}{4}$	3	19	9
		19	$11\frac{1}{4}$
£	64	15	$11\frac{1}{4}$, answer.

32. 875 ells of Irish cloth, at 1 s. $4\frac{3}{4}$d. per ell?

$\frac{1}{20}$	875		
$\frac{1}{4}$	43	15	
$\frac{1}{2}$	10	18	9
$\frac{1}{6}$	5	9	$4\frac{1}{2}$
		18	$2\frac{3}{4}$
£	61	1	$4\frac{1}{4}$, answer.

33. What

33. What cost 879 ells of Irish cloth, at 1 s. $5\frac{1}{4}$d. per ell?

$\frac{1}{20}$	879		
$\frac{1}{3}$	43	19	
$\frac{1}{4}$	14	13	
$\frac{1}{4}$	3	13	3
		18	$3\frac{3}{4}$
£	63	3	$6\frac{3}{4}$ answer.

34. 871 ells ditto, at 1 s. $5\frac{3}{4}$d.?

$\frac{1}{20}$	871		
$\frac{1}{2}+\frac{1}{8}$	43	11	
	14	10	4
$\frac{1}{6}$	5	8	$10\frac{1}{2}$
		18	$1\frac{3}{4}$
£	64	8	$4\frac{1}{4}$, answer.

35. 171 ells ditto, at 1 s. $6\frac{1}{4}$d.?

$\frac{1}{20}$	171		
$\frac{1}{3}$	8	11	
$\frac{1}{2}$	2	17	
$\frac{1}{8}$	1	8	6
		3	$6\frac{3}{4}$
£	13	–	–$\frac{3}{4}$, answer.

36. 137 ells ditto, at 1 s. $6\frac{3}{4}$d.?

$\frac{1}{20}$	137		
$\frac{1}{2}$	6	17	
$\frac{1}{8}$	3	8	6
		8	$6\frac{3}{4}$
£	10	14	–$\frac{3}{4}$, answer.

37. 875 ells ditto, at 1 s. $7\frac{1}{4}$d.?

$\frac{1}{20}$	875		
$\frac{1}{2}$	43	15	
$\frac{1}{6}$	21	17	6
$\frac{1}{4}$	3	12	11
		18	$2\frac{3}{4}$
£	70	3	$7\frac{3}{4}$ answer.

38. 173 ells ditto, at 1 s. $7\frac{3}{4}$d.?

$\frac{1}{20}$	173		
$\frac{1}{2}$	8	13	
$\frac{1}{4}$	4	6	6
$\frac{1}{6}$	1	1	$7\frac{1}{2}$
		3	$7\frac{1}{4}$
£	14	4	$8\frac{3}{4}$ answer.

39. 375 ells ditto, at 1 s. $8\frac{1}{4}$d.?

$\frac{1}{20}$	375		
$\frac{1}{2}$	18	15	
$\frac{1}{3}$	9	7	6
$\frac{1}{8}$	3	2	6
		7	$9\frac{3}{4}$
£	31	12	$9\frac{3}{4}$, answer.

40. 721 ells ditto, at 1 s. $8\frac{3}{4}$d.?

$\frac{1}{20}$	721		
$\frac{1}{2}$	36	1	
$\frac{1}{3}+\frac{1}{8}$	18	–	6
	6	–	2
	2	5	–$\frac{3}{4}$
£	62	6	$8\frac{3}{4}$, answer.

41. What

41. What cost 307 ells of Irish cloth, at 1 s. $9\frac{1}{4}$d. per ell?

$\frac{1}{20}$	307		
$\frac{1}{2}$	15	7	
$\frac{1}{2}$	7	13	6
$\frac{1}{4}$	3	16	9
		19	$2\frac{1}{4}$
£	27	16	$5\frac{1}{4}$, answer.

42. 317 ells ditto, at 1 s. $9\frac{1}{4}$d.?

$\frac{1}{20}$	317		
$\frac{1}{2}$	15	17	
$\frac{1}{2}$	7	18	6
$\frac{1}{12}$	3	19	3
		6	$7\frac{1}{4}$
£	28	1	$4\frac{1}{4}$, answer.

43. 107 ells ditto, at 1s. $10\frac{1}{4}$d.?

$\frac{1}{12}$	107		
$\frac{1}{10}$	8	18	4
$\frac{1}{8}$		17	10
		2	$2\frac{3}{4}$
£	9	18	$4\frac{3}{4}$, answer.

44. 199 ells ditto, at 1 s. $10\frac{3}{4}$d.

$\frac{1}{12}$	199		
$\frac{1}{8}$	16	11	8
$\frac{1}{10}$	2	1	$5\frac{1}{2}$
		4	$1\frac{3}{4}$
£	18	17	$3\frac{1}{4}$, answer.

45. 147 ells ditto, at 1 s. $11\frac{1}{4}$d.?

$\frac{1}{12}+\frac{1}{80}$	147		
	12	5	
$\frac{1}{12}$	1	16	9
		3	$-\frac{3}{4}$
£	14	4	$9\frac{3}{4}$, answ.

46. 175 ells ditto, at 1 s. $11\frac{3}{4}$d.?

$\frac{1}{12}+\frac{1}{80}$	175		
	14	11	8
$\frac{1}{4}$	2	3	9
		10	$11\frac{1}{4}$
£	17	6	$4\frac{1}{4}$, answ.

CASE VII.

When the integer is pounds, shillings, pence, and farthings,

RULE,

Multiply the given quantity by the pounds; and proceed with the shillings, pence, and farthings, as in the foregoing cases.

1. What cost 137 yards of brocade, at 1 l. 17 s. 6¼d. per yard?

$\frac{7}{10}+\frac{1}{6}+\frac{1}{120}$	137		
14 s. -d =	95	18	
3 4 =	22	16	8
2 ($\frac{1}{8}$) =	1	2	10
		2	10¼
Answer £	257	-	4¼

2. 2710 cwt. of sugar, at 2 l. 3 s. 7 d.?

$\frac{1}{6}+\frac{1}{80}$	2710		
	× 2		
	5420		
3s. 4d. =	451	13	4
3 ($\frac{1}{6}$) =	33	17	6
	5	12	11
Answer £	5911	3	9

3. 741 cwt. ditto, at 2 l. 13 s. 7¼d.?

$\frac{1}{10}$	741		
	× 2		
$\frac{1}{3}$	1482		
13s. 4d. =	494		
3 ($\frac{1}{12}$) =	9	5	3
		15	5¼
Answer	986	-	8¼

4. 947 cwt. of hops, at 4l. 15s. 10¼d.?

$\frac{7}{10}+\frac{1}{12}$	947		
	× 4		
	3788		
14s. -d. =	662	18	-
1 8 ($\frac{1}{10}$ =	78	18	4
$\frac{1}{8}$	7	17	10
		19	8¾
Answer £	4538	13	10¼

5. 457 last of colefeed, at 14l. 17s. 9½d. per last?

$\frac{7}{10}+\frac{1}{6}$	457		
	× 14		
	6398		
14 s. -d. =	319	18	
3 4 ($\frac{1}{8}$) =	76	3	4
$\frac{1}{10}$	9	10	5
		19	-½
Answer £	6804	10	9½

6. 375 cwt. of hops, at 3l. 7s. 11¼d.?

$\frac{1}{6}$	375		
	× 3		
$\frac{1}{8}$	1125		
7s. 6d. =	140	12	6
4 $\frac{1}{4}$	6	5	-
$\frac{1}{4}$	1	11	3
		7	9¾
Answer £	1273	1	6¾

CASE VIII.

When the given quantities are of feveral denominations,

RULE,

RULE,

Find the value of the integers, as in the foregoing cafes; and for the leffer denomination in the given quantity, if they are the aliquot part of an integer, divide the given price thereby; but if they are not aliquot parts, divide them into fuch, or of each other, as you can moft conveniently; then add all together, their fum will be the anfwer.

ALIQUOT PARTS *in* CLOTH MEASURE.

One yard the integer.

Qrs.	n.	
2	–	= $\frac{1}{2}$
1	–	= $\frac{1}{4}$
–	2	= $\frac{1}{8}$

One quarter the integer.

Nail.		
1	–	= $\frac{1}{4}$
1	–	= $\frac{1}{2}$

One ell Englifh integer.

Qrs.	n.	
2	2	= $\frac{1}{2}$
1	1	= $\frac{1}{4}$
1	–	= $\frac{1}{5}$
–	2	= $\frac{1}{10}$
–	1	= $\frac{1}{20}$

One French ell integer.

Qrs.	n.	
3	–	= $\frac{1}{2}$
2	–	= $\frac{1}{3}$
1	2	= $\frac{1}{4}$
1	–	= $\frac{1}{6}$
–	3	= $\frac{1}{8}$
–	2	= $\frac{1}{12}$

One Flemifh ell integer.

Qrs.	n.	
1	2	= $\frac{1}{2}$
1	–	= $\frac{1}{3}$
–	3	= $\frac{1}{4}$
–	2	= $\frac{1}{6}$
–	1	= $\frac{1}{12}$

1. 713 yds. 3 qrs. 2 n. of kerfey, at 7s. $9\frac{1}{2}$d. per yard?

$\frac{1}{3}+\frac{1}{20}$			713	3	3
6	8	=	237	13	4
1	–	=	35	13	–
–	$1\frac{1}{2}$	=	4	9	$1\frac{1}{2}$
Qrs.	2	=		3	$10\frac{3}{4}$
	1	=		1	$11\frac{1}{4}$
Nails	3	=		1	$5\frac{1}{2}$
Anfwer £			278	2	9

2. 17 ells E. – qrs. 2 n. of gold brocade, at 3l. 10s. 9d.?

$\frac{1}{2}$	17	–	2			
	3					
		s.	d.		s.	d.
	51	–	–			
$\frac{1}{20}$	8	10	–	=	10	–
		8	6	=	–	6
$\frac{1}{2}$		4	3	=	–	3
2 nails		7	$-\frac{3}{4}$	=	$\frac{1}{10}$ price.	
£	60	9	$9\frac{3}{4}$, anfwer.			

3. What cost 19 French ells, – qrs. 3 n. of Bruffels lace, at 3l. 19s. 11d?

	3	19	11
			3
	11	19	9
			6
	71	18	6
3 n. = $\frac{1}{8}$	3	19	11
		9	$11\frac{3}{4}$
£	76	8	$4\frac{3}{4}$, anfw.

4. What cost 71 French ells 1 qr. ditto, at 2l. 17s. $8\frac{1}{2}$d.?

	F. E.	qr.			s.	d.
$\frac{1}{2}+\frac{1}{3}+\frac{1}{20}$	71	1				
	2					
	142					
	35	13	–	=	10	–
1 qr.	23	13	4	=	6	8
	3	11	–	=	1	–
$\frac{1}{24}$	–	2	$11\frac{1}{2}$	=	–	$-\frac{1}{2}$
	–	9	$7\frac{1}{4}$	=	$\frac{1}{6}$ price	
£	205	6	$10\frac{3}{4}$.			

5. What cost 709 French ells, 5 qrs. 3 n. of ditto, at 14s. $7\frac{1}{4}$d.?

	F. E.	qr.	n.		s.	d.
$\frac{7}{10}+\frac{1}{40}$	709	5	3			
	496	6	–	=	14	–
$\frac{1}{6}$	17	14	6	=	–	6
$\frac{1}{4}$	2	19	1	=	–	1
	–	14	$9\frac{1}{4}$	=	–	$-\frac{1}{4}$
3 – =	–	7	$3\frac{1}{2}$	=	$\frac{1}{2}$ } p.	
2 – =	–	4	$10\frac{1}{4}$	=	$\frac{1}{3}$ }	
– 3 = $\frac{1}{4}$	–	1	$9\frac{3}{4}$			
£	518	8.	$3\frac{1}{4}$			

6. What cost 719 ells Flemish, 2 qrs. 3 n. of fine Holland, at 1 l. 10 s. $9\frac{1}{4}$d.?

	E. Flemish.				
$\frac{1}{2}+\frac{1}{30}$	719				
	359	10	–	=	10s.
$\frac{1}{8}$	23	19	4	=	– 8d.
$\frac{1}{4}$	2	19	11	=	– 1
qr. n.	–	14	$11\frac{3}{4}$	=	– –
1 2 =	–	15	$4\frac{1}{2}$	=	$\frac{1}{2}$ } p.
1 – $\frac{1}{4}$	–	10	3	=	$\frac{1}{3}$ }
– 1 =	–	2	$6\frac{3}{4}$	=	
£	1107	12	5, anfw.		

7. What cost 4 pieces of ribbon, each 17 yards, 1 qr. 3 nails, at 1s. $1\frac{1}{2}$d. per yard?

	Y.	qr.	n.	
	17	1	3	
			4	
$\frac{1}{20}$	69	3	–, at $13\frac{1}{2}$d.	
qrs. $\frac{1}{8}$	3	9	–	
2 – =	–	8	$7\frac{1}{2}$	
1 – =	–	–	$6\frac{3}{4}$	} $\frac{1}{2}$ } p.
	–	–	$3\frac{1}{4}$	} $\frac{1}{4}$ }
£	3	18	$5\frac{1}{2}$, anfwer.	

8. What cost 13 ells, 2 qrs. 2 n. of Holland, at 3s. $7\frac{1}{2}$d. per ell English?

$\frac{1}{2}$	–	3	$7\frac{1}{2}$			
		13				
	2	7	$1\frac{1}{2}$		qr.	n.
		1	$9\frac{3}{4}$	=	2	2
£	2	8	$11\frac{1}{4}$, anfwer.			

ALIQUOT

ALIQUOT PARTS *in* TROY WEIGHT.

One ounce the integer.

dwt.	gr.		
10	–	=	$\frac{1}{2}$
6	16	=	$\frac{1}{3}$
5	–	=	$\frac{1}{4}$
4	–	=	$\frac{1}{5}$
3	8	=	$\frac{1}{6}$
2	12	=	$\frac{1}{8}$
2	–	=	$\frac{1}{10}$
1	16	=	$\frac{1}{12}$
1	–	=	$\frac{1}{20}$

One pennyweight integer.

gr.		
12	=	$\frac{1}{2}$
8	=	$\frac{1}{3}$
6	=	$\frac{1}{4}$
4	=	$\frac{1}{6}$
3	=	$\frac{1}{8}$
2	–	$\frac{1}{12}$

N.B. 4l. per oz. is 2d. per grain.

9. A silver gilt punch bowl, weight 49 oz. 2 dwt. 12 gr. what comes it to at 8s. 11¾d. per ounce?

$\frac{4}{10} + \frac{1}{30} + \frac{1}{80}$

		oz.	dwt.	gr.	
		49	2	12	
		19	12	–	at 8s. –d.
		1	12	8	at – 8
dwt. gr.	¼	–	12	3	at – 3
2 12		–	3	–¾	
		–	1	1¼	= ⅛ price.
		£ 22	1	1,	answer.

10. A pair of chased silver salts, weight 7 oz. 5 dwt. at 8s. 9¾d. per ounce?

¼		8	9¾
			7
	3	1	8¼
		2	2¼
	£ 3	3	10½, answer.

11. I demand the value of a service of gold plate, weight 971 oz. 15 dwt. 16 gr. at 3l. 19s. 11¾d. per ounce?

		oz.	dwt.	gr.	
		971	15	16	
		3			
		2913			
		873	18		
		80	18	4	
	¼	12	2	9	
dwt.		3	–	8¼	
10	–	1	19	11¾	= ½ oz.
4	–		15	11¾	
1	16		6	7¾	
		£ 3886	2	4½	

ALIQUOT PARTS *in* AVERDUPOISE WEIGHT.

One tun the integer.

Cwt.	qr.	lb.	
10	–	–	= $\frac{1}{2}$
5	–	–	= $\frac{1}{4}$
4	–	–	= $\frac{1}{5}$
2	3	12	= $\frac{1}{7}$
2	2	–	= $\frac{1}{8}$
2	–	–	= $\frac{1}{10}$
1	–	–	= $\frac{1}{20}$

One hundred integer.

Qr.		lb.	
2	=	56	= $\frac{1}{2}$
1	=	28	= $\frac{1}{4}$
		16	= $\frac{1}{7}$
		14	= $\frac{1}{8}$

$\frac{1}{2}$ Cwt. = 56 lb. integer.

Qr.		lb.	
1	=	28	= $\frac{1}{2}$
		14	= $\frac{1}{4}$
		8	= $\frac{1}{7}$
		7	= $\frac{1}{8}$

$\frac{1}{4}$ Cwt. or 28 lb. integer.

	14	= $\frac{1}{2}$
	7	= $\frac{1}{4}$
	4	= $\frac{1}{7}$
2	8	= $\frac{1}{8}$

One pound the integer.

oz.	
8	= $\frac{1}{2}$
4	= $\frac{1}{4}$
2	= $\frac{1}{8}$

One ounce the integer.

dr.	
8	= $\frac{1}{2}$
4	= $\frac{1}{4}$
2	= $\frac{1}{8}$

12. What cost 73 cwt. 1 qr. of sugar, at 3 l. 15 s. 7d. per cwt.?

	Cwt.		
$\frac{6}{10} + \frac{1}{6} + \frac{1}{80}$.	73	1	
	3		
	219		
	43	16	
	12	3	4
	–	18	3
1 qr. =	–	18	$10\frac{3}{4}$ = $\frac{1}{4}$ p.
	£ 276	16	$5\frac{3}{4}$, answ.

13. What cost 731 cwt. 3 qrs. of hops, at 3l. 18s. $7\frac{1}{2}$d. per cwt.?

	Cwt.	qr.	
$\frac{9}{10} + \frac{1}{40}$	731	3	
	3		
	2193		
	657	18	
$\frac{1}{4}$	18	5	6
	4	11	$4\frac{1}{2}$
	1	19	$3\frac{3}{4}$ = $\frac{1}{2}$ } p.
	–	19	$7\frac{7}{8}$ = $\frac{1}{4}$ } p.
	£ 2870	13	10, answer.

14. What

14. What cost 7 cwt. 2 qrs. 10 lb. of treacle, at 1l. 17s. 8d. per cwt.?

	l.	s.	d.
$\frac{1}{2}$	1	17	8
			7
	13	3	8
$\frac{1}{7}$	–	18	10
$\frac{1}{4}$	–	2	$8\frac{1}{4}$
	–	–	8

£ 14 5 10^{1}, answer.

15. 17 cwt. 1 qr. 12 lb. at 1l. 19s. 8d. per cwt.?

$\frac{9}{10}+\frac{1}{12}$	17			
	5	6		
qr lb.	1	8	4	
1 —	–	9	11	= $\frac{1}{4}$ price.
7	–	2	5	= $\frac{1}{4}$ of last.
4	–	1	5	= $\frac{1}{7}$ of dit.
1	–	–	4	= $\frac{1}{4}$ of last.

£ 34 8 6, answer.

17. What cost 17 hogsheads of treacle, at 1l. 12s. 7d. per cwt. each hogshead weighing 5 cwt. 2 qrs. 8 lb.?

	Cwt.	qr.	lb.
	5	2	8
			3
	16	2	24
			6
	100	1	4
	5	2	8
$\frac{6}{10}+\frac{1}{4}$	94	1	24
	56	8	–
$\frac{1}{6}$	2	7	–
qr. lb.	–	7	10
2 –	–	16	$3\frac{1}{2}=\frac{1}{2}$
16	–	4	$7 = \frac{1}{7}$
8	–	2	$3\frac{3}{4}$

£ 154 6 1, answer.

16. What cost the freight of 7 ton, 13 cwt. 3 qrs. 19 lb. at 14l. 17s. 9d. per ton?

$\frac{1}{2}+\frac{1}{7}$	14	17	9
			7
	104	4	3
$\frac{1}{20}$	7	8	$10\frac{1}{2}$
	2	2	$6\frac{1}{4}$
		14	$10\frac{1}{2}$
			11 = $\frac{1}{2}$ of $\frac{1}{8}$ cwt.

£ 114 11 5, answer.

$\frac{1}{2}$	14	$0\frac{1}{2}$
$\frac{1}{8}$	7	$5\frac{1}{4}$
	–	11 d.

 18. What

18. What cost the freight of 37 tons, 19 cwt. 3 qrs. at 19l. 19s. 2d. per ton?

	T.	cwt.	qrs.	
	37	19	3	
	19			
	l.	s.	d.	
$\frac{1}{20}$	703	–	–	at 19 – – per ton.
	35	3	–	at – 19 –
cwt.	–	6	2	at – – 2
10 –	9	19	7	$=\frac{1}{2}$ } price.
5 –	4	19	$9\frac{1}{2}$	$=\frac{1}{4}$ } price.
$4=\frac{1}{8}$	3	19	10	$=\frac{1}{5}$ } price.
$\frac{1}{2}$		9	$11\frac{3}{4}$	
		4	$11\frac{3}{4}$	
	£ 758	3	4,	answer.

Aliquot Parts in Land Measure.

One acre the integer.

R.	P.	
2	–	$= \frac{1}{2}$
1	–	$= \frac{1}{4}$
–	32	$= \frac{1}{5}$
–	20	$= \frac{1}{8}$
–	16	$= \frac{1}{10}$

One rood the integer.

Poles.	
20	$= \frac{1}{2}$
10	$= \frac{1}{4}$
8	$= \frac{1}{5}$
5	$= \frac{1}{8}$
4	$= \frac{1}{10}$
2	$= \frac{1}{20}$

19. What is the rent of 713 acres, 3 roods, 39 perches of flax-land, at 3l. 17s. 6d. per acre?

		A.	R.	P.	
$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$		713	3	39	
		×3			
		2139			
		356	10	–	
		178	5	–	
R. P.		89	2	6	
2 –		1	18	9	$=\frac{1}{2}$ } price.
1 –		–	19	$4\frac{1}{2}$	$=\frac{1}{4}$ } price.
20	$\frac{1}{2}$	–	9	$8\frac{1}{4}$	$=\frac{1}{8}$ } price.
10	$\frac{1}{2}$	–	4	10	
5		–	2	5	
4		–	1	$11\frac{1}{4}$	
		£ 2766	14	6,	answer.

20. What

20. What is the rent of 17 acres, 3 roods, and 35 perches of flax-land, at 4 l. per acre?

	A.	R.	P.
	17	3	35
	4		
R. P.	68		
2 –	2	–	–
1 –	1	–	–
– 20	–	10	–
– 10	–	5	–
– 5	–	2	6
	£71	17	6, answer.

21. 17 acres, – roods, 10 perches, at 2 l. 13 s. 6d. per acre?

	A.	R.	P.
$\frac{6}{10} + \frac{1}{20}$	17	–	10
	2		
	34		
	10	4	
$\frac{1}{2}$	–	17	–
	–	8	6
Per. 10	–	3	4 = $\frac{1}{4}$ of $\frac{1}{4}$
	£45	12	10, answer.

N. B. This belongs to Example 21.

$\frac{1}{4}$	2	13	6
$\frac{1}{4}$		13	$4\frac{1}{2}$
		3	4

Captain JOHN ELFORD,

Bought of THOMAS CHEESEMONGER.

1773, September the 12th.		Cwt.	qr.	lb.		l.	s.	d.		l.	s.	d.
Old Cheshire cheeses, 12		5	2	24	at	1	17	4	per cwt.	10	13	4
Glocester cheeses, 45		4	2	10	at	1	12	6		7	9	1¼
Firkins of butter, 12		–	–	–	at	1	10	–	each	18	–	–
Stilton cheeses, 93		2	3	25	at	2	16	10	per cwt.	8	8	11¼
Flitches of bacon, 7	weight 49 stone				at	–	3	4	per stone	8	3	4
Suffolk butter, 12 weys					at	8	3	8	each	98	4	–
									£	150	18	8½

Mr. JONATHAN MARRIOT,

Bought of GEORGE GROCER and Company.

1773, August 25.	Cwt.	qrs	lb.		l.	s	d.		l.	s.	d.
Sugar, 2 hogsheads	17	2	17	at	1	13	10	per cwt.	29	17	2½
Raisins, 11 barrels	12	1	19	at	1	14	5		21	7	5
Tobacco, 1 hogshead	4	–	12	at	4	19	4		20	7	11½
Rice, 1 barrel	1	–	15	at	2	16	4		3	3	10½
Pepper, 1 bag	1	3	19	at	3	12	4		6	18	10¼
Brimstone	2	1	19	at	1	19	1		4	14	6¼
Bees-wax, 4 cakes	2	2	12	at	1	18	4		4	19	11¼
								£	19	9	9¾

Cwt.

Cwt. qr. lb. l. s. d.
5 2 24 at 1 17 4
5
9 6 8
— 18 8
$\frac{1}{7}$ — 5 4
— 2 8
£10 13 4

4 2 10 at 1 12 6
4
6 10 —
$\frac{1}{7}$ — 16 3
$\frac{1}{4}$ — 2 3¾
— — 6¾
£7 9 1½
$\frac{1}{7}$

2 3 25 at 2 16 10
2
5 13 8
1 8 5
$\frac{1}{4}$ — 14 2½
$\frac{1}{8}$ — 8 1¼
— 3 6½
— 1 —
£8 8 11¼

17 2 17 at 1 13 10
4
6 15 4
4
27 1 4
1 13 10
$\frac{1}{8}$, $\frac{1}{7}$ — 16 11
$\frac{1}{4}$ — 2 5
— 2 1¼
— — 7¼
£29 17 2½

Cwt. qr. lb. $\frac{1}{7}$ l. s. d.
12 1 19 at 1 14 5
12
20 13 —
— 8 7¼
$\frac{1}{8}$ — 4 11
$\frac{1}{2}$ — — 7¼
— — 3½
£21 7 5

4 — 12 at 4 19 4
4
19 17 4
$\frac{1}{7}$) 2 9 8½ — 7 1
3 6½
£20 7 11½

1 — 15 at 2 16 4
$\frac{1}{8}$, $\frac{1}{7}$) 1l. 8s. 2d. 4 —¼
— 3 6¼
£3 3 10½

1 3 19 at $\frac{1}{7}$ 3 12 4
1 16 2
— 18 1
$\frac{1}{8}$ — 10 4
$\frac{1}{2}$ — 1 3½
— — 7¾
£6 18 10¼

2 1 19 at $\frac{1}{4}$ 1 19 1
2
3 18 2
$\frac{1}{8}$, $\frac{1}{7}$) 19s. 6½d. — 9 9½
$\frac{1}{2}$ — 2 9½
— 2 5¼
— 1 4¾

2 2 12 at $\frac{1}{2}$ 1 18 4
2
3 16 8
$\frac{1}{7}$ — 19 2
— 2 8¾
— 1 4¼
£4 19 11

The

The Honourable Sir Michael Newton,

Bought of Thomas Goldsmith.

1773.			Oz.	dwts.	gr.		s.	d.		l.	s.	d.
May	31.	A silver set of castors - - - - - -	25	10	10	at	7	9	per ounce	9	17	9
July	7.	Half a dozen soup-plates - - - - -	85	14	15	at	6	6	- - -	27	17	2¾
	10.	A silver tea-pot and lamp - - - - -	29	16	15	at	6	4	- - -	9	8	10¾
August	9.	A large punch-bowl - - - - - -	67	—	16	at	6	10	- - -	22	18	-
Nov.	6.	A dozen of silver spoons - - - - -	33	11	10	at	6	2	- - -	10	6	11¼
	11.	A dozen of desert knives, forks, and spoons, with a shagreen case - -							- - -	40	-	-
									£	120	8	10¾

	oz.	dwts.	gr.	s.	d.		
	25	10	10 at	7	9	per ounce.	
			5		5		
			1	18	9		
					5		
			9	13	9	oz.	
1/5, 1/4 | 7 9				1	11¼	5	
				1	6½	4	
				-	4½	1	gr.
				-	1½	-	8
				-	-¼	-	2
			£ 9	17	9		
	85	14	15 at	6	6	per ounce.	
					12		
			3	18	-		
					7		
			27	6	-		
1/5				6	6	dwt.	gr.
				3	3	10	-
				1	3½	4	-
1/4				-	1¾	-	12
					¼	-	3
			£ 27	27	2½		
	29	16	15 at	6	4	per ounce.	
					3		
				19	-		
					9		
			8	11	-		
				12	8	dwt.	gr.
				3	2	10	-
1/5				1	7	5	-
1/4				-	3¾	1	-
					1¾	-	12
					¼	-	3
			£ 9	8	10¾		
	67	-	16 at	6	10	per ounce.	
					6		
			2	1	-		
					11		
			22	11	-		
4 dwt.				6	10	dwt.	gr.
is 1/12 | 1 8½					1½	-	12
					½	-	4
			£ 22	18	-		
	33	11	10 at	6	2		
					3		
				18	6		
					11		
			10	3	6	dwt.	gr.
1/10				3	1	10	-
1/5					3½	1	-
1/4					1	-	8
					-¼	-	2
			£ 10	6	11¼		

The Right Honourable RICHARD Earl of CASTLEMAIN, Debtor

To BENJ. BUILDER, for Work and Materials in his House at Henley Park, Surry.

1773.			Quantity		l.	s.	d	Rate	l.	s.	d.
March	27.	Oaken timber	12 load	at	2	5	—	a ton	33	15	—
	30.	Fir timber	35 ton	at	1	12	10	a load	45	19	4
April	5.	Oaken plank	96 feet	at	—	—	$3\frac{1}{2}$	per foot	1	8	—
	16.	Norway deals	690	at	6	15	-	per hundred	38	16	3
		Sixpenny nails	29 hundred	at	—	3	10	per thousand	5	11	2
May	5.	Ten groat nails	3 hundred	at	1	4	10	ditto	—	7	$5\frac{1}{4}$
June	28.	Work for myself	90 days	at	—	3	4		15	-	—
		Ditto for three men	90 days	at	—	2	6	each	33	15	—
		Wainscot	73 yards 7 feet	at	—	3	2	per yard agreed	11	13	$7\frac{1}{2}$
		Double quarter	58 feet	at	—	—	4	per foot	—	19	4
July	30.	Christiana deals	135	at	7	10	—	per hundred	8	8	9
		Riga timber	180 feet, cubed	at	—	1	1	per foot	9	15	—
									£ 205	8	$10\frac{3}{4}$

1. What

1. What will the carriage of 17 cwt. 3 qrs. 11 lb. come to, at the rate of 7s. the hundred?

7s. per cwt.
17

£	s.	d.		cwt.	qr.	lb.
£ 5	19	–	Carriage of	17	–	–
–	3	6		–	2	–
–	1	9		–	1	–
–	–	$5\frac{1}{4}$		–	–	7
–	–	3		–	–	4
£ 6	4	$11\frac{1}{4}$, the anſwer.	Cwt.	17	3	11

2. A draper bought 56 pieces of kerſey, each piece containing 34 ells Engliſh, at the rate of 5s. 4d. per ell Flemiſh. What did the whole come to?

56 pieces
34

224
168

4) 1904 ells Engliſh in all,
476

2380 yards in all.

3) 5s. 4d per ell Flemiſh.
1 $9\frac{1}{3}$

7 $1\frac{1}{3}$ per yard.

	Pence.	s.	d.	l.	s.	d.
$\frac{1}{10}$ $\frac{1}{4}$)	2380 =	198	4 =	9	18	4

595
238
9 18 4
3 6 $1\frac{1}{4}$

846 4 $5\frac{1}{4}$, the anſwer.

3. If one ounce of ſilver (plate) bullion coſt 5 s. $4\frac{1}{2}$ d. what will be the value of 14 ingots, each weighing 28 oz. 15 pwts, 12 grains?

Oz.

	Oz.	dwt.	gr.
	28	15	12
			2
	57	11	–
	–	7	–
$\frac{1}{4}$	402	17	– at 5 s. $4\frac{1}{2}$d. per oz.
$\frac{1}{4}$	80	8	
$\frac{1}{3}$	20	2	
$\frac{1}{8}$	6	14	
$\frac{1}{6}$		16	9
		4	$6\frac{1}{2}$,
	108	5	3^{1}, the anſwer required.

$\frac{1}{2}$	5	$4\frac{1}{2}$
$\frac{1}{5}, \frac{1}{2}$	2	$8\frac{1}{4}$
	1	4
	–	$6\frac{1}{4}$
	4	$6\frac{1}{2}$, p. of 17 dwt.

CASE IX.

DUODECIMALS.

Duodecimals are ſo called, becauſe they decreaſe by twelves from the place of feet, towards the right-hand; the inches I call primes, the next ſeconds, thirds, &c. according to their diſtance from feet.

This rule is ſometimes called croſs-multiplication.

Rules for multiplying Duodecimals.

Having under the multiplicand written the correſponding denomination of the multiplier; multiply each term in the multiplicand, beginning at the loweſt, by the feet in the multiplier; write each reſult under its reſpective term, and carry an unit for every 12, from each lower denomination to its next ſuperior.

2. In the ſame manner multiply each term in the multiplicand by the prime (or inches) in the multiplier, and write down the reſult of each term, one place removed to the right-hand of thoſe in the multiplicand.

3. In the like manner multiply with the ſeconds (or parts of an inch) ſetting down the reſult, one place ſtill further to the right-hand; and the ſum of all theſe give the product required.

Let it be required to multiply 9 feet $7\frac{3}{4}$ inches by 7 feet $10\frac{1}{4}$ inches.

Thus

	F.	inch.		f.	′	″
Thus	9	$7\frac{3}{4}$	=	9	7	9
And	7	$10\frac{1}{4}$	=	7	10	3

67	6	3	-	-	multiplic. ×	7 feet.
8	-	5	6	-		10 primes.
	2	4	11	3		3 seconds.
75	9	1	5	3,	the product.	

It will often happen, that the feet in the given multiplicand are so many, that to multiply them by the less denominations, and take $\frac{1}{12}$th of the product as before directed, will require some work to be done on spare paper, which may be avoided by observing the following

RULE.

Multiply the feet first; then instead of multiplying by the primes or inches, take an aliquot part of the multiplicand, according to their corresponding inches; thus, for 1 prime or inch, take $\frac{1}{12}$ of the multiplicand, for 2 inches, take $\frac{1}{6}$, for 3 take $\frac{1}{4}$, for 4 take $\frac{1}{3}$, for 5 take $\frac{1}{4}$, $+ \frac{1}{6}$ or $\frac{1}{3} + \frac{1}{12}$, for 6 take $\frac{1}{2}$, for 7 take $\frac{1}{3} + \frac{1}{4}$ or $\frac{1}{2} + \frac{1}{12}$, for 8 take $\frac{1}{3} + \frac{1}{3}$, or $\frac{1}{2} + \frac{1}{6}$, for 9 take $\frac{1}{2} + \frac{1}{4}$, for 10 take $\frac{1}{2} + \frac{1}{3}$, and for 11 inches take $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$; and in like manner for seconds or parts, only observing that the last quotes are only $\frac{1}{12}$ part of the foregoing, and must accordingly be put one place further toward the right-hand.

Let it be required to multiply 368 feet $7\frac{1}{2}$ inches, by 9 feet $4\frac{3}{4}$ inches?

Feet.	′	″			
368	7	6			
9	4	9			
3317	7	6			
122	10	6	-	-	= $\frac{1}{3}$, or 4 primes.
15	4	3	9	-	= $\frac{1}{2}$ of $\frac{1}{12}$, or 6 seconds.
7	8	1	10	6	= $\frac{1}{2}$ of the last, or 3 seconds.
3463	6	5	7	6	

279 Feet.

Feet	′	″			
279	5	3			
7	9	6			
1956	–	9	–	–	
139	8	7	6	–	= $\frac{1}{2}$ or 6 primes.
69	10	3	9	–	= $\frac{1}{2}$ of the laſt, or 3 primes.
11	7	8	7	6	= $\frac{1}{6}$ of the laſt, or 6 ſeconds.
2177	3	4	10	6	

But if the multiplier alſo be a large number, multiply the feet into each other; then for the primes and ſeconds in the multiplier, proceed as in the laſt examples; and for the primes and ſeconds in the multiplicand, take aliquot parts of the feet in the multiplier; the ſum of all will be the anſwer required.

Feet.	′	″			
187	10	3			
73	7	9			
561					
1309					
93	11	1	6	–	= $\frac{1}{2}$ for 6 primes.
15	7	10	3	–	= $\frac{1}{6}$ of the laſt for 1 prime.
7	9	11	1	6	= $\frac{1}{2}$ of the laſt for 6 ſeconds.
3	10	11	6	9	= $\frac{1}{2}$ of the laſt for three ſeconds.
36	6	–	–	–	= $\frac{1}{2}$ of 73 feet for 6 primes } in the
24	4	–	–	–	= $\frac{1}{3}$ of 73 feet for 4 primes } multi-
1	6	3	–	–	= $\frac{1}{4}$ of 73 primes for 3″ } plicand.
13834	8	1	5	3	

CHAPTER IV.

VULGAR FRACTIONS.

SECT. I.

NOTATION.

A FRACTION, or broken number, is that which repreſents a part of any thing propoſed, and is expreſſed by two numbers, placed one above the other, with a line drawn betwixt them,

Thus $\left\{ \dfrac{3}{4} \begin{array}{l}\text{Numerator.}\\ \text{Denominator.}\end{array} \right.$

The denominator, or number placed underneath the line, denotes how many equal parts the integer or whole thing is supposed to be divided into, being only the divisor in division; and the numerator or number placed above the line, shews how many of these parts are contained in the fraction.

A vulgar fraction is either proper, improper, simple, or compound.

A proper fraction is such, whose numerator is less than its denominator, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$, or $\frac{11}{231}$, &c.

An improper fraction is such, whose numerator is equal to, or greater than its denominator, as $\frac{5}{5}$, $\frac{18}{7}$, $\frac{271}{11}$, $\frac{387}{5}$, &c.

Here note, that if the numerator and denominator are equal, the fraction is equal to an integer.

A simple fraction hath only one numerator and denominator, whether it be proper or improper, as $\frac{1}{3}$, $\frac{4}{5}$, $\frac{8}{8}$, $\frac{25}{4}$, &c.

A compound fraction, or fraction of a fraction, hath several numerators and denominators connected together by the particle of, as $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{3}{5}$, by which is meant first, that the integer or whole thing is divided into five equal parts, three of which parts make $\frac{3}{5}$, which fraction is divided into eight equal parts, and seven of those parts taken, viz. $\frac{7}{8}$ of $\frac{3}{5}$; then this fraction is divided into seven equal parts, and two of those parts taken, viz. $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{3}{5}$.

Suppose, for instance, a pound sterling to be so divided.

5	20 s.
	4 s. × 3 = 12 s. = $\frac{3}{5}$ £.
8	12 s.
	1 s. 6 d. × 7 = 10 s. 6 d. = $\frac{7}{8}$ of $\frac{3}{5}$ of £ 1.
7	10 s. 6 d.
	1 s. 6 d. × 2 = 3 s. = $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{3}{5}$ of a pound sterl.

A mixed number is a whole number with a fraction annexed, as $5\frac{3}{7}$, which is read five and three-sevenths; $21\frac{1}{2}$ is twenty-one and one-half, &c.

SECT.

SECT. II.

Reduction *of* Vulgar Fractions.

IN order to facilitate the doctrine of vulgar fractions, I ſhall premiſe the following

AXIOM.

If both the numerator and denominator of a fraction be multiplied or divided by one and the ſame number, the fraction will retain the ſame value.

Viz. $\frac{7}{9} \times \frac{3}{3} = \frac{21}{27}$, and $\frac{12}{16} \div \frac{4}{4} = \frac{3}{4}$; that is, if the numerator 7 and the denominator 9 be each multiplied by the ſame number, viz. by 3, the produced fraction, viz. $\frac{21}{27}$, and the propoſed one $\frac{9}{7}$ are equal, as the numerator and denominator of the firſt are in the ſame proportion as the numerator and denominator of the ſecond.

Alſo if the numerator 12, and the denominator 16, be each divided by the ſame number 4, the fractions $\frac{3}{4}$ and $\frac{12}{16}$ for the ſame reaſon are equal.

CASE I.

To reduce a compound fraction into a ſingle one.

RULE.

Multiply all the numerators into one another for a numerator, and all the denominators into one another for the denominator.

1. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{4}{11}$ into a ſingle fraction.

$\frac{3 \times 2 \times 5 \times 4}{4 \times 3 \times 6 \times 11} = \frac{120}{792}$, the ſingle fraction required.

If a numerator of one term in a compound fraction be equal to a denominator in another term, cancel or reject both, and divide thoſe numerators and denominators which are diviſible by each other, or by the ſame number; which quotients multiplied into the remaining numerators and denominators, reduce the compound fraction to a ſingle one in its loweſt terms.

Let the laſt example, viz. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{4}{11}$ be reduced into a ſingle fraction, and its loweſt terms.

$$\frac{\cancel{3}}{\cancel{4}} \text{ of } \frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{5}{\underset{3}{\cancel{6}}} \text{ of } \frac{4}{11} = \frac{5}{33} = \frac{120}{792}.$$

2. Let $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{4}{9}$ of $\frac{3}{7}$ be reduced into a ſingle fraction in its loweſt terms.

$$\frac{1}{2} \text{ of } \frac{\cancel{7}}{\underset{2}{\cancel{8}}} \text{ of } \frac{\cancel{4}}{\underset{3}{\cancel{9}}} \text{ of } \frac{\cancel{3}}{\cancel{7}} = \frac{1}{12}, \text{ as was required.}$$

CASE II.

To reduce mixed numbers and integers into improper fractions.

I ſhall divide this caſe into three parts.

I. If the integer has no aſſigned denominator.

RULE.

An unit ſubſcribed muſt be the denominator.

Thus $7 = \frac{7}{1}$, $12 = \frac{12}{1}$, $56 = \frac{56}{1}$, $248 = \frac{248}{1}$, &c.

II. If the integer have an aſſigned denominator.

RULE.

Multiply the integer by the aſſigned denominator, the product is the numerator to the aſſigned denominator.

Reduce 17 into a fraction whoſe denominator ſhall be 12.

Thus $17 \times 12 = 204$ numerator, $\therefore \frac{204}{12} = 17$.

III. If the integer have a fraction annexed.

RULE.

Multiply the integer by the denominator, and to the product add the numerator; the ſum is the numerator to the denominator of the annexed fraction.

Let

Let $7\frac{7}{8}$, $21\frac{19}{27}$ and $119\frac{35}{38}$ be reduced into improper fractions.

First, $7 \times 8 + 7 = 63$, $21 \times 27 + 19 = 586$, and $119 \times 38 + 35 = 4557$.

Therefore $7\frac{7}{8} = \frac{63}{8}$, $21\frac{19}{27} = \frac{586}{27}$, and $119\frac{35}{38} = \frac{4557}{38}$.

CASE III.

To reduce an improper fraction into its equivalent, whole, or mixed number.

RULE.

Divide the numerator by the denominator, the quotient gives the integer, and under the remainder, if any, subscribe the denominator.

Reduce $\frac{63}{8}$, $\frac{586}{27}$, $\frac{4557}{38}$, into their equal, whole, or mixed numbers.

$8\,)\,\underline{63}\,(\,7\frac{7}{8} = \frac{63}{8}$, $27\,)\,\underline{586}\,(\,21\frac{19}{27} = \frac{586}{27}$, and
$\quad 7 \qquad\qquad\qquad 19$

$38\,)\,\underline{4557}\,(\,119\frac{35}{38} = \frac{4557}{38}$.
$\quad 35$

2. Let $\frac{204}{12}$, $\frac{364}{7}$, and $\frac{208}{3}$, be reduced into their equivalent, whole, or mixed numbers.

$12\,)\,204\,(\,17 = \frac{204}{12}$, $7\,)\,364\,(\,52 = \frac{364}{7}$, and $3\,)\,108\,(\,36 = \frac{108}{3}$.

CASE IV.

To abbreviate or reduce fractions into their lowest or least denomination.

If the numerator and denominator are even numbers, take half the one, and half the other, as often as may be; and when either of them fall out to be an odd number, then divide them by any number that you can discover will divide both numerator and denominator without any remainder.

Or, by finding the greatest common measure by the following

RULE.

Divide the greater number by the lesser, and that divisor by the remainder (if there be any) and so on continually until there be no remainder left. Then will the last divisor be the greatest common measure, which if it happen to be 1, then are they prime numbers, and are already in their lowest terms; but if otherwise, divide the numbers by the last divisor, and their quotients will be their least terms required.

1. Let $\frac{192}{336}$ be reduced into its lowest terms.

$$2)\frac{192}{336}\left(\frac{96}{168}\right)\frac{48}{84}\left(\frac{24}{42}\right)\overset{3}{\frac{12}{21}}\left(\frac{4}{7}\right. \cdots = \frac{192}{336}.$$

By finding the common measure.

$$192)\,336\,(1 \qquad 144)\,192\,(1 \qquad 48)\,144\,(3$$
$$144 \qquad\qquad 48 \qquad\qquad 144$$

Common measure $48)\frac{192}{336}\left(\frac{4}{7}\right.$.

2. What is $\frac{1036}{1184}$ in its lowest terms?

$$2)\frac{1036}{1184}\overset{2)}{\left(\frac{518}{592}\right)}\overset{37)}{\frac{259}{296}}\left(\frac{7}{8}\right. = \frac{1036}{1184}.$$

By finding the common measure.

$$1036)\frac{1184}{184}\left(1 \quad 1036\left(7\right.\right. \qquad 148)\frac{1036}{1184}\left(\frac{7}{8}\right.,\text{ as before.}$$

CASE V.

To alter or change different fractions into one denomination, retaining the same value.

RULE.

Multiply all the denominators into each other for a new and common denominator, and each numerator into all the denominators but its own for a new numerator.

1. Reduce $\frac{3}{4}$, $\frac{7}{9}$, and $\frac{2}{5}$, into fractions, having one common denominator.

First,

First, $4 \times 9 \times 5 = 180$, common denominator.

$$\text{Also}\quad \left.\begin{aligned} 3 \times 9 \times 5 &= 135 \\ 7 \times 4 \times 5 &= 140 \\ 2 \times 4 \times 9 &= 72 \end{aligned}\right\} \text{numerators.}$$

Therefore $\frac{3}{4} = \frac{135}{180}$, $\frac{7}{9} = \frac{140}{180}$, and $\frac{2}{5} = \frac{72}{180}$.

2. Reduce $\frac{2}{3}$, $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{2}{5}$, and $3\frac{5}{7}$, into fractions, having one common denominator.

First, $\frac{1}{\cancel{3}}$ of $\frac{\cancel{3}}{\cancel{4}_2}$ of $\frac{\cancel{2}}{5} = \frac{1}{10}$, and $3\frac{5}{7} = \frac{26}{7}$.

The fractions reduced to single ones will be $\frac{2}{3}$, $\frac{1}{10}$, and $\frac{26}{7}$.

First, $3 \times 10 \times 7 = 210$, common denominator.

$$\text{Also}\quad \left.\begin{aligned} 2 \times 10 \times 7 &= 140 \\ 1 \times 7 \times 3 &= 21 \\ 26 \times 3 \times 10 &= 780 \end{aligned}\right\} \text{numerators.}$$

$\therefore \frac{2}{3} = \frac{140}{210}$, $3\frac{5}{7} = \frac{26}{7} = \frac{780}{210}$. And $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{2}{5} = \frac{1}{10} = \frac{21}{210}$.

If there be two denominators already alike, you need multiply but by one of them, as in the following example:

3. Reduce $\frac{7}{8}$, $\frac{1}{5}$ of $\frac{7}{8}$ of $\frac{5}{7}$, 5, and $25\frac{3}{5}$ into fractions, having one common denominator.

First $\frac{1}{\cancel{5}}$ of $\frac{\cancel{7}}{8}$ of $\frac{\cancel{5}}{\cancel{7}} = \frac{1}{8}$, $5 = \frac{5}{1}$, and $25\frac{3}{5} = \frac{128}{40}$.

In single ones, $\frac{7}{8}$, $\frac{1}{8}$, $\frac{5}{1}$, $\frac{128}{5}$.

$8 \times 1 \times 5 = 40$, common denominator.

$$\left.\begin{aligned} 7 \times 1 \times 5 &= 35 \\ 1 \times 5 &= 5 \\ 5 \times 8 \times 5 &= 200 \\ 128 \times 8 &= 1024 \end{aligned}\right\} \text{numerators.}$$

Viz $\frac{7}{8} = \frac{35}{40}$, $\frac{1}{5}$ of $\frac{7}{8}$ of $\frac{5}{7} = \frac{1}{8} = \frac{5}{40}$, $5 = \frac{5}{1} = \frac{200}{40}$.

And $25\frac{3}{5} = \frac{128}{5} = \frac{1024}{40}$.

 2. When

2. When there are only two fractions to be reduced, if one of the denominators is a multiple of the other, divide; and by the quote multiply the numerator and denominator of that fraction which hath the least denominator, and the fraction thus found will be equivalent to the given ones.

Reduce $\frac{3}{7}$ and $\frac{17}{28}$ to a common denominator.

First, $28 \div 7 = 4$. Then $\frac{3 \times 4}{7 \times 4} = \frac{12}{28}$.

$\therefore$ $\frac{12}{28}$ and $\frac{17}{28}$ are the fractions required.

3. Or if both of the denominators have a common multiple, divide each of the denominators thereby, and multiply the contrary numerators and denominators by each contrary quotient.

Let $\frac{5}{6}$ and $\frac{3}{8}$ be fractions proposed to be reduced.

As 2 will measure 6 and 8, their respective quotes being 3 and 4.

Then $3 \times 8 = 4 \times 6 = 24$, the common denominator.

Also $5 \times 4 = 20$, and $3 \times 3 = 9$, the numerators.

$\therefore$ $\frac{5}{6} = \frac{20}{24}$, and $\frac{3}{8} = \frac{9}{24}$, the fraction required.

Reduce $\frac{7}{20}$ and $\frac{11}{15}$ to a common denominator.

Divide by 5 . . . 4 and 3 are the quotes.

Then 20×3, or $15 \times 4 = 60$, the common denominator.

Also $7 \times 3 = 21$, and $11 \times 4 = 44$, numerators.

Therefore $\frac{7}{20} = \frac{21}{60}$, and $\frac{11}{15} = \frac{44}{60}$, are the fractions required.

CASE VI.

To reduce a fraction to an equivalent one of any other assigned denominator, viz. to find a numerator, which, with the assigned denominator, will make a fraction equivalent to the proposed one, when possible.

RULE.

Multiply the assigned denominator by the numerator of the proposed fraction, and divide the product by the denominator; the quote (if there be no remainder) is the numerator sought.

Reduce $\frac{3}{4}$ to an equivalent fraction, having for its denominator 28.

Thus

Thus $28 \times 3 = 84$; then $84 \div 4 = 21$, the numerator; that is, $\frac{21}{28} = \frac{3}{4}$.

Whenever the denominator aſſigned is diviſible (without a remainder) by the denominator of the given fraction, the thing is poſſible, otherwiſe not.

CASE VII.

To find whether one fraction be greater or leſs in value than another.

RULE.

Multiply the numerators into each other's denominator, and if the products are equal, the fractions are ſo; otherwiſe the numerator of the greateſt fraction multiplied by the denominator of the other, will be the greateſt product.

Which is the fraction of the greateſt value, viz. $\frac{7}{9}$, or $\frac{5}{6}$?

Thus $7 \times 6 = 42$; but $5 \times 9 = 45$, conſequently $\frac{5}{6}$ is the fraction of the greater value.

Let $\frac{3}{4}$ and $\frac{21}{28}$ be fractions propoſed.

Then $3 \times 28 = 84$; and $4 \times 21 = 84$. Here the products, and alſo the value of the fractions, are equal.

CASE VIII.

To reduce coins, weights, meaſures, &c. into fractions.

RULE.

Reduce the coin, weight, &c. into the loweſt name mentioned for a numerator; and put the number of thoſe parts contained in an unit of the integer, to which the propoſed fraction is to be reduced for the denominator; then reduce the fraction into its loweſt terms.

Reduce 7 s. 3 d. into a fraction, a pound being the integer.

12

87 pence, the fraction will be $\frac{87}{240}$ l.

$8)\frac{87}{240}(= \frac{29}{20}$ l. in its loweſt terms = 7s. 3d.

Reduce 4s. $7\frac{3}{4}$ d. into a fraction, a pound being the integer.

12

55

4 $\frac{223}{960}$ l. = 4 s. $7\frac{3}{4}$ d. as was required.

223 farthings.

Reduce

Rednce $4\frac{1}{2}$ d. into the fraction of a shilling.

$$\begin{array}{l} 4 \\ \hline 18 \text{ farthings} \end{array}$$

$6)\frac{18}{48}(= \frac{3}{8} = 4\frac{1}{2}$ d. as was required.

Reduce 3 cwt. 2 qrs. 21 lb. into a fraction, 1 cwt. being the integer.

$$\begin{array}{l} 3 \text{ cwt. } 2 \text{ qrs. } 21 \text{ lb.} \\ 4 \\ \hline 14 \\ 28 \\ \hline 112 \\ 28 \\ \hline 413 \end{array}$$

$7)\frac{413}{112} = \frac{59}{16} =$ 3cwt. 2qr. 21 lb. as req.

Reduce 27 oz. 17 pwt. 18 gr. into a fraction, one ounce troy being the integer.

$$\begin{array}{rrr} \text{oz.} & \text{pwt.} & \text{gr.} \\ 27 & 17 & 18 \\ 20 & & \\ \hline 557 & & \\ 24 & & \\ \hline 2228 & & \\ 1114 & & \\ \hline \end{array}$$

$6)\frac{13386}{480} = \frac{2232}{80} \; 4) = \frac{558}{20} =$ 27 oz. 17 pwt. 18 gr.

CASE IX.

To reduce a fraction of an unit of a higher denomination to an equivalent fraction of an unit of a lower species of the same kind with the higher.

RULE.

Multiply the numerator of the given fraction, by the number of units in the next inferior species that make an unit of the denomination of your fraction, and that product multiply by

by the number of units in the next inferior denomination that make an unit of the laſt denomination, and thus proceed till you come to the loweſt you deſign; then make the laſt product a numerator to the denominator of the fraction given.

1. Reduce $\frac{3}{14}$ l. to an equivalent fraction in the denomination of 1 d.

Firſt, $3 \times 20 = 60$, and $60 \times 12 = 720$, numerator.

$$\overset{\text{D.}}{\frac{720}{14}} = \overset{\text{D.}}{\frac{360}{7}} = \overset{£.}{\frac{3}{14}}, \text{ as was required.}$$

2. Reduce $\frac{4}{5}$ of a ſhilling to the fraction of a farthing.

Firſt, $4 \times 12 = 48$, and $48 \times 4 = 192$, numerator.

$$\therefore \overset{\text{qr.}}{\frac{192}{5}} = \frac{4}{5} \text{ of a ſhilling, as was required.}$$

3. Reduce $\frac{2}{3}$ cwt. to the fraction of 1 lb.

Thus $4 \times 2 = 8$, and $8 \times 28 = 224$, numerator.

$$\therefore \overset{\text{lb.}}{\frac{224}{3}} = \frac{2}{3} \text{ cwt. as was required.}$$

CASE X.

To reduce a fraction of an unit of a lower denomination to an equivalent fraction in the denomination of an higher.

RULE.

Multiply the denominator by the number of units in the given fraction that is equal to an unit of the next ſuperior denomination, and the product by ſuch a number of units of its denomination, as is equal to an unit of the next above it; and thus go on till you come to the higheſt ſpecies required, and the laſt product is a denominator to the numerator of the fraction given.

1. Reduce $\frac{5}{8}$ of a farthing into the fraction of 1 l.

$$8 \times 4 \times 12 \times 20 = 7680, \text{ denominator.}$$

So that $\frac{5}{8}$ of a farthing $= \frac{5}{7680} = \frac{1}{1536}$ l.

Or

Or by compound fractions, $\frac{5}{8}$ of a farthing $= \frac{\cancel{5}}{8}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{\cancel{20}_{4}} = \frac{1}{1536}$.

2. Reduce $\frac{4}{7}$ oz. into the fraction of 1 cwt.

$7 \times 16 \times 28 \times 4 = 12544$, denominator.

$$\therefore \frac{4}{12544}\text{ cwt.} = \frac{1}{3136}\text{ cwt.} = \frac{4}{7}\text{ oz.}$$

That is, $\frac{\cancel{4}}{7}$ of $\frac{1}{16}$ of $\frac{1}{28}$ of $\frac{1}{4} = \frac{1}{3136}$ cwt. $= \frac{4}{7}$ oz., as before.

CASE XI.

To find the value of a fraction in coin, weight, measure, time, &c.

RULE.

Multiply the numerator of the given fraction by the number of units of the next inferior species that makes one of the denomination of your fraction, and divide the product by the denominator; the quotient is so many integers of that lower species; and if there is a remainder, proceed as before, still reducing and dividing, till you come to the lowest species; and the several quotients, with the remainder, (if any, which is always the numerator of a fraction of the lowest species) are the answer.

1. What is the value of $\frac{5}{7}$ of a pound?

```
     5
  × 20
  ------   s.  d.  qr.
7)100(14   3   1 5/7, answer.
     2
  × 12
  ------
    24
     3
   × 4
  ------
    12
   (5)
```

2. What is the value of $\frac{17}{178}$ cwt.?

```
          17
           4
          --
          68
        × 28
        ----
         544
         136
        ------  lb. oz. dr.
178)1904(10  11  2 30/89, ans.
         124
        × 16
        ----
        1984
         204
          26       2) 60/178 (30/89
        × 16
        ----
         416
        (60)
```

3. What

3. What is the value of $\frac{4}{5}$ of a shilling?

$$\begin{array}{l} \quad 4 \\ \times 12 \\ \hline \qquad\quad \text{d.} \quad \text{f.} \\ 5)48(9 \quad 2\tfrac{2}{5}\text{, answer,} \\ \quad 3 \\ \times 4 \\ \hline \quad 12 \\ \quad (2) \end{array}$$

4. What is the value of $\frac{3}{8}$ of a degree?

$$\begin{array}{l} \quad 3 \\ \times 60 \\ \hline 8)180(22' \; 30''\text{, answer.} \\ \quad 4 \\ \quad 60 \\ \hline \quad 240 \end{array}$$

5. What is the value of $\frac{6}{7}$ hundred weight?

$$\begin{array}{l} \quad 6 \\ \quad 4 \\ \hline \qquad\quad \text{qr.} \quad \text{lb.} \\ 7)24(3 \quad 12\text{, answer.} \\ \quad 3 \\ \times 28 \\ \hline \quad 84 \end{array}$$

6. What is the value of $\frac{2}{3}$ of $\frac{5}{7}$ of a year?

$\frac{2}{3}$ of $\frac{5}{7} = \frac{10}{21}$.

Seconds in a year $= 31556937 \times 10 = 315569370$.

$$\begin{array}{l} \qquad\qquad 60) \\ 21)315569370(15027112\tfrac{18}{21} = \tfrac{6}{7}\text{ seconds.} \\ \quad 105 \qquad\quad 60 \,|\, 250451 \text{ minutes } 52''\tfrac{6}{7}. \\ \quad\; 56 \\ \quad 149 \qquad\qquad 24 \,|\, 4174 \text{ hours } 11' \; 52''\tfrac{6}{7}. \\ \qquad 23 \\ \qquad 27 \qquad\qquad\quad 173 \text{ days } 22 \text{ h. } 11' \; 52''\tfrac{6}{7}\text{, answer.} \\ \qquad 60 \\ \hline \qquad 18 \end{array}$$

If the fraction to be valued be an improper one, divide the numerator by the denominator, and the quotient is an integer of the same species with the fraction; then reduce the remainder as before.

7. What is the value of $\frac{77}{8}$ of an ounce troy?

$\frac{77}{8})$

$$\frac{77}{8}\Big)\,77\,(\;9\quad 12\quad 12,\ \text{the answer.}$$

oz. dwt. gr.

$$\begin{array}{r} 5 \\ \times 20 \\ \hline 100 \\ 4 \\ \times 24 \\ \hline 96 \\ \hline \end{array}$$

* *

SECT. III.

ADDITION of FRACTIONS.

IN order to prepare fractions for addition or subtraction, all compound fractions must be reduced to single ones; and if they are of different denominations, they must be brought into the same denomination, and reduced, so as all the fractions shall have one common denominator.

RULE.

Add together all the numerators, for a new numerator; under which subscribe the common denominator.

1. Add $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{5}$ together.

First $\frac{1}{3}=\frac{10}{30}$, $\frac{1}{2}=\frac{15}{30}$, and $\frac{2}{5}=\frac{12}{30}$, per reduction.
Then $10+15+12=37$, the new numerator.
$\therefore\ \frac{1}{3}+\frac{1}{2}+\frac{2}{5}=\frac{37}{30}=1\frac{7}{30}$, the sum required.

2. Add $3+\frac{5}{8}+\frac{7}{8}+\frac{4}{5}$ of $\frac{7}{8}+7$ into one sum.
First,

$$\left.\begin{array}{l} \frac{5}{8} = \frac{25}{40} \\ \frac{7}{8} = \frac{35}{40} \\ \frac{4}{5}\text{ of }\frac{7}{8} = \frac{28}{40} \end{array}\right\}\text{ per reduction.}$$

Then $25+35+28=88$, and $\frac{88}{40}=2\frac{1}{5}$.
$\therefore\ 3+7+2\frac{1}{5}=12\frac{1}{5}$, the sum required.

3. Add $\frac{2}{7}$ of 15 l. + $3\frac{3}{7}$ l. + $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of a pound + $\frac{2}{3}$ of $\frac{3}{7}$ of a shilling into one sum.

$$\left.\begin{array}{l} \text{First, } \frac{2}{7} \text{ of } 15\text{l.} = \frac{30}{7} = 4\frac{2}{7} \\ \qquad\qquad\qquad\qquad\qquad 3\frac{3}{7} \\ \frac{1}{\cancel{3}} \text{ of } \frac{\cancel{5}}{7} \text{ of } \frac{\cancel{3}}{\cancel{5}} \quad = \quad \frac{1}{7} \\ \frac{2}{\cancel{3}} \text{ of } \frac{\cancel{3}}{7}\text{s.} = \frac{2}{7}\text{s.} = \frac{1}{70} \end{array}\right\} \text{Reduced into pounds and fractions of a pound sterling.}$$

And $\frac{2}{7} = \frac{20}{70}$, $\frac{3}{7} = \frac{30}{70}$, $\frac{1}{7} = \frac{10}{70}$, and $\frac{1}{70}$, with one common denominator.

Then $20 + 30 + 10 + 1 = 61$, numerator.

$\therefore 4 + 3 + \frac{61}{70} = 7\frac{61}{70} = 7$ l. 17 s. $5\frac{1}{7}$ d. the answer.

* *

SECT. IV.

SUBTRACTION *of* FRACTIONS.

THE fractions being prepared, as before directed in addition, then,

RULE.

Subtract one numerator from the other, and their difference will be a new numerator, under which subscribe the common denominator.

1. Subtract $\frac{2}{9}$ of $\frac{3}{7}$, from $\frac{2}{3}$.

First $\frac{2}{\underset{3}{\cancel{9}}}$ of $\frac{\cancel{3}}{7} = \frac{2}{21}$, and $\frac{2}{3} = \frac{14}{21}$.

$\therefore \frac{14}{21} - \frac{2}{21} = \frac{12}{21} = \frac{4}{7}$, the answer required.

2. What number is that, from which if you deduct the $\frac{1}{25}$ of $\frac{7}{8}$, and to the remainder add $\frac{1}{16}$ of $\frac{47}{19}$, the sum will be 3?

First, $\frac{1}{16}$ of $\frac{47}{19} = \frac{47}{304}$. Then $3 = \frac{912}{304}$; and $\frac{912}{304} - \frac{47}{304}$ $= \frac{865}{304} = \frac{21625}{7600}$.

Also

Also $\frac{1}{25}$ of $\frac{7}{8} = \frac{266}{7600}$ ∴ $\frac{21625}{7600} + \frac{266}{7600} = 2\frac{6691}{7600}$, the anf.

3 What number is that, to which if you add $\frac{1}{11}$ of 12 + $\frac{1}{19}$ of 27, and from the total subtract $\frac{1}{3}$ of $7\frac{1}{2} - \frac{29}{30}$ of $1\frac{1}{4}$, the remainder shall be 8?

First, $\frac{1}{3}$ of $7\frac{1}{2} = \frac{60}{24}$ and $\frac{29}{30}$ of $1\frac{1}{4} = \frac{29}{24}$ ∴ $\frac{60}{24} - \frac{20}{24} = 1\frac{16}{24} = 1\frac{2}{3}$

Then $8 + 1\frac{2}{3} = 9\frac{2}{3}$. Also $\frac{1}{11}$ of $\frac{12}{1} = \frac{228}{209}$; and $\frac{1}{19}$ of $\frac{27}{1} = \frac{297}{209}$.

∴ $\frac{228}{209} + \frac{297}{209} = \frac{525}{209} = 2\frac{107}{209}$. Lastly, $9\frac{2}{3} - 2\frac{107}{209} = 7\frac{97}{627}$ the number sought.

SECT. V.

Multiplication of Fractions.

TO prepare fractions for either multiplication or division, reduce compound fractions to single ones; bring mixed numbers into improper fractions, and express whole numbers fraction-wise; also reduce fractions into their lowest terms. Then,

RULE,

Multiply the numerators into one another for a new numerator, and the denominators one into another for a new denominator.

1. Multiply $\frac{3}{7}$ into $\frac{5}{6}$. First, $3 \times 5 = 15$, and $7 \times 6 = 42$.

Answer, $\frac{3}{7} \times \frac{5}{6} = \frac{15}{42} = \frac{5}{14}$.

2. Multiply $\frac{7}{11}$ into $\frac{2}{5}$ of $\frac{5}{7}$.

First, $\frac{2}{\cancel{5}}$ of $\frac{\cancel{5}}{7} = \frac{2}{7}$. Then $7 \times 2 = 14$, and $11 \times 7 = 77$.

Answer, $\frac{7}{11} \times \frac{2}{5}$ of $\frac{5}{7} = \frac{14}{77} = \frac{2}{11}$.

3. Mul-

3. Multiply $7\frac{4}{9}$ into $5\frac{3}{4}$. First $7\frac{4}{9}=\frac{67}{9}$, and $5\frac{3}{4}=\frac{23}{4}$. Then $67\times 23=1541$, and $9\times 4=36$.

Answer $7\frac{4}{9}=5\frac{3}{4}=\frac{1541}{36}=42\frac{29}{36}$.

4. Multiply $2\frac{1}{2}$ by $\frac{1}{8}$, and this product by 2, and this again by $\frac{1}{3}$ of $\frac{5}{6}$.

First $2\frac{1}{2}=\frac{5}{2}$, $2=\frac{2}{11}$ and $\frac{1}{3}$ of $\frac{5}{8}=\frac{5}{18}$.

Then $\frac{5}{2}\times\frac{1}{8}\times\frac{5}{18}=\frac{25}{144}$, the answer.

Hence it may be observed, that if the multiplier be a proper fraction, the product will always be less than the multiplicand.

SECT. VI.

DIVISION *of* FRACTIONS.

THE fractions being prepared as directed for multiplication; division may be thus performed.

RULE.

Multiply the number of the dividend into the denominator of the dividing fraction for a numerator, and the other numerator and denominator together for a new denominator.

1. Divide $\frac{3}{4}$ by $\frac{2}{5}$ . . . . $\frac{2}{5}$) $\frac{3}{4}$ ($\frac{15}{8}=1\frac{7}{8}$, answer.

2. Divide $\frac{5}{7}$ of a pound by $\frac{2}{3}$ of a shilling. First $\frac{2}{3}$ of a shilling $\frac{2}{60}$ of a pound.

Then $\frac{2}{60}=\frac{1}{30}$) $\frac{5}{7}$ ($\frac{150}{7}=21$ l. 8 s. $6\frac{3}{4}$p. + $\frac{3}{4}$ farthings.

3. Divide $\frac{3}{5}$ by 7. Thus $\frac{7}{1}$) $\frac{3}{5}$ ($\frac{3}{3}$, the answer required.

4. Divide $4\frac{2}{3}$ by $5\frac{3}{7}$. First $4\frac{2}{3}=\frac{14}{3}$, and $5\frac{3}{7}=\frac{38}{7}$.

Then $\frac{38}{3})\frac{14}{3}(\frac{9}{114} = \frac{49}{57}$, the anſwer required.

5. Divide $\frac{2}{3}$ of $\frac{3}{7}$ by $\frac{5}{6}$ of $\frac{3}{4}$. Firſt $\frac{2}{3}$ of $\frac{3}{7} = \frac{2}{7}$, and $\frac{5}{6}$ of $\frac{3}{4} = \frac{5}{8}$.

Then $\frac{5}{8})\frac{2}{7}(\frac{16}{35}$, the quotient ſought.

If the diviſor and dividend have both the ſame denominator, the quotient may be found by dividing one numerator by another.

6. Divide $3\frac{3}{4}$ by $\frac{3}{4}$ . . $\frac{3}{4})\frac{15}{4}(5$, the anſwer.

7. Divide $\frac{5}{17}$ by $\frac{13}{17}$ . . . $\frac{13}{17})(\frac{5}{17}(\frac{5}{13}$, the anſwer.

2. If the diviſor and dividend have each the ſame numerator; divide one of the denominators by the other, which will give the quotient required.

8. Divide $\frac{4}{17}$ by $\frac{4}{5}$ . . . . $\frac{4}{5})\frac{4}{17}(\frac{5}{17}$, the anſwer.

9. Divide $\frac{7}{9}$ by $\frac{7}{23}$ . . . . $\frac{7}{23})\frac{7}{9}(2\frac{5}{9}$, the anſwer.

3. If the numerator and denominator of the dividend can be dividend without a remainder, by the numerator and denominator of the diviſor, their quotients will anſwer the queſtion.

10. Divide $\frac{9}{28}$ by $\frac{3}{7}$ . . . . $\frac{3}{7})\frac{9}{28}(\frac{3}{4}$, the anſwer.

4. If a number can be found that will divide both the numerators, or both the denominators (viz. thoſe of the diviſor and dividend) without a remainder; uſe thoſe quotients inſtead of the given numerators and denominators, which will give the reſult in its loweſt terms.

11. Divide $\frac{12}{35}$ by $\frac{6}{14}$ . . . . $\frac{2}{7}\ \frac{2}{14})\frac{4}{5}\ \frac{8}{15}$, the anſwer.

QUESTIONS

QUESTIONS *to exercise* VULGAR FRACTIONS.

1. A Lad having got 4000 nuts, in his return was met by Mad Tom, who took from him $\frac{5}{8}$ of $\frac{2}{3}$ of his whole ſtock. Raving Ned lights on him afterwards, and forces $\frac{2}{5}$ of $\frac{5}{8}$ of the remainder from him. Unlucky Poſitive Jack found him, and required $\frac{7}{10}$ of $\frac{17}{20}$ of what he had left. Smiling Dolly was by promiſe to have $\frac{3}{4}$ of a quarter of what nuts he brought home. How many then had the boy left?

$$\begin{array}{rcll} & & 4000 & \\ \frac{5}{8} \text{ of } \frac{2}{3} \text{ of } 4000 & = & 1666\frac{2}{3} & \text{Mad Tom took.} \\ & & 2333\frac{1}{3} & \text{left.} \\ \frac{2}{5} \text{ of } \frac{5}{8} \text{ of } \frac{7000}{3} & = & 583\frac{1}{3} & \text{Raving Ned took.} \\ \hline & & 1750 & \text{left.} \\ \frac{7}{10} \text{ of } \frac{17}{20} \text{ of } 1750 & = & 1041\frac{1}{4} & \text{Poſitive Jack took.} \\ \hline & & 708\frac{48}{64} & \text{left.} \\ \frac{3}{4} \text{ of } \frac{1}{4} \text{ of } \frac{2835}{4} & = & 132\frac{57}{64} & \text{Smiling Dolly had.} \\ \hline & & 575\frac{55}{64}, & \text{the anſwer.} \end{array}$$

2. There is a number, which if divided by $\frac{16}{3}$ of $_{16}$, will quote $\frac{361}{37}$; pray what is the ſquare of that number, $\frac{16}{3}$ of $\frac{3}{16} = \frac{1}{1}$, which neither multiplies nor divides.

$$\therefore \frac{361}{37} \times \frac{361}{37} = \frac{130321}{1369} = 95\frac{266}{1369}, \text{ the anſwer.}$$

3. There is a number, which if multiplied by $\frac{3}{4}$ of $\frac{7}{9}$ of $2\frac{3}{4}$, will produce no more than 1; what is the cube of that number?

$$\frac{3}{4} \text{ of } \frac{7}{9} \text{ of } \frac{11}{4} = \frac{77}{48})\ \frac{1}{1}\ (\frac{48}{77}.$$

$$\text{Then } \frac{48}{77} \times \frac{48}{77} \times \frac{48}{77} = \frac{110592}{456583}, \text{ the anſwer.}$$

4. Four figures of 9 may be ſo placed and diſpoſed of, as to denote and read for 100, neither more nor leſs: Pray how is that to be done?

Anſwer $99\frac{9}{9} = 100$.

 5. Kitty

5. Kitty told her brother George, that though her fortune on her marriage took 19312 l. out of the family, it was but $\frac{3}{5}$ of two years rent, Heaven be praised, of his yearly income; pray what was that?

$$\frac{3}{5}\Big)\frac{19312}{1}\Big(\frac{96560}{3}$$

$$\frac{2}{1}\Big)\frac{96560}{3}\Big(\frac{96560}{6} = 16093\text{ l. } 6\text{ s. } 8\text{ d. per annum.}$$

6. A merry young fellow in a small time got the better of $\frac{1}{3}$ of his fortune; by advice of his friends he then gave 2200 l. for an exempt's place in the Guards; his profusion continued till he had no more than 880 guineas left, which he found by computation was just $\frac{3}{20}$ part of the money after the commission was bought; pray what was his fortune at first?

$$810 \text{ guineas} = 924\text{ l.}$$

$$\text{Then } \frac{3}{20}\Big)\frac{924}{1}\Big(\frac{18180}{3} = 6160\text{ l.}$$

$$2200 + 6160 = 8360 = \tfrac{4}{5} \text{ of his whole fortune.}$$

$$\frac{4}{5}\Big)\frac{8360}{1}\Big(\frac{41800}{4} = 10450\text{ l. the answer.}$$

7. A certain captain sends out $\frac{1}{3}$ of his soldiers + 10, and there remained $\frac{1}{2}$ + 15: how many soldiers had he?

$\frac{1}{3}$ or $\frac{2}{6} + 10 =$ what he sent out.
And $\frac{1}{2} + \frac{3}{6} + 15 =$ what remained.
Their sum $\frac{5}{6} + 25 =$ number of soldiers.
Hence $25 = \frac{1}{6}$ of the soldiers.
$\therefore 25 \times 6 = 150$, the answer required.

8. A certain gentleman hires a servant, and promises him 24 pounds yearly wages, together with a cloak. At eight months end the servant obtains leave to go away, and instead of his wages receives a cloak + 13 pounds; how much did the cloak cost?

Ashby's Analyst.

As

As 8 months $= \frac{2}{3}$ year; therefore, at 8 months end, his due is $\frac{2}{3}$ of 24 l. (= 16 l.) + $\frac{2}{3}$ of the cloak.

Then 16 l. — 13 l. = 3 l. = value of $\frac{1}{3}$ of the cloak.

∴ 3 l. × 3 = 9 l. the anſwer required.

9. If a man gains 30 crowns a week, how much muſt he ſpend a week to have 500 crowns, together with the expence of four weeks, remaining at the year's end?

Aſhby's Analyſt.

Firſt 30 × 52 = 1560 crowns gained in a year.
Alſo 1560 — 500 = 1060, the dividend.
And 52 + 4 = 56, the diviſor.
∴ 56)1060(18$\frac{13}{14}$ crowns ſpent = 4 l. 14s. 7$\frac{5}{7}$d. per week.
And 30 — 18$\frac{13}{14}$ = 11$\frac{1}{14}$ crowns = 2 l. 15s. 4$\frac{2}{7}$d.

10. A country ſpark addreſt a charming ſhe,
In whom all lovely features did agree!
But he not ſkill'd i'th' art (you may preſage,)
Was too ſolicitous to know her age.
The lady ſmil'd at this prepoſt'rous rule
Of courtſhip; but to ſatisfy the fool,
Made him this anſwer with a gen'rous air,
(A lofty charm peculiar to the fair)
My age is that, if multiply'd by three,
And two-ſevenths of that product tripled be,
The ſquare-root of two-ninths of that is four;
And now farewel, I'll never ſee you more.—
Your fond impertinence has caus'd this rage;
'Tis clowniſh ſure to aſk a woman's age.
So you're deſir'd to aſſiſt him, or perchance,
The ſpark muſt ſtill remain in ignorance. *Ladies Diary.*

Firſt 4 × 4 = 16. Then $\frac{2}{9}\Big)\frac{16}{1}\Big(\frac{144}{2} = 72.$

Alſo 3) 72 (24, and $\frac{2}{7}\Big)\frac{24}{1}\Big(\frac{168}{2} = 84.$

∴ 3) 84 (28, the anſwer required.

11. A perſon having about him a certain number of crowns, ſaid, If $\frac{1}{4} + \frac{1}{3} + \frac{1}{6}$, of what he had, were added together, they would make juſt 45; how many crowns had he about him?

$$\frac{1}{4} = \frac{3}{12}, \ \frac{1}{3} = \frac{4}{12}, \text{ and } \frac{1}{6} = \frac{2}{12}.$$

 Then

Then $\frac{3}{12}+\frac{4}{12}+\frac{2}{12}=\frac{9}{12}=\frac{3}{4}=45$.

$\therefore \frac{3}{4}\Big)\frac{45}{1}\Big(\frac{180}{3}=60$, the anſwer.

12. A ſchoolmaſter being aſked how many ſcholars he had, anſwered: If I had as many, and $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many, I ſhould have 99; how many had he?

First $1=\frac{4}{4}$, $\frac{1}{2}=\frac{2}{4}$.

Then $\frac{4}{4}+\frac{4}{4}+\frac{2}{4}+\frac{1}{4}=\frac{11}{4}=99$, per queſt.

$\therefore \frac{11}{4}\Big)\frac{99}{1}\Big(\frac{396}{11}=36$ ſcholars. Q. E. F.

13. When I wrote this, if to my age you add,
$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ (thereof) with $\frac{3}{5}$ more,
The number 25 will then be had;
Ingenious Tyro's, pray my age explore.

Firſt $1=\frac{30}{30}$, $\frac{1}{2}=\frac{15}{30}$, $\frac{1}{3}=\frac{10}{30}$, and $\frac{1}{5}=\frac{6}{30}$.

Alſo $\frac{30}{30}+\frac{15}{30}+\frac{10}{30}+\frac{6}{30}=\frac{61}{30}$.

And $25-\frac{3}{5}=24\frac{2}{5}=\frac{122}{5}=\frac{732}{30}$.

$\therefore \frac{61}{30}\Big)\frac{732}{30}\Big($12 years, the anſwer required.

14. What number is that, which added to its $\frac{1}{2}$ + its $\frac{1}{4}$ + 3, makes 108?

Firſt $\frac{1}{1}=\frac{4}{4}$, $\frac{1}{2}=\frac{2}{4}$. Alſo $108-3=105$.

Then will $\frac{4}{4}+\frac{2}{4}+\frac{1}{4}=\frac{7}{4}=105$.

$\therefore \frac{7}{4}\Big)\frac{105}{1}\Big(\frac{420}{7}=60$, the anſwer.

15. Admit there is 212 l. 14 s. 7 d. to be divided amongſt a captain, four men, and a boy; the captain to have a ſhare and half; the men each a ſhare, and the boy $\frac{1}{3}$ of a ſhare; what ought each perſon to have?

$1\frac{1}{2}$

$1\frac{1}{2} = \frac{3}{2} = \frac{9}{6}$ capt. $\frac{4}{1} = \frac{24}{6}$ men, and $\frac{2}{6}$ boy.

Then $\frac{9}{6} + \frac{24}{6} + \frac{2}{6} = \frac{35}{6} =$ 212l. 14s. 7d. = 51055d.

$\therefore \frac{35}{6}\Big)\frac{51055}{1}\Big(\frac{306330}{35} = 8752\frac{2}{7} =$ 36 l. 8 s. $4\frac{2}{7}$d.

					l.	s.	d.		
And 2)36	9	$4\frac{2}{7}$	}	=	54	14	$-\frac{2}{7}$	captain.	Q. E. F.
+ 18	4	$8\frac{1}{7}$	}						
36	9	$4\frac{2}{7} \times 4$		=	145	17	$5\frac{1}{7}$	men.	
3)36	9	$4\frac{2}{7}$ (		=	12	3	$1\frac{3}{4}$	boy.	
					£ 212	14	7		

16. There is a ciftern with three unequal cocks, containing 60 gallons of water; and if the greateft cock be opened, it will be empty in one hour; and if the fecond cock be opened, it will be empty in two hours; if the third be opened, it will be empty in three hours: now I demand in what time it will be empty, if all run together?

The firft empties 1, The fecond - - $\frac{1}{2}$, And the third - $\frac{1}{3}$ } ciftern in an hour = { $\frac{6}{6}$, $\frac{3}{6}$, $\frac{2}{6}$ } their fum = $\frac{11}{6}$ all running.

$\therefore \frac{11}{6}\Big)\frac{60}{1}\Big(\frac{360}{11} = 32\frac{8}{11}$ minutes, the anfwer.

17. A gentleman has an orchard of fruit trees, one-half of the trees bearing apples, one-fourth pears, one-fixth plums, and fifty of them bearing cherries; how many fruit trees in all grow in the faid orchard?

Firft $\frac{1}{2} = \frac{6}{12}$ apples, $\frac{1}{4} = \frac{3}{12}$ pears, $\frac{1}{6} = \frac{2}{12}$ plums.

Then $\frac{6}{12} + \frac{3}{12} + \frac{2}{12} = \frac{11}{12}$. Alfo $\frac{12}{12} - \frac{11}{12} = \frac{1}{12}$ cherries $= 50$.

$\therefore$ $50 \times 6 = 300$ apples.
Alfo $50 \times 3 = 150$ pears.
Again $50 \times 2 = 100$ plums.
And - - - - - - - 50 cherries.

In all - 600. Q. E. F.

18. Five persons discoursing about their ages, said the second to the first, my age is the double of your's; and said the third to the first, my age is as much, and $\frac{1}{4}$ as much as your's; then said the fourth to the second and third, my age is as much as both yours added together; but said the fifth, my age is three times as much as the age of the first, and the sum of all our ages make just 168 years; what was the age of each?

$$\begin{array}{r} 1 \\ 2 \\ 1\frac{1}{4} \\ 3\frac{1}{4} \\ 3 \\ \hline \end{array}$$

$10\frac{1}{2} = \frac{21}{2}$ divisor for the first person's age.

$$\left.\begin{array}{llcl} \therefore \frac{21}{2}) & \frac{168}{1} (\frac{336}{21} & = & 16 \text{ first} \\ \text{Also -} & 16 \times 2 & = & 32 \text{ second} \\ \text{Again} & 16 \times 1\frac{1}{4} & = & 20 \text{ third} \\ \text{Likewise} & 32 + 20 & = & 52 \text{ fourth} \\ \text{And -} & 16 \times 3 & = & 48 \text{ fifth} \end{array}\right\} \text{person's age.}$$

Sum 168

* *

CHAPTER V.

DECIMAL FRACTIONS.

SECT. I.

NOTATION.

THE word decimal is derived from *decem* (ten) and denotes the nature of its numbers; because the integer, or whole thing, whether it be coin, weight, measure, time, &c. is supposed to be divided into ten equal parts, and every one of those parts into ten other equal parts, &c. *ad infinitum*.

The

The integer being thus divided by imagination into 10, 100, 1000, 10000, &c. is the denominator to the decimal fraction:

Thus $\frac{3}{10}$, $\frac{5}{100}$, $\frac{17}{1000}$, $\frac{51}{10000}$, &c.

These denominators are seldom or never set down, but only the numerators; and when the numerators do not consist of so many places as the denominator hath cyphers, the said places in the numerator must be supplied by cyphers prefixed on the left-hand. So $\frac{3}{10}$ is wrote .3, $\frac{5}{100}$ is .05, $\frac{17}{1000}$ is .017, and $\frac{51}{10000}$ is .0051, &c.

Also mixed numbers are expressed thus, viz. 8.7 is 8 and 7 tenths, 59.017 is 59 and 17 thousandths, or parts of a thousand, &c.

Cyphers at the end, namely at the right-hand of a decimal, do neither augment or diminish its value; for 5, .50, .500, .5000, and .50000, are decimals having the same value, being each equal to $\frac{1}{2}$, as may be found by abbreviation of vulgar fractions.

Cyphers prefixed to decimals, decrease their value in a ten-fold proportion, by removing them further from the integer.

$$\text{Thus}\left\{\begin{array}{l} .5 = 5 \text{ tenth parts.} \\ .05 = 5 \text{ parts of an hundred.} \\ .005 = 5 \text{ parts of a thousand.} \\ .0005 = 5 \text{ parts of ten thousand.} \\ .00005 = 5 \text{ parts of an hundred thousand, \&c.} \end{array}\right.$$

In whole numbers, the first place above (that is, on the left-hand of) the place of units, signifies tens of units; but in fractions, the first place beneath (that is, on the right-hand of) the place of units, denotes tenth parts of 1, or unity, and is called the first place of decimal parts, or place of primes; likewise the second place above the place of units, signifies hundreds of units; but the second place beneath the place of units, expresses hundredth parts of unity, and is called the second place of decimals, or place of seconds; so that as the value of the places in integers ascend in a ten-fold proportion from the place of units towards the left-hand, so the value of the places of decimals descend in a ten-fold proportion beneath the place of units towards the right-hand.

A TA-

A TABLE for NOTATION of INTEGERS and DECIMALS.

8	7	2	3	6	5	.	8	2	3	5	3	7	8	5
Hundreds of thousands, &c.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units place.		Tenth.	Hundredth parts.	Thousandth parts.	Parts of ten thousand.	Parts of an hundred thousand.	Parts of a million.	Parts of ten millions.	Parts of an hundred millions.

It may be observed by the foregoing table, that the places of integers, or whole numbers, are separated from the decimal parts by a point, that the numbers on the left-hand of the point expresses 872365 integers, or units; and that the number on the right-hand of the point shews .82353785 parts of 1 (or an integer) supposed to be divided into 100000000 equal parts.

Hence, if the separating point, in any mixed or fractional number, be moved one place towards the left-hand, then every figure, and consequently the whole expression is but a a tenth part of what it was before; that is, it is divided by 10; if it be moved two places, it is divided by 100; if three places by 1000, &c. But if the separating point be moved towards the right-hand, then the whole expression is multiplied by 10, 100, 1000, &c. according as it is moved one, two, or three places.

There are several ways of reading or expressing a decimal, as supposing the decimal parts in the table were to be read in words, viz. .82353785.

First, They may be reduced to, and expressed as vulgar fractions, viz. $\frac{82353785}{100000000}$.

Secondly, By calling them primes, seconds, &c. according to their distance from the separating point, viz. 8 primes, 2 seconds, 3 thirds, 5 fourths, 3 fifths, 7 sixths, 8 sevenths, and 5 eighths.

Thirdly, Thus 82 millions, 353 thousand, 785 eighths.

Fourthly, Or thus, 8, 2, 3, 5, 3, 7, 8, 5 of a decimal.

SECT.

SECT. II.

REDUCTION *of* DECIMALS.

CASE I.

To reduce a vulgar fraction into a decimal.

RULE.

Annex cyphers to the numerator, till it be equal to, or greater than the denominator; then divide by the denominator, and the quotient will be the decimal ſought.

If, after you have made uſe of all the cyphers annexed to the numerator, there be a remainder, annex cyphers thereto, and continue your diviſion, till it divide off or arrive to what degree of exactneſs you think proper.

Always obſerve to ſet a point betwixt the numerator and the cyphers annexed thereto, and that the quotient have as many places as you annex cyphers to the numerator and remainders; and if it be deficient, let the want be ſupplied by prefixing as many cyphers to the quotient as it falls ſhort.

EXAMPLE.

Reduce $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$, into decimals. Thus,

2)1.0(.5 . . . 4(1.00(.25 . . . 4)3.00(.75.

Alſo 8)1.000(.125, and 16)1.0000(.0625.

2. Reduce $\frac{1}{5}$, $\frac{2}{25}$, and $\frac{3}{125}$, into decimals.

5)1.0(.2 $= \frac{1}{5}$ . . 25)2.00(.08 $= \frac{2}{25}$ . . 125)3.000(.024 $= \frac{3}{125}$.

Thoſe decimals that are reduced from ſuch a vulgar fraction, whoſe numerator with cyphers annexed is an aliquot part of, or can be meaſured by its denominator, are finite or terminate decimals; as the decimals reſulting from the foregoing examples.

No fraction will produce a finite decimal, but ſuch whoſe denominator is 2 or five, and their multiples.

But ſuch as are produced from a vulgar fraction, whoſe numerator with cyphers annexed is no aliquot part of, or cannot be meaſured by its denominator, will be indeterminate or endleſs.

In circulating decimals, if one figure only repeats, it is called a ſingle repetend; as for example,

3. Sup-

3. Suppose the decimal of $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$, &c. was required.

9)1.0000 9)2.0000 9)3.0000

.1111, &c. $= \frac{1}{9}$. .2222, &c. $= \frac{2}{9}$. .3333 $= \frac{3}{9} = \frac{1}{3}$.

To avoid the trouble of writing down unnecessary figures, a single repetend is denoted by the repeating digit dashed; that is, the decimal .11111, &c. $= .\cancel{1} = \frac{1}{9}$ .22222, &c. $= .\cancel{2} = \frac{2}{9}$. Also .33333, &c. $= .\cancel{3} = \frac{3}{9} = \frac{1}{3}$, $.\cancel{4} = \frac{4}{9}$, $.\cancel{5} = \frac{5}{9}$, $.\cancel{6} = \frac{6}{9} = \frac{2}{3}$, $.\cancel{7} = \frac{7}{9}$, and $.\cancel{8} = \frac{8}{9}$.

4. Suppose it was required to reduce $\frac{1}{12}$, $\frac{5}{36}$, and $\frac{31}{960}$, into decimals.

12)1.000 36)5.000 960)31.0000000

$.08\cancel{3} = \frac{1}{12}$ $.13\cancel{8} = \frac{5}{36}$ $.032291\cancel{6} = \frac{31}{960}$.

The decimals resulting from these last examples are called mixed single repetends.

5. Let $\frac{2}{11}$, $\frac{3}{7}$, and $\frac{17}{286}$, be reduced into decimals.

11)2.000000 7)3.00000 286)17.0000000

.181818, &c. $= .\cancel{1}\cancel{8}$ $.\cancel{4}2857\cancel{1}$ $.0\cancel{5}9440\cancel{5}$.

Those decimals in which two or more figures circulate, are called compound repetends; and the manner of distinguishing them, is by dashing the first and last figure of the repetend, by which means we make one place of the repetend sufficient, as in the last example.

In a compound repetend, any one of the circulating figures may be made the first of the repetend; for instance, in the repetend $8.6\cancel{3}2\cancel{5}325325$, &c. it may be made $8.63\cancel{2}5\cancel{3}$, or $8.632\cancel{5}3\cancel{2}$. And by this means any two or more repetends may be made to begin and end in the same place; and then they are said to be conterminous.

5. Let $\frac{13}{3731}$ be reduced to a decimal.

3731)

```
3731)13.000(.0034843205, &c.
      18070
       31460
        16120
         11960
          7670
           20800
           -----
            2145
```

The decimal resulting from the last example is called an approximate decimal, having some places true, and the rest uncertain; these approximating decimals are sometimes written with the signs + or —, to denote whether the last figure is greater or less than just: thus .0034843205 +, or .00348843206 —; the first signifies that the decimal is greater than .0034843205 by some uncertain figures; and the second, viz. 0034843206—, denotes that the true decimal exceeds .0034843205, and is less than .003484320c6.

CASE II.

To reduce coins, weights, measures, &c. into decimals.

RULE I.

Reduce the different species into one, viz. the lowest denomination they consist of for a dividend; then reduce the integer into the same denomination for a divisor; the result will be the decimal required.

RULE II.

Write the given denominations or parts orderly under each other, the inferior or least parts being uppermost, let these be the dividends.

Against each part on the left-hand write the number thereof contained in one of its superior; let these be divisors.

Then beginning with the upper one, write the quotient of each division as decimal parts on the right-hand of the dividend next below it; and let this mixed number be divided by its divisor, &c. till all be finished, and the last quotient will be the decimal sought.

RULE

RULE III.

The decimal may be readily found by the rule of practice, namely, by considering the next inferior denomination as aliquot parts of the integer; and those still lower as aliquot parts of the superior ones, or of each other; the sum of all those aliquot parts will be the decimal required.

Ex. 1. Let $3\frac{3}{4}$ d. be reduced to a decimal, a pound sterl. being the integer.

By Rule I.	By Rule II.
$3\frac{3}{4}$ d.	4 | 3.00
4	12 | 3.75
———	20 | 0.3125
960)15.0(.015625 l. = $3\frac{3}{4}$.	———
540	0.15625
600	———
240	The decimal as before = $3\frac{3}{4}$ d.
480	

By Rule III.

$3\text{ d} = \frac{1}{80}$ of 1 l. = .0125
$\frac{3}{4}\text{ d} = \frac{1}{4}$ of 3 d. = .003125

Sum £ .015625

The decimal sought.

2. What decimal of a pound is 5 s. $7\frac{3}{4}$ d.

	By Rule II.
5 s. $7\frac{3}{4}$ d.	4 | 3 00
12	12 | 7.75
———	20 | 5.64583
67	0.2822916 = 5 s. $7\frac{3}{4}$ d.
4	
———	
96.0)27.1(.28229, &c.	
790	
220	
280	
880	
———	
(16)	

By

By Rule III.

5 s. $= \frac{1}{4}$ of 1 l. $= .25$
6 d. $= \frac{1}{10}$ of 5 s. $= .025$
$1\frac{1}{2} = \frac{1}{4}$ of 6 d. $= .00625$
$\frac{1}{4} = \frac{1}{6}$ of $1\frac{1}{2}$ d. $= .001041\dot{6}$

Viz. 5 s. $7\frac{3}{4}$ d $=$ £ $.282291\dot{6}$

3. What decimal of a pound is equal to 19 s. 11 d.?

19 s. 11 d.
12

$24.0)23.9(.9958\dot{3} =$ 19s. 11d.

By Rule II.

12	11.000
20	$19.91\dot{6}$
	$0.9958\dot{3} =$ 19 s. 11 d.

By Rule III.

19 s. — d. $=$ 95
— 6 $= \frac{1}{2}$ of 1 s. $= .025$
— 3 $= \frac{1}{2}$ of 6 d. $= .0125$
— 2 $= \frac{1}{3}$ of 6 d. $= .0083\dot{3}$

19 s. 11 d. $=$ £ $.9958\dot{3}$

4. What decimal part of an hundred weight is 2 qrs. 12 lb. 12 oz.?

qr. lb. oz.
2 12 12
28

68
16

1792)1100.0(.6138393
2480
6880
15040
7040
16640
512

By Rule II.

16 viz.	4	12 .. 4) 3.0
28 —	4	12.75 .. 7) 3.1875
	4	2.4553571 +
		0.61383 —

By

By Rule III.

			qr.	lb.	oz.
$\frac{1}{2}$	= .5	=	2	—	—
$\frac{1}{7}$ of 2 qr.	= .0714286	=	—	8	—
$\frac{1}{2}$ of 8 lb.	= .0357143	=	—	4	—
$\frac{1}{8}$ of 4 lb.	= .00446428	=	—	—	8
$\frac{1}{2}$ of 8 oz.	= .00223214	=	—	—	4
	.6138393	=	2	12	12, as before.

5. What decimal part of a pound Troy are 10 oz. 18dwt. 16 qrs?

```
oz. dwt. qr.
10  18  16
24
----
218
 24
----
 872
436
----              oz. dwt. qr.
576.0)524.8(.91 = 10  18  16
 640
 ---
  64
```

By Rule II.

```
24{ 4|16
    6|4.0
   20|18.6
   12|10.93
      .91 lb. troy.
```

6. What decimal part of a degree of a circle are 48′ 37″ 54‴?

```
48′ 37″ 54‴
60
-----
2917
  60
-----
216.000)175.074(.81052
 227
 1140
  600
 -----
 1680
```

By Rule II.

```
60|54.0
60|37.9
60|48.6316
   0.81052 = 48′ 37″ 54‴
```

7. What

7. What decimal part of a foot = $10\frac{1}{4}$ inches?

$10\frac{1}{4}$ inches.
4

```
48) 41.0(.85416
    260
    200
     80
     320
```

By Rule II.

```
 4 | 1.00
12 | 10.25
   | 0.85416 = 10¼ inches.
```

8. What decimal part of a gallon of ale = 133 cubic inches?

```
282) 133.0(.4716312 —
      2020
       460
       1780
        5880
         340
          580, &c.
```

9. What decimal part of a year = 217 days, 7 hours, 18 minutes?

```
 d.   h.   m.
217   7   18
 24
----
 868
434
-----
5215
  60
------
312918
    60
--------
31556937) 18775080.0(.5949588833667
          299661150
          156487170
           302504220
            185817870
             &c.
```

CASE III.

To reduce any decimal into the equivalent known parts of coin, weight, or measure

RULE.

RULE.

Multiply the given number by the number of units contained in the next inferior denomination, cutting off as many figures from the product as the given decimal consists of; then multiply the remaining parts (if any) by the next lower denomination, cutting off as before; and thus proceed till you have converted your decimals, or come to the lowest part; and the several figures to the left-hand of the separating points will be the several parts of the quantity required.

What known parts of coin are equal to .015625 l.?

```
 .015625
      20
 -------
 .312500
     12
 -------
3.750000   Answer, 3¾ d.
 4
3.000000
 -------
```

What known parts of coin are equal .282291$\dot{6}$ l.?

```
   .282291ṡ6
         20
s. ---------
  5.6458333̇
         12
d. ---------
  7.7500000
  4            Answer, 5s. 7¾ d.
   ---------
  3.0000000
   ---------
```

What known coin equals .9958$\dot{3}$ l.?

```
    .99583̇
       20
s.  ------
  19.91666̇
       12   Answer, 19 s. 11 d.
d.  ------
  11.00000
```

What

What known weight is .6138393 of a cwt.?

$$\begin{array}{rl} & .6138393 \\ & 4 \\ \hline \text{qr.} & 2.4553572 \\ & 28 \\ \hline & 36428576 \\ & 9107144 \\ \hline & 12.7499916 \\ & 16 \\ \hline & 11.9998656 = 12 \text{ oz. very near.} \end{array}$$

Answer, 2 qr. 12 lb. 12 oz.

CASE IV.

To reduce a decimal into its least equivalent vulgar fraction.

1st. If the decimal be finite,

RULE.*

Under the given decimal write an unit, with as many cyphers as the decimal consists of places; then divide both the numerator and denominator by the greatest common measure, which gives the least equivalent vulgar fraction required.

1. Required the least vulgar fractions equivalent to .5, .25, .75, .125, and .0625?

Answer $.5 = \frac{5}{10} = \frac{1}{2}$, $.25 = \frac{25}{100} = \frac{1}{4}$, $.75 = \frac{75}{100} = \frac{3}{4}$, $.125 = \frac{125}{1000} = \frac{1}{8}$, and $.0625 = \frac{625}{10000} = \frac{1}{16}$.

2. What is the least vulgar fraction equal to .625 and .5625?

Answer, $.625 = \frac{625}{1000} = \frac{5}{8}$, and $.5625 = \frac{5625}{10000} = \frac{9}{16}$.

How to find the greatest common measure is taught before in vulgar fractions, so shall give only one example to refresh the learner's memory.

Let

Let $\frac{5625}{10000}$ be reduced to its lowest or least equivalent fraction.

$$
\begin{array}{l}
5625)\,10000\,(1 \\
\quad\quad 4375 \;\; 5625\,(1 \\
\quad\quad\quad\quad 1250\,(4375\,(3 \\
\quad\quad\quad\quad\quad\quad 625 \;\; 1250\,(2 \\
\quad\quad\quad\quad\quad\quad\quad\quad \overline{} \\
\quad\quad\quad\quad\quad\quad\quad\quad (0)
\end{array}
$$

$5625\,)\frac{5625}{10000}\frac{9}{16}$, as before.

2d. If the given decimal be a repetend,

RULE.

The decimal is the numerator of a vulgar fraction, whose denominator consists of as many nines as there are recurring places in the given decimal; both which divide by their greatest common measure (as before) and their quotient will be the least equivalent vulgar fraction.

1. Required the least vulgar fraction equivalent to $\dot{6}$?

Answer, $0\dot{6} = \frac{6}{9} = \frac{2}{3}$.

2. What is the least vulgar fraction equal to $.\dot{7}6923\dot{0}$?

The greatest common measure to $\frac{769230}{999999}$ is found to be $.76923$.

Therefore, $.76923\,)\frac{769230}{999999}\left(\frac{10}{13}\right.$, the answer required.

3. What is the least vulgar fraction equal to $\dot{1}6\dot{2}$?

The greatest common measure to $\frac{162}{999}$ is 27.

Therefore, $27\,)\frac{162}{999}\left(\frac{6}{37}\right.$, the answer required.

3. When the given decimal is part final, and part a circulate,

RULE.

To as many nines as there are figures in the repetend, annex as many cyphers as there are finite places for a denominator;

minator; then multiply the nines in the said denominator by the finite part, and to the product add the repeating decimal for a numerator; these divided by their greatest common measure, will give the least equivalent fraction.

What is the least equal vulgar fraction to $.5\dot{3}$? $\overline{9 \times 5} + 3 = 48$ numerator; $.90$ being the denominator?

$\therefore \frac{48}{90} = \frac{8}{15}$, the least vulgar fraction required.

What is the least vulgar fraction equal to $.5\dot{9}2\dot{3}$?

First, $9990 =$ denominator, and $999 \times 5 + 923 = 5920$, numerator.

$\therefore 5\dot{9}2\dot{3} = \frac{5920}{9990} = \frac{16}{27}$, as was required.

What is tne least vulgar fraction equal to $.008\dot{4}9713\dot{3}$?

First 999999000 is the denominator.

Likewise $\overline{8 \times 999999} + 497133 = 8497125$, numerator.

And 102375, the greatest common measure.

$\therefore 102375) \frac{8497125}{999999000} (\frac{83}{9768}$, the answer.

A general rule for reducing decimal into vulgar fractions.

Under the given decimal set an unit, with as many cyphers as there are places in the given decimal; then set the finite decimal as a numerator, even under the lowest figures of the first numerator, with its proper denominator; lastly, subtract the under numerator from the upper one, and the under denominator from the upper one, the remainder will be a vulgar fraction equivalent to the given decimal, which reduce to its lowest terms.

1. What is the vulgar fraction equivalent to $.13\dot{8}$?

$$\frac{138}{1000}$$
$$\frac{13}{100}$$
$$\frac{125}{900} = .12\dot{8} = \frac{5}{36}.$$

 Re-

2. Required the vulgar fraction equal to .0084̇97133̇?

$$\text{From } \frac{8497133}{1000000000}$$

$$\text{Take - - - } \frac{8}{1000}$$

$$\text{Laves } \frac{8497125}{999999000} = .0084̇97133̇ = \frac{83}{9708}, \text{ as was required.}$$

✱ ✱

SECT. III.

Addition *of* Decimals.

WHEN decimal fractions are to be added together, observe that the commas, or separating points in each expression, be placed directly underneath each other; for then primes, seconds, thirds, &c. will fall under those of the same name; and in mixed numbers, units will fall under units, tens under tens, &c.

CASE I.

To add finite decimals.

RULE.

Add as in whole numbers, and from the sum or difference, cut off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Let .3746 + 137.5 + 1.34 + 375 + 1.85 + .0736285 + 87396.4 + 8.7386429 + 127 + 5.375, be added together.

.3746

·3746
137·5···
··1·347·
375······
··1.85··
····0736285
873964······
····8.7386429
127·······
5 275

83053 5583714

CASE II.

To add decimals wherein are single repetends.

RULE.

Make every line end at the same place, filling up the vacancies by the repeating digits, and annexing a cypher or cyphers to the finite terms; then add as before, only increase the sum of the right-hand row with as many units as it contains nines; and the figure in the sum, under that place, will be a repetend.

Let $3.\dot{6} + 78.347\dot{6} + 735.\dot{3} + .375 + .2\dot{7} + 187.\dot{4}$, be added together.

$$\begin{array}{r} 3.\dot{6}66\dot{6} \\ 78.347\dot{6} \\ 735.\dot{3}333 \\ \cdots.3750 \\ \cdots.2\dot{7}77 \\ 187.\dot{4}44\dot{4} \\ \hline 1005.444\dot{8} \end{array}$$

SECT.

SECT. IV.

Subtraction *of* Decimals.

CASE I.

To subtract finite decimals.

RULE.

HAVING first set down the greater of the two numbers given (whether it be a whole number, mixed number, or decimal) set down the less under it, according to the directions given in addition; then subtract as in whole numbers, imagining all the vacant places filled with cyphers.

From 375.5 take 86.47284. Also from 87.569245 take 19.87.

Minuend	375.5	87.569245
Subtrahend	86.47264	19.87
Remainder	289.02716	67.699245

From 1 take .732594. And from 684 take 9.3275.

From	1.0	684. . . . .
Subtr.	.732594	9.3275
Rem.	.267406	674.6725

Let 375.5 be diminished or made less by .97637387, and shew their difference.

Minuend	375.5 . . . .
Subtrahend	.97637387
Difference	374.52362613, or remainder.

CASE

CASE II.

To subtract decimals that have repetends.

RULE.

Make the repetends similar and conterminous, and subtract as in the last case; observing only, if the repetend of the number to be subtracted, be greater than the repetend of the number it is to be taken from, then the right-hand figure of the remainder must be less by unity, than it would be, if the expressions were finite; and the repetend in the remainder will consist of as many places as there are in the other two numbers.

Let $57.7\dot{3}$ be lessened by $18.9541\dot{6}$, and $51.52\dot{8}$ by $.\dot{6}$.

From	$57.7333\dot{3}$	51.528
Take	$18.9541\dot{6}$	$.66\dot{6}$
Rem.	$38.7791\dot{6}$	$50.86\dot{1}$.

Let $47.4\dot{1}7817\dot{8}$ be made less by $15.\dot{5}\dot{6}$, and $49.\dot{5}2\dot{8}$ by 38.4736.

From	$47.4\dot{1}7817\dot{8}$	$49.5285\dot{2}8\dot{5}$
Take	$15.5\dot{6}5656\dot{5}$	$38.4736\dot{0}0\dot{0}$
Rem.	$31.8\dot{5}2161\dot{7}$	$11.0549\dot{2}85$

From $43.8\dot{5}4026\dot{0}$ take $20.\dot{9}2\dot{5}$, and from $49.5\dot{3}$ take $42.\dot{7}5\dot{9}$.

From	$43.8\dot{5}4026\dot{0}$	$49.5\dot{3}3\dot{3}$
Subt.	$20.9\dot{2}5925\dot{9}$	$42.7\dot{5}9\dot{7}$
Rem.	$22.9\dot{2}8100\dot{1}$	$6.7\dot{7}3\dot{5}$

SECT.

SECT. V.

Multitiplication *of* Decimals.

CASE I.

When both factors are finite decimals, whether they are single, or joined with integers,

RULE.

Multiply them as if they were all whole numbers, and from the product (towards the right-hand) cut off so many places for decimal parts in the product, as there were in both the multiplier and multiplicand counted together. But if it so happen that there are not so many places in the product, supply the defect by prefixing cyphers.

```
   8.7537          864          27.576
    .3275          .78            6.23
  -------         ----          -------
   437685         6912           82728
  612759         6048           55152
 175074          ------        165456
262611           673.92        ---------
----------       ------        171.79848
2 86683675                     ---------
----------
```

```
     .57386            .27345
      .8237             .273
    -------            -------
     401702             82035
    172158             191415
   114772              54690
  459088              ---------
 ------------         .07465185
 .472688482           ---------
 ------------
```

CASE

CASE II.

Two decimal fractions being given, to reserve in their product any assigned number of places.

RULE.

Set the unit's place of the multiplier directly under that figure of the decimal part of the multiplicand, whose place you would reserve in the product, and invert the order of all its other places; that is, write the decimals on the left-hand, and the integers, if any, on the right.

Then in multiplying, always begin at that figure of the multiplicand which stands over the figure wherewith you are then multiplying, setting down the first figure of each particular product directly underneath one another, due regard being had to the increase which would arise out of the two next figures, to the right-hand of that figure in the multiplicand, which you then begin with; carrying one from 5 to 15; two from 15 to 25; three from 25 to 35, &c. and the sum of these lines will give the product.

Let 73.8429753 be multiplied into 4.628754, reserving only five places of decimal parts in the product.

73.8429753, the multiplicand, as usual.
457826.4, the multiplier inverted, with the unit's place set under the 5th place in decimals, denoting that there will be five places of parts in the product.

```
 29537190
 4430579
  147686
   59074
    5169
     369
      30
---------
341.80097
```

The work at large.

```
      73.8429753
       4.628754
---------------------
       29|537190 12
      369|2148765
     5169|008271
    59074|38024
   147685|9506
  4430578|518
 29537190|12
---------------------
341.80096|72917762
```

Let

Let 843.7527 be multiplied into 8634.175, reserving only the integers in the product.

```
 843.7527              843.7527
 578.4368              8634.875
 --------          ----------------
 6750022                  4|2187635
  506251                 59|962689
   25313                675|00216
    3375               3375|0108
     675              25312|581
      59             502651|62
       4            6750021|6
 -------           ----------------
 7285699           72856699|0954125
```

CASE III.

If the right-hand figure of the multiplicand be a circulate,

RULE.

In multiplying increase the right-hand figure of each resulting line by as many units as there are nines in the product of the first figure in that line, and the right hand figure of each line will be a circulate; and before you add them together, make them all end at the same place.

```
 .1726            835.273
     6              .7484
 ------         -----------
 1.0366           3341093
                 66821866
                334109333
               5846913333
               -----------
              625.1185626
```

CASE IV.

If the right-hand figure of the multiplier be a circulate,

RULE.

Multiply by it as by a finite digit, setting the product one place extraordinary towards the left-hand; then divide that

that product by 9, continuing the quotient (if needful) till it arrives at a circulate; then beginning at the place under the right-hand figure of the multiplicand, cut off for decimal parts.

$$\begin{array}{r} 63.274 \\ .11\dot{6} \\ \hline 9)379644 \\ \hline 421822\dot{6} \\ 189822 \\ 63274 \\ 63274 \\ \hline 7.1921442\dot{6} \\ \hline \end{array} \qquad \begin{array}{r} .47375 \\ .876 \\ \hline 9)284250 \\ \hline 31583\dot{3} \\ 331625 \\ 379000 \\ \hline .4153208\dot{3} \\ \hline \end{array}$$

CASE V.

When the multiplicand and multiplier are each a single circulate,

RULE.

The first line (or that produced by multiplying by the circulate in the multiplier) must be managed as in Case III. only the right-hand figure must be increased by as many units as there are nines in the product of the first figure of that line, the products of the rest must be managed as directed in Case II.

$$\begin{array}{r} 3.72\dot{3} \\ .2\dot{6} \\ \hline 9)2234\dot{0} \\ \hline 24822 \\ 74466 \\ \hline .99288 \\ \hline \end{array} \qquad \begin{array}{r} 8574.\dot{3} \\ 87.\dot{5} \\ \hline 9)42871\dot{6} \\ \hline 47635\dot{1}8\dot{5} \\ 60020\dot{3}333 \\ 68594\dot{6}6666 \\ \hline 750730.5\dot{1}8\dot{5} \\ \hline \end{array}$$

CASE VI.

If the multiplicand be a compound repetend, and the multiplier a finite number,

RULE.

RULE.

In multiplying, obſerve to add to the right-hand place of the product, ſo many units as there are tens in the product of the left-hand place of the repetend; and the product ſhall contain a repetend, whoſe places are equal to thoſe in the multiplicand; and if there are more places of figures in the multiplier than one, make all the ſeveral products conterminous towards the right-hand, as in Caſe II. and IV.

$$\begin{array}{r} 3.\dot{9}7\dot{3} \\ 8 \\ \hline 31.\dot{7}9\dot{1} \end{array} \qquad \begin{array}{r} 78\dot{6}.492\dot{8} \\ .005 \\ \hline 3.93\dot{2}464\dot{3} \end{array}$$

$$\begin{array}{r} 732.\dot{5}8\dot{6} \\ 43.7 \\ \hline 5128\dot{1}0\dot{6} \\ 2197\dot{7}5\dot{9}7 \\ 2931\dot{3}4\dot{6}34 \\ \hline 32024.0\dot{3}3\dot{8} \end{array} \qquad \begin{array}{r} 86.\dot{3}207\dot{3} \\ 37.135 \\ \hline 431\dot{6}036\dot{6} \\ 258\dot{9}621\dot{9}9 \\ 86\dot{3}207\dot{3}32 \\ 604\dot{2}4513245 \\ 25\dot{9}621\dot{9}9621 \\ \hline 3205.520\dot{4}276\dot{5} \end{array}$$

CASE VII.

It the multiplier be a compound repetend,

RULE.

Multiply each figure of the repetend, and add the ſeveral products together; then add the reſult in this manner; ſet the left-hand figure ſo many places forward as exceeds the number of places in the repetend by one, and the reſt of the figures in order after it; and thus proceed, till the reſult laſt added be carried beyond the firſt; laſtly, add the ſeveral reſults together, beginning under the right-hand place of the firſt; and from thence daſh as many figures for a repetend, as the repetend of the multiplier conſiſts of.

834.

834.75	49640.54
$\dot{3}.2\dot{7}$	$.\dot{7}050\dot{3}$
584325	14892162
166950	24820270
250425	34748378
27296325	349980699162
272963	34998069
272	349
$2732.3\dot{6}4\dot{8}$	$34968.41\dot{9}900\dot{3}$

If the multiplier hath any terminate places joined with the repetend, and if the repetend be ſmall, and theſe many, multiply and add the products of the repetend firſt; then multiply by the terminate figures, and add their products to the ſum of the products of the repetend; and to this laſt reſult, add the ſaid ſum of the repetend products.

874.37
$1.2\dot{7}\dot{4}$

349748
612059

Sum 6470338 of the products of the repetend.
174874
87437

1113.94738
647033
6470
64

$1114.600\dot{6}\dot{4}$

But if the terminate figures are few, and the places of the repetend many, ſubtract the terminate figures from thoſe of the repetend, and multiply by the remainder as a repetend.

1735.8072
32.4738
32
Remains 324704, the new multiplier.

69432288
121506504
69432288
34716114
52074216
5636235410688
5636235410
563623
56
56367.9902597

CASE VIII.

If both factors have compound repetends,

RULE,

Proceed as in the two last cases; for as the places of the repetend in the product will be uncertain as to their number, they can only be determined (in any manner fit for practice) by continuing and repeating the first product, which will contain a certain repetend equal in places to that of the multiplicand.

Multiply 67.824
into 5.278
52
5.223
203474
1356406
135640664
339124124
354.2490660
3 5424906
354249
3542
35
357.8273333, &c. Here the fourth place of parts comes out a single repetend, viz. 3.

Again.

Again. Multiply 3.148
into 4.797
4

```
         4293

         9438
       283090
       620090
     12581818

     13503438363636363
       1350343636363636
          13503436363
              135034
                 135

     13.5769533169533
```

Examples of this kind, though very accurate, yet are more curious than useful; as they may be easier done exact enough for business, by the contracted way of multiplication taught Case II.

SECT. VI.

Division of Decimals.

IN any of the following cases in division, if the dividend be greater than the divisor, the quotient will be either a whole or a mixt number; but when the dividend is less than the divisor, the quotient must necessarily be a fraction; for a less number is contained in a greater once at the least, but the greater is not contained once in the less.

CASE I.

When the divisor and dividend are both finite decimals.

RULE.

Divide as in whole numbers, and from the right-hand of the quotient point off for decimals so many places as the decimal places in the dividend exceed those in the divisor; and those to the left, if any, are integers; but if the places of the quotient are not so many as this rule requires, supply

 the

ply the defect by prefixing cyphers to the quotient; but if the decimal places in the divisor be more than those in the dividend, annex cyphers to the dividend to make them equal, and the quotient will be integers until all those cyphers are used.

```
87·364 ) 714 025972 ( 8.173
           15·1139
            637757        ·7875 ) 441.0000 ( 560
             262092                  47250
            ------                   -----
            ......                   .....

179 ) 48624097 (.00271643
      1282....
        294          .2628 ) 27.0000 ( 100.55865
        1150                  1500
          769                  15750
          537                   23250
          ---                    17700
          ...                     15900
                                   24750
                                   -----
                                     585
```

CASE II.

To contract the work of division, when the divisor consists of many decimal places.

RULE.

Having determined the value of the quotient figures, let each remainder be a new dividend; and for every such dividend, point off one figure from the right-hand of the divisor; observing at each multiplication to have regard to the increase of the figures so cut off, as in contracted multiplication.

```
.67268479 ) 56·00000000 ( 83·2485
 .......      218521680
               16716243
                3262547
                 571808
                  33661
                  ...27
                  -----
```

384.672158)

```
384.672158) 14169.206603851 ( 36,8345
  ...        2629041863
             3210089158
              132711894
              17310247
               1923361
               .......

9.365407 ) 87.076326 ( 9.2976554
            2787663
             914582
              71696
               6138
                519
                 51
                  4
```

If any whole, mixed, or decimal number is given to be divided by 10, 100, 1000, &c. only remove the separating point towards the left-hand so many places as there are cyphers in the divisor; also in multiplication the separating point is moved to the right-hand so many places as there are cyphers in the multiplier.

EXAMPLES in

Multiplication.	Division.
.7865 × 10 = 7.865	10) 7865 (786.5
.7865 × 100 = 78.65	100) 7865 (78.65
.7865 × 1000 = 786.5	1000) 7865 (7.865
.7865 × 10000 = 7865	10000) 7865 (.7865
.7865 × 100000 = 78650	100000) 7865 (.07865

CASE III.

If the dividend be a repetend.

RULE.

If it be a single repetend, bring down the circulating figure until the quotient either repeats, or is as exact as required; but if the repetend in the dividend be a compound one, then bring down the circulating figures in the same order they stand in; and when you have got through them all, bring down the first figure in the repetend over again;

 and

and so proceed until your quotient either repeats, or becomes as exact as is necessary.

```
.7484 ) 625118.5626 ( 835273.3
        26398
         39465        137 ) .566 ( .0041362533
          20456              186
           54882              496
            24946              856
          -------               346
            2494                 726
                                  416
                                 -----
                                  56

41.764 ) 1761.3840 ( 42.17, &c. ad infinitum.
           90804
            77760
            309961
           --------
             17613

764.5 ) 319.28007111 ( .4176328
          13480
           58350
            48357
             24871 . . .  }
              19361       }  ad infinitum.
               40712      }
              -------
               2487
```

CASE IV.

If the divisor be a single repetend.

RULE.

If the divisor be only a single repetend, place the dividend under itself, but one place forward towards the right-hand, which subtract from the dividend; the remainder will be a new dividend, which divide by the divisor, in the same manner as if it was a terminate number. But if the divisor consists of terminate numbers, joined to the repetend, subtract those terminate numbers from the divisor, and subtract the dividend as before directed, and the remainder will be a new divisor and dividend.

Divide

Divide 134.2$\dot{6}$ by .$\dot{6}$.

1342

.6) 120.84 (201.4, the true quotient.

Divide 234.$\dot{6}$ by .$\dot{7}$.

234

.7) 211.2 (301.$\dot{7}$1428$\dot{5}$, &c. *ad infinitum*.

Divide 6.2511856 2$\dot{6}$ by 875.27$\dot{3}$.

875.27$\dot{3}$) 6.25118562$\dot{6}$
87527 625118562

787.746) 5.626067064 (.00714198
1118450
3307046
1560624
7728780
6390660

88692

Divide 856.988 by 4.8$\dot{6}$.

4.86) 856.988
48 85.6988

4.38) 771.2892 (176.0934247
3332
2668
4092
1500
1860
1080
2040
3080

14

CASE V.

If a compound repetend is found in your divisor only, or in both your divisor and dividend,

RULE,

Set the divisor and dividend under themselves, each so many places towards the right-hand, as there are places in the repetend of the divisor; which subtract as in the last case, and the remainder will be a new divisor and dividend. But if the divisor is a compound repetend without any terminate figures, divide by it as a terminate number; first subtracting the dividend from itself, as before directed.

Divide 13.5169533 by 4.297.

4.297) 13.5169533169
4 135169533

4.293) 13.5034363636 (3.145
6244
19513
23416 &c. *ad infinitum.*
1951

Divide 5264.4574579 30 by 42.3430.

42.3430) 5264.4574279 30
42 526445745

42.3394) 5263.931007194 (124.32701, true quotient.
1029991
1832030
1384540
1143587
2967991
423394

......

Divide 395.273014 by .317.

395273

.317) 394.878341 (1245.673
778......
1447
1798
2133
2314
951

...

The

The following lemmas and corollaries may be of uſe in illuſtrating the different methods and peculiar proceſſes uſed in the arithmetic of circulating numbers.

LEMMA I.

A ſeries of nines infinitely continued, is equal to unity or one, in the next left-hand place. Thus, 0.9999, &c $= 1$, and .0999, &c. $= .1$; alſo, .00999, &c. $= .01$; and 73.999, &c. $= 74$.

DEMONSTRATION. It is evident that $.9 = \frac{9}{10}$ wants only $\frac{1}{10}$ of unity, .99 wants only $\frac{1}{100}$, and .999 wants only $\frac{1}{1000}$; ſo that if the ſeries were continued to infinity, the difference between that ſeries, of nines and an unit, would be equal to unity divided by infinity; that is, nothing at all.

Q. E. D.

LEMMA II.

Any ſingle repetend divided by 10, and the quotient ſubtracted from the ſaid repetend, the remainder will be the ſame number complete or terminate.

DEMONSTRATION. Let the given repetend be 3.333, &c. $3.333 \div 10 = .333$, and $3.333 - .333 = 3$. Alſo, 54.444, &c. $\div 10 = 5.444$, &c. and $54.444 - 5.444 = 49$.

COROLLARY I.

Hence it follows, that if a compound repetend be divided by an unit, with ſo many cyphers annexed as are equal to the places of the repetend, and the quotient ſubtracted from the ſaid repetend, the remainder will be the ſame number complete or terminate, that conſtituted the repetend. Thus, $\dot{3}2\dot{5}.325 \div 1000 = \dot{3}2\dot{5}$; and $\dot{3}2\dot{5}.325 - .\dot{3}2\dot{5} = 325$; and $42.\dot{3}43\dot{6} \div 10000$, will be $.0042\dot{3}43\dot{6}$; and $42.\dot{3}43\dot{6} - .0042\dot{3}43\dot{6}$, will be 42.3394.

COROLLARY II.

Hence alſo, if any repetend be divided by an unit with as many cyphers as it contains places, and the quotient multiplied by as many nines as the repetend contains places, the reſult will be the ſame as before; that is, the ſame number terminate or complete; for any number divided by 10, and the quotient ſubtracted, the remainder is the ſame as the quotient multiplied by nine.

1. Thus, $6.666 \div 10 = .6666$; and $.6666$, &c. $\times 9 = 5.999$, &c. $= 6$.

2. Again, $\dot{3}2\dot{5} \times 999 = 324.999$, &c. $= 325 = \dot{3}2\dot{5}.325 - \dot{3}2\dot{5}$.

3. And $\dot{3}2\dot{5}.325 \div 1000 = .\dot{3}2\dot{5}$.

COROLLARY III.

It is evident from the last corollary, that a single repetend is to the same number terminate or complete, as 10 is to 9; a compound repetend of two places, as 100 to 99, and a compound repetend of three places is to the same number terminate or complete, as 1000 to 999, &c. And by the converse of the said corollary it must follow, that any number multiplied by 1, with as many cyphers as it contains figures, and the product divided by as many nines, will give the same number perpetually circulating.

Thus, $6 \times 10 = 60$, and $60 \div 9 = 6.66\dot{6}$, &c.
And $325 \times 1000 = 325000$, and $325000 - 999 = \dot{3}2\dot{5}$.

COROLLARY IV.

Hence also, if any number be divided by as many nines as it contains figures, and the quotient added to the said number, the result will be the same as before; for any number multiplied by 10, and the product divided by 9, the quotient must be equal to $\frac{1}{9}$ of the same number added to itself.

Thus, the quotient of $6 \div 9 + 6 = 6.666$, &c,
And the quotient of $325 \div 999$ added to $325 = \dot{3}2\dot{5}$.

LEMMA III.

Any number divided by 9, 99, 999, &c, will be equal to the sum of the quotients of the same number continually divided by 10, 100, 1000, &c.

Thus, 717 divided by 10, 100, &c,

```
71.7
 7.17         9)717(79.6̇
  717
   717
    717
     717
---------
79.6666, &c.
```

Also,

Alfo,

$\dot{2}3684\dot{7} \div 1000$, &c.	999) 236847 (237.$\cancel{0}$8$\cancel{4}$
236.847	3704
.236847	7077
236847	8400
236	4080
237.$\cancel{0}\cancel{8}\cancel{4}$084	84

From thefe lemmas and corollaries appear the reafon of multiplication and divifion of fingle as well as compound repetends.

I fhall here add the following ufeful propofition, viz.

To perform the work of multiplication by divifion, or of divifion by multiplication.

RULE.

Divide an unit, with cyphers annexed, by the given multiplier or divifor, the quotient will be the divifor or multiplier fought.

Let 27576 × 625. I would alfo have a divifor which will give a quotient equal to the product of thofe numbers.

27576	625) 1.000 (.0016	) 27576.0000 (17235000
625	3750	115
137880	....	37
55152		56
165456		80
17235000		..

Let 67392 be divided by 78, and find a multiplier, which being multiplied into the fame number, fhall produce a number equal to the quotient,

78) 67392 (864
499
312
...

78) 1.00 (.0128205
220
640
160
400
10

67392
× .0128205
336960
134784
539136
134784
67392
863.9991360
8639991360
8639
863.9999999999999 = 864.

QUESTIONS *to exercise* DECIMALS.

1. A grocer bought two chests of sugar, the one weighing net 18 cwt. 3 qrs. 14 lb. at 2 l. 9 s. 8 d. per cwt.; the other weighed net 18 cwt. 1 qr. 21 lb. at $4\frac{1}{2}$ d. per pound, which he mingled together; now I desire to know how much a hunded weight of this mixture is worth?

l.	s.	d.	
2	9	8	per cwt.
		6	
14	18	–	
	3		
44	14	–	
1	4	10	$\frac{1}{2}$
–	12	5	$\frac{1}{2}$
–	6	$2\frac{1}{2}$	
46	17	$5\frac{1}{2}$	
38	14	$4\frac{1}{2}$	
£85	11	10	whole cost.

12 | 10
20 | 11 8.3
£ 85.5916

1 qr. = .25
7 lb. = .0625
37.3125

$4\frac{1}{2}$ d.
× 4
1 6
× 7
10 6
× 4
2 2 – a cwt.
6
12 12 –
3
37 16 –
10 6
5 3
2 $7\frac{1}{2}$
38 14 $4\frac{1}{2}$

cwt.

cwt.	qr.	lb.
18	3	14
18	1	21
37	1	7

37 1 7 = 37.3125) 85.5916 (2.293914
1096666 | 20
3504166 | 5.878280
146041 | 12
34102 | 10.539360
522 | 4
149 | 2.157440.

Answer, 2 l. 5 s. $10\frac{1}{2}$ d.

2. What quantity of water will you add to a pipe of mountain wine, value 33 l. to reduce the first cost to 4 s. 6 d. the gallon?

12 | 6
20 | 4.5

0.225) 33.000 (146.6
1050 (126 gallons in a pipe.
1500
(150)20.6 gal. the answer.

3. If a cubic inch of oil olive be .52835 decimal parts of an ounce averdupoise, what quantity of oil, weighing $7\frac{1}{2}$ pounds per gallon, will be contained in a cask, allowed to hold $13\frac{1}{3}$ gallons of water, each 282 solid inches.

13.3 × 282 = 3760 cubic inches, content of the cask.
3760 × .52835 = 1986.596 oz. weight of the oil.
16) 1986.596 (124.12255 lb. averdupoise.
38
66
25 7.5) 124.16225 (16.555 galls. the ans.
99 491
36 416
40 412
80 372

4. A person was possessed of a $\frac{3}{5}$ share of a copper-mine, and sold $\frac{3}{4}$ of his interest therein for 1710 l.; what was the reputed value of the whole property at the same rate?

$\frac{3}{4}$ of $\frac{3}{5} = \frac{9}{20} = .45$.
.45) 1710.00 (3800 l. the answer required.
360

5. If

5. If I buy 14 yards of cloth for 10 guineas, how many ells Flemish can I buy for 283l. 17s. 6d. at the same rate?

l. s.
7 | 10 10
2 | 1 10
4 | 15 -d. per yard.
3 9
11 3 = .5925 per ell Flemish.

By inspection, 283l. 17s. 6d. = 283.875l.

.5625) 283.875 (504.6 = 604 ells, 2 qrs. the answer.
26250
37500
37500
...

6. Goliah of Gath is said to have been six cubits and a half, or a span, high; this answers to 10 feet, 4.592 inches; pray what was the length of the cubit in British measure?

12) 4.592 (.3826
10.3826 feet, the height of Goliah.
6.5) 10.3826 (1.5973241 = 1 foot, 7.168 inches, the ans.

7. A factor bought 84 pieces of stuff, which cost 537l. 12s. at 5s. 4d. per yard; I demand the number of yards in all, and how many yards in each piece?

12 | 4
20 | 5.3
£ .26 = 5s. 4d.

20 | 12
537.6 = 537l. 12s.

.26) 537.6 (2016 yards in all, and 7 x 12 = 84.
2 53.76
.24) 483.84
144
...

12 | 2016
7 | 168
24 yards in each piece.

CHAP.

CHAPTER VI.

EVOLUTION.

OR,

Extracting the Roots *out of all* Single Powers.

EVOLUTION is the unravelling or unfolding any proposed number into the parts of which it was made up of or composed.

If any number is multiplied into itself, that product is called a square number.

Thus the square numbers 4, 9, 25, 36, &c. are each of them composed of two equal numbers, viz; $2 \times 2 = 4$, $3 \times 3 = 9$, $4 \times 4 = 16$.

If any number be multiplied into itself, and that product be multiplied into the same number, the second product is called a cube number.

Thus the cube numbers, 8, 27, 64, &c. are each composed of $2 \times 2 = 8$, $3 \times 3 \times 3 = 27$, $4 \times 4 \times 4 = 64$, &c.

These powers exist in nature, viz. a root is represented by a line or side, having but one dimension, viz. only length: the square is a plane figure of two dimensions, viz. length, breadth; and the cube of three, viz. length, breadth, and thickness.

All the superior powers have no existence in nature, but are composed of a multiplication of any number four or more times into itself.

Thus, $2 \times 2 \times 2 \times 2 = 16$, the biquadrate, whose root is 2.

Or, $3 \times 3 \times 3 \times 3 \times 3 = 243$, the sursolid, whose root is 3, have no existence in nature, but may be understood as a series of numbers in geometrical progression.

When any number is proposed to have the root extracted, the first work is to prepare it by points set over (or under) their proper figures, according as the given power, whose root is sought, doth require; which for the square is 2, for the cube is 3, for the biquadrate 4, &c. always beginning those points over the place of unity towards the left-hand, if the given numbers be integers, and descend towards the right-hand in decimal parts.

Thus

Thus for the ſquare-root 5837429643847

Cube 5837429643847

Biquadrate 5837429643847

Surſolid 5837429643847

Or in decimals,

Thus for the ſquare-root 0.5327943847 28

Cube 0 532794384728

Biquadrate 0.532794384728

Surſolid 0.532794384728

A TABLE OF POWERS.

	Square or ſecond power.	Cube or third power.	Biquadrate, or ſquare ſquared, or 4th power.	Surſolid, or the fifth power.	Cube ſquared, or the ſquare cubed, or the ſixth power.	Second ſurſolid, or ſeventh power.	Biquadrate ſquared, or the eighth power.	Cube cubed, or the ninth power.
	In	Ind	Inde.	Index	Index.	Index.	Index.	Index.
	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

SECT.

SECT. I.

To extract the SQUARE-ROOT.

RULE.

HAVING pointed the given resolvend as before directed, find the greatest square that is contained in the first period towards the left-hand, setting down the root as a quotient, and subtract that square out of the first period.

2. To the remainder bring down the two figures under the next point for a dividend. N.B. This is always to be repeated.

3. Double the quotient for a divisor, enquiring how often it may be had in that dividend (excepting the last figure) and set down the quotient figure, which annex to the divisor. This must also be repeated, as a new divisor must be found to every figure.

4. Then multiply the whole increased divisor, and subtract the product from the dividend. Proceed thus till all the periods are brought down.

5. After doubling the quotient every time for a divisor, always annex the last quotient figure to the divisor for a new divisor.

6. If there be a remainder after you have finished your periods, bring down a pair of cyphers for decimals; proceeding as before directed, till the root is as exact as is required.

N. B. You must always, in mixed numbers, cut off as many whole numbers in the root, as there are periods of whole numbers, and as many decimals as there are periods of decimals.

1. Extract the square root of 393129.

```
          .  .  .
         393129(627
         36
         ---
   122) 331
        244
        ----
  1247) 8729
        8729
        ----
        ....
```

Extract

2. Extrnct the square root of 7587341933786503919510 5089.

7587341933786503919510 5089 (8710534962783
64

167) 1187
1169

1741) 1834
1741

174205) 931933
871025

1742103) 6090478
5226309

17421064) 86456965
69684256

174210689) 1677270903
1567896201

174210698) 109374702
4848283
1364070
144895
5527

The five last figures in the example above are found by the contracted method of division.

3. What is the square root of 1850701.764025 ?

1850701.764025 (1360,405, the root required.
1

23) 85
69

266) 1607
1596

27204) 110176
108816

2720805) 13604025
13604025

4. Extract

4. Extract the ſquare-root of .001234.

```
  . . . .
0 001234(.0351283362, the root very near.
     9
   -----
65)334
   325
   -----
701)  900
      701
      -----
7022) 19900
      14044
      ------
70248) 585600
       591984
       ------
        23616
         2542
          435
           14
```

5. Extract the ſquare-root of 2.

```
2 ( 1.414213527, root nearly.
1
-----
24) 100
     96
    -----
281)  400
      281
      -----
2824)  11900
       11296
       ------
28282)   60400
         56564
         ------
282841)  383600
         282841
         ------
282842)  100759
          15906
            764
            198
```

RULE II.

First deduct the greatest square, placing the root in the quotient as before.

2. Divide the whole remainder by 2, and point it a-new: this may be called a new dividend.

3. Make the root of the first square a divisor, enquiring how often it may be found in a new dividend, to the next figure, reserving the figure under the next point, for half the square of the quotient figure.

4. Multiply the divisor into it, adding to that product the tens of the half square, if any.

5. Annex the quotient figure to the last divisor for a new divisor, with which proceed as with the last until all be finished.

6. Extract the square root of 3272869681.

$$
\begin{array}{l}
\dot{3}2\dot{7}2\dot{8}6\dot{9}6\dot{8}1\ (\ 5,\ \text{first single root.}\\
25\\
\hline
2)772869681\\
\hline
\end{array}
$$

$$
\begin{array}{lll}
5) & \dot{3}8\dot{6}4\dot{3}4\dot{8}4\dot{0}.\dot{5}\ (57209 & \\
+7 & 374.5 = \overline{5\times 7} + \tfrac{1}{2}\ \text{square of } 7,\ \text{viz. } \tfrac{49}{2} = 24.5 & \\
\hline
57 & 1193 & \\
+ & 2)1142 = \overline{57\times 3} + \tfrac{1}{2}\ \text{square of } 2 & \\
\hline
572 & 5148405 & \\
+09 & 5148405 = \overline{572\times 9} + \tfrac{1}{2}\ \text{square } 9 = 40.5. & \\
\hline
\end{array}
$$

7. What is the square-root of $\frac{7}{9} = .\dot{7}$?

$$
\begin{array}{l}
0.77777\dot{7}\ (.8\\
\ 64\\
\hline
2)13777\dot{7}\\
\hline
.068888,\ \&c.\\
\hline
\end{array}
$$

```
.8) .068888 (.881917103688207
 +8  672
 ----  ----
.88)   16888
 +1     8805
 ----   -----
.881)   808388
  + 9   793305
 -----  -------
.8819)   1508388
   +1     881905
 ------   -------
.88191)   62648388
    + 7   61733945
 -------  ---------
 .881917)   91444388
     + 1    88191705
 --------   ---------
 .8819171 )   3252683888
       + 03   2645751345
 ----------   -----------
 .881917103    606932543
   ......       77792281
                 7238913
                  182570
                    6193
```

SECT. II.

Some USES *of the* SQUARE-ROOT.

CASE I.

TO find a mean proportional between any two given numbers.

RULE.

Extract the square-root of the product of the two numbers, which root will be the mean proportional sought.

Required a mean proportional between 16 and 256.

First $16 \times 256 = 4096$; also, $\sqrt{4096} = 64$.

```
                              36
                              ---
∴ 16 : 64 :: 64 : 256.   124)496
                             496
```

CASE II.

To find the ſide of a ſquare equal in area to any given ſuperficies.

RULE.

Find the ſquare-root of the given ſuperficies, which is the ſide of the ſquare ſought.

Suppoſe I have a circular, elliptical, or irregular fiſh-pond, containing in ſurface 9 acres, 2 roods, 15 perches, and would have a ſquare one of the ſame content; I deſire to know how many yards the ſide muſt be?

```
     A. R. P.
     9  2 15
     4
    --
    38
    40
   ----
   1535 perches.
5.5 × 5.5 = 30.25 ſquare yards in 1 perch.
   -----
    7675
   3070
  4605
  --------
   . . . .                          yds. foot. inch.
   46433.75(215.484918=215  1  5.467.
   4                                 Q. E. F.
   -
 41)64
    41
    --
 425)2333
     2125
     ----
 4304) 20875
       17216
       -----
 43088) 365900
        344704
        ------
 430964) 2119600
         -------
          395744
          ------
            7876
            ----
            3566
```

CASE

CASE III.

Having the area of a circle, to find its diameter.

RULE.

Multiply the ſquare-root of the area by 1.12837, the product will be the diameter.

In the midſt of a meadow, well ſtored with graſs,
I took juſt an acre to tether my horſe:
How long muſt the cord be, that feeding all round,
He may'nt graze leſs or more than this acre of ground.

4840 ſquare yards in an acre.

$\sqrt{4840} = 69.57$
36
———
129) 1240
1161
———
1385) 7900
6925
———
13907) 97500
97349
———

∴ 1.12837
× 75.96
———
677022
101553
5642
790
———
2) 78.5007
———
$39\frac{1}{4}$ yds. length of the tether and horſe.

The periphery of any circular figure may be found by multiplying the ſquare-root of the area by 3.5449.

Thus $69.57 \times 3.5449 = 246$ yards, 1 foot, $10\frac{1}{4}$ inches, the perimeter of the before-mentioned acre of land.

CASE IV.

Any two ſides of a right-angled triangle being given, to find the remaining ſide.

RULE.

As the ſquare of the hypothenuſe, or longeſt ſide of a right-angled triangle, is equal to the ſum of the ſquares of the other two ſides; conſequently the difference of the ſquares of the hypothenuſe, and of either of the other ſides, is the ſquare of the remaining ſide.

1. As I was walking out one day,
Which happen'd on the firſt of May;

 As

As luck would have it, I did 'ſpy
A may-pole raiſed up on high;
The which, at firſt, me much ſurpriz'd,
Not being before-hand advertiz'd
Of ſuch a ſtrange uncommon ſight;
I ſaid I would not ſtir that night,
Nor reſt content, until I'd found
Its height exact from off the ground:
But when theſe words I juſt had ſpoke,
A blaſt of wind the May-pole broke;
Whoſe broken piece I found to be
Exact in length yards ſixty-three;
Which, by its fall, broke up a hole,
Twice fifteen yards from off the pole;
But this being all that I can do,
The May-pole now being broke in two
Unequal parts, to aid a friend,
Ye ladies pray an anſwer ſend. *Ladies Diary.*

Firſt $63 \times 63 = 3969$
Alſo $30 \times 30 = 900$

$\sqrt{3069} = 55.3985$ piece ſtanding.

$\therefore 63 + 55.3985 = 118.3985$ yards $= 118$ yards, 1 foot, $2\frac{1}{4}$ inches, height of the pole.

2. A caſtle wall there was, whoſe height was found
To be an hundred feet from th' top to th' ground;
Againſt the wall a ladder ſtood upright,
Of the ſame length the caſtle was in height
A waggiſh fellow did the ladder ſlide,
(The bottom of it) ten feet from the ſide.
Now I would know how far the top did fall,
By pulling out the ladder from the wall?

$100 \times 100 = 10000$
$10 \times 10 = 100$

$\sqrt{9900} = 99.49874$

$\therefore 100 - 99.49874 = .50125 =$ a very little more than 6 inches.

3. I want the length of a ſhoar, that being to ſtrut 11 feet from the upright of a building, will ſupport a jamb 23 feet, 10 inches from the ground?

$$11 \times 11 = 121$$
$$23.8\dot{3} \times 23.8\dot{3} = 568.02\dot{7}$$

feet. inches.

$$\sqrt{689.02\dot{7}} = 26 \quad 2.9918. \quad Q.E.F.$$

4. The height of an elm, growing in the middle of a circular iſland 30 feet in diameter, plumbs 53 feet; and a line ſtretched from the top of the tree, ſtraight to the outer edge of the water, 112 feet: what then is the breadth of the moat, ſuppoſing the land on either ſide of the water to be level?

$$112 \times 112 = 12544$$
$$53 \times 53 = 2809$$

$$\sqrt{9735} = 98.666.$$

$\because 30 \div 2 = 15$, and $98\frac{2}{3} - 15 = 83\frac{2}{3}$, breadth of the moat required.

5. Two ſhips ſet ſail from the ſame port, one of them goes due eaſt 50 leagues, the other due north 84; how far are they aſunder?

$$50 \times 50 = 2500$$
$$84 \times 84 = 7056$$

$$\sqrt{9556} = 97.75, \text{ or } 97\tfrac{3}{4} \text{ leagues.}$$

6. A line 27 yards long will exactly reach from the top of a fort, on the oppoſite bank of a river, known to be 23 yards broad: the height of the wall is required.

$$27 \times 3 = 81 \text{ feet}, \quad 23 \times 3 = 69 \text{ feet.}$$
$$81 \times 81 = 6561$$
$$69 \times 69 = 4761$$

$$\sqrt{1800} = 42.426 = 42 \text{ feet}, \ 5 \text{ inches.}$$

7. Suppoſe a light-houſe built on the top of a rock; the diſtance between the place of obſervation, and that part of the rock level with the eye, and directly under the building, is given 310 fathoms; the diſtance from the top of the rock to the place of obſervation, is 423 fathoms, and from the top of the building 425; the height of the edifice is required.

$425 \times 425 = 180625$
$310 \times 310 = 96100$

$\sqrt{84525} = 290.73187$, light-house and rock.

Also $423 \times 423 = 178929$
$- 96100$

$\sqrt{82829} = 287.80027$, rock.

$\because 290.73187 - 287.80027 = 2.9316 = 17.59$ feet. Q. E. F. the height of the light-house.

8. A ladder 40 feet long may be so planted, that it shall reach a window 33 feet from the ground, on one side the street; and without moving it at the foot, will do the same by a window 21 feet high, on the other side: the breadth of the street is required.

$40 \times 40 = 1600$
$33 \times 33 = 1089$

$\sqrt{511} = 22.6$

$21 \times 21 = 411$

$\sqrt{1159} = 3404$

Answer, 56.64 feet. Q. E. F.

9. An ancient bath was found, of a triangular form, the sum of whose three equal sides was 125 feet; the area of the bottom is required.

$3)\,125\,(41.\dot{6}$, each side.
$2)\,41.\dot{6}\,(20.8\dot{3}$, half the side.

Then $41.\dot{6} \times 41.6 = 1736.\dot{1}11$
Also $20.8\dot{3} \times 20.83 = 434.027$

$\sqrt{1302.084} = 36.084$, perp.

Then $41.\dot{6} \times 36.084 = 1503.5$.
$\because 2)1503.5(= 751.75$ feet, the area required.

10. The paving of a triangular court, at 18 d. per foot, came to 100 l. the longest of the three sides was 88 feet: what then was the sum of the other two equal sides?

18 d.

$18\text{ d.} = .075\text{l.})100.000(1333.\dot{3}$ feet, the area.
$\frac{88}{2} = 44)1333.\dot{3}(30.\dot{3}$, perpend,
$44 \times 44 = 1936$
$30.\dot{3} \times 30.\dot{3} = 918.\dot{7}\dot{7}$

$\sqrt{2854.\dot{7}\dot{7}} = 53.425$, either of the two [equal sides.
$\therefore 53.425 \times 2 = 106.85$ feet, the answer.

11. I would plant 10 acres of hop-ground, which muſt be done, either in the ſquare order, or as the number 4 ſtands on the dice, or in the quincunx order, as the number 5; the three neareſt binds, in both caſes, muſt be ſet lineally juſt 6 feet aſunder: how many plants more will be required for the laſt order than for the firſt, admitting the form of the plat to lay the moſt advantageous for the plantation in either caſe?

$6 \times 6 = 36$ ſquare feet each plant in the ſquare order.
$3 \times 3 = 9$

$\sqrt{27} = 5.19615$ perpend.
Alſo $6 \times 5.19615 = 31.1769$ feet, each in the quincunx order.
In one acre are 43560 ſquare feet.
$10 \times 43560 = 435600$
$31.1769)435600(13972$ plants in the quincunx } order.
$36)435600(12100$ plants in the ſquare. }

Difference $= 1872$. Q. E. F.

12. The quarry of glaſs is $3\frac{3}{4}$ inches on every ſide, and as much croſs the middle, coſt 1 d.; the ſquare is $5\frac{1}{4}$ inches by $3\frac{1}{2}$, and coſt $1\frac{1}{2}$ d.; what will be ſaved, glazing 1000 feet, the cheapeſt of the two ways, ſuppoſe the leading of the lights be nearly equal in either kind of work?

$3.75 \times 3.75 = 14.0625$
$1.875 \times 1.875 = 3.515625$

$\sqrt{10.546875} = 3.2476$ perpend.
Then $5.25 \times 3.5 = 18.375$ } area of the { ſquare
Alſo $3.2476 \times 3.75 = 12.1785$ } area of the { quarry.
$1000 \times 144 = 144000$ ſquare inches.
$18.375)144000.000(7836.7$ ſquares.
$12.1785)144000.0000(11824$ quarries.

$\frac{1}{8}$	7836.7		$\frac{1}{12}$	11824
	d.			d.
$\frac{1}{20}$	979 7		$\frac{1}{20}$	985 4
£	48 19 7 cost in squares.			49 5 4 cost in quarries.

∴ 49 l. 5 s. 4 d. — 48 l. 19 s. 7 d. = 5 s. 9 d. advantage in squares.

13. A summer-house is a cube of 10 feet in the clear, the cornice of which projects just 15 inches on a side, and being of timber and stucco, the sides are 6 inches thick, so that the whole front of the roof, from out to out, is $13\frac{1}{2}$ feet. This is hipped from each of the corners to the center, and being truly pediment pitch, it raises $\frac{2}{9}$ of the front, or three feet; I would, by the help of these dimensions, measure the slating, without venturing to climb for more, and compute the cost at $3\frac{1}{2}$ d. per square foot.

First $10 + 2\frac{1}{2} + 1 = 13\frac{1}{2} = \frac{27}{2} =$ whole breadth.

Then $\frac{27}{2} \times \frac{2}{9} = 3$ feet, the rise of the roof.

2) 13.5 (6.75 = half the breadth.

Also $6.75 \times 6.75 = 45.5625$

$3 \times 3 = 9$

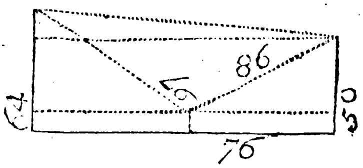
$\sqrt{54.562} = 7.38664.$

Then $13.5 \times 2 \times 7.38664 = 199.4393$, area of the roof.

$\frac{1}{80}$	199.4393
$\frac{1}{6}$	2.49274
	.41545
£	2.90819 = 2 l. 18 s. 2 d. the answer.

14. There are two columns in the ruins of Persepolis left standing upright; one is 64 feet above the plane, the other 50; between these, in a right line, stands an ancient statue, the head whereof is 97 feet from the summit of the higher, and 86 feet from the top of the lower column; and the center of the figure's base just 76 feet from the foot of that column. By these notices the distance of the top of the columns may be, by numbers, easily found.

First



First 86 × 86 = 7396
Also 76 × 76 = 5776

√1620 = 40.24922.

Then 50 − 40.24922

9.75078, height of the statue.

Also 64 − 9.75078 = 54.24922.

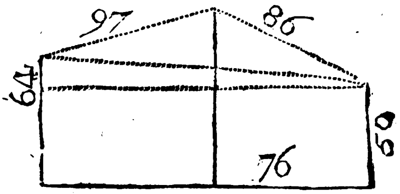
Again, 97 × 97 . . . = 9409
54.24922 × 54.24922 = 2942.97787

√6466.02213 = 80.4121.

80.4121 + 76 = 156.4121, distance of the columns.
64 − 50 = 14, difference of their heights.
156.4121 × 156.4121 = 24464.745
14 × 14 . . = 196

√24660.745 = 157. Q. E. F.

But if the statue be an enormous Colossus, higher than the towers, the work will be thus:



First 86 x 86 = 7396
Also 76 × 76 = 5776

√1620 = 40.24922

Then 50 + 40.24922, height of the statue = 90.24922.

And 90.24922 − 64 = 26.24922, higher than the other column.

Again 97 × 97 = 9409
Also 26.24922 × 26.24922 = 689.0583

√8719.9417 = 93.3806

∴ 76 + 93.3806 = 169.3806, distance of the columns.
Also 64 − 50 = 14, difference of their altitudes.
Lastly 169.3806 × 169.3806 = 28689.7944316
14 × 14 = 196.

√28885.7944316 = 169.958.
Q. E. F.

15. The

15. The ſemi-diameter of the earth being 3984.58 miles, and the perpendicular height of a mountain three miles; how far will it be ſeen at ſea, or on plain ground, ſuppoſing the eye of the ſpectator to be on the ſurface of the ground or water?

3984.58 ſemidiameter.
3 height.

$3987.58 \times 3987.58 = 15900794.2564$
$3984.58 \times 3984.58 = 15876877.7764$

$\sqrt{23916.4800} = 154.64$ miles. Q. E. F.

CASE V.

Given three ſides of a triangle, to find the area.

RULE.

From half the ſum of the three ſides, ſubtract each ſide ſeverally; let the half ſum, and the three differences, be multiplied continually; the ſquare root of the product will be the area required.

1. Having a fiſh-pond of a triangular form, whoſe three ſides meaſure 400 yards, 348, and 312; what quantity of ground does it cover?

Firſt $400 + 348 + 312 = 1060$.

Alſo $\frac{1060}{2} = 530$ yards, half the ſum of the three ſides.

And $\left.\begin{array}{l} 530 - 400 = 130 \\ 530 - 348 = 182 \\ 530 - 312 = 218 \end{array}\right\}$ differences.

Alſo $530 \times 130 \times 182 \times 218 = 2733676400$.

$\sqrt{2733676400} = 52284.571338$ ſquare yards.

4840) 52284.571338 (10.8026 = 10 a. 3 r. 8 p. Q. E. F.

2. A field of a triangular form, whoſe ſides are 380, 420, and 765 yards, lets for 55 s. per acre; how much does the whole bring in per annum?

Firſt $380 + 420 + 765 = 1565$.

Alſo $\frac{1565}{2} = 782.5$ yards, half the ſum of the three ſides.

And $\left.\begin{array}{l} 782.5 - 380 = 402.5 \\ 782.5 - 420 = 362.5 \\ 782.5 - 765 = 67.5 \end{array}\right\}$ differences.

Alſo $782.5 \times 402.5 \times 362.5 \times 67.5 = 1998003710.9375$.

$\sqrt{1998003710.9375} = 44699.034$ ſquare yards.

4840) 44699.034 (9.2374 = 9 acres, 38 perches.

Anſwer, 25 l. 8 s. $-\frac{3}{4}$d. per annum.

SECT.

SECT. III.

To extract the Cube Root.

RULE I.

HAVING pointed the given resolvend into periods of three figures, as before directed,

Seek the greatest cube in the left-hand period; write the root in the quotient, and the cube under the period; which subtract, and to the remainder bring down the next period; call this a new resolvend, under which draw a line.

2. Under this resolvend write the triple square of the root, so that the units in the latter stand under the place of hundreds in the former; and under the said triple square write the triple root, removed one place to the right; the sum of these is the divisor, under which draw a line.

3. Seek how oft this divisor may be had in the new resolvend (its right-hand place excepted) and write the result in the quotient.

4. Under the divisor write the product of the triple square of the root by the last quotient figure, setting down the units place of this line under that of tens to the divisor; under this line write the product of the triple root by the square of the last quotient figure; let this line be removed one place beyond the right in the former; and under this line, removed one place forward to the right, write down the cube of the last quotient figure, the sum of these three lines call the subtrahend, under which draw a line.

5. Subtract the subtrahend from the new resolvend; to this remainder bring down the next period for another resolvend; the divisor must be the triple square of the quotient, added to the triple thereof, &c. as before directed

Extract

Extract the cube-root of 122615327232.

122615327232 (4968	
64	
58615	new resolvend.
48	triple square of 4.
12	triple of 4.
492	divisor.
432	
972	
729	
53649	subtrahend.
4966327	resolvend.
7203	triple square of 49.
147	triple of 49.
72177	divisor.
43218	triple square of 49 × 6.
5292	triple of 49 × square of 6.
216	cube of 6.
4374936	subtrahend.
591391232	resolvend.
738048	triple square of 496.
1488	triple of 496.
7381968	divisor.
5904384	triple square of 496 × 8.
95232	triple of 496 × square of 8.
512	cube of 8.
591391232	subtrahend = last resolvend; so that 4968 is the true cube root of 122615327232.

RULE.

RULE II.

1. The resolvend being pointed into proper periods, find the nearest less root of the figures of the first punctuation on the left-hand; subtract its cube from the number given; to the remainder annex the next figure for a new resolvend.

2. Take $\frac{1}{3}$ of the resolvend for a dividend.

3. And for a divisor take the square of the root added to half the root (or rather added to the product of the root, and the next quotient figure, leaving out the last figure of the product.)

4. Divide the said dividend by that divisor, the quotient is the second figure of the root.

5. Begin the operation a-new; viz. cube the two figures of the root, and subtract the cube from the given number, annexing another figure for the resolvend.

6. Take the third part of the resolvend for a dividend, and the square of the root added to half the root (or rather added to the product of the root and the next quotient figure, striking off the last figure of the product) for a divisor.

7. The division gives another figure of the root; but the division is to be continued on to two figures, by the contraction in division of decimals, or otherwise.

8. Repeating the operation with four figures in the root, you will get four more by a new division, which gives eight figures in the root; and from 8 to 16, &c. always double.

9. Note, when the cube exceeds the number given, a less figure must be writ in the quotient; and observe every division gives one figure, and the rest are found by continuing the division, and dropping a figure of the divisor every time.

10. If after all the periods, both in whole numbers and decimals, are brought down, the extraction may be continued as far as you please, by still adding ternaries of cyphers. At last cut off as many places of whole numbers as there are points in whole numbers, and the like for decimals.

11. If you desire the last quotient to go true to more places of figures, add half the last quotient to the last root, and square the sum for a divisor, and divide over again.

Extract

Extract the root out of 9239864750 6217.

```
92398647506217 (45
64
———              16 square.
3)283             64 cube.
16) 94 (5         4 × 5 = 20.
+2) 90
——  ———
18)  4
      92398
      91125
     ———————
    3) 12736            square of 45 =  2025.
     ———————
2025) 4245 (208              cube = 91125.
   9  4068                  45 × 2 = 90.
 ———  ————
2034)  177   root 45208 in whole numbers,
  ..   163          its square 2043763264,
      ————               cube 92394449638912.
       14
        92398647506217                 45208
        92344449638912                     6
       ————————————————               ——————
        3)4197867305 0                271248
       ————————————————
2043763264)1399289101.6 (68465392
  + 27124   12262742328
   ———————  ———————————— root 45208.68465.
2049760388)  1730148688
             1635032310
            ————————————
               95116378
              ——————————
```

RULE III.

1. After the given resolvend is truly pointed, seek the greatest cube in the left-hand period; write the root in the quotient; subtract the cube from the period, as directed in the other rule; and to the remainder bring down all the remaining periods in the given number, for a new resolvend.

2. To

2. To the root (or quotient) annex as many cyphers as there are remaining periods; multiply this by 3; by this product divide the resolvend, and point the quotient into periods of two places (beginning at units) observing that there be no more points than there were periods brought down to the resolvend.

3. Make the root (found in the first period of the given numbers) a divisor; see how often it may be had in the left-hand period of the quotient (excepting the place under the point) and the figure resulting write in the quotient (to the right-hand of the root first found) and on the right of the divisor; multiply this increased divisor by the last quotient figure; to the remainder bring down the next period; divide this by the last divisor.

Extract the cube root of 8302348000000.

```
 8302348000000
 8
 ---
6,0000) 30234800,0000
    202) 5039133( 024
         404      2000
         ---      ----
   2024)9991      2024
        8096
        ----
```

The cube of 2024 = 8291469824

```
 8302348000000 ) 20240 × 3 = 60720, divisor.
 8291469824
 ----------
6072,0) 1087817600,0 (179153,42556

  20248) 179153.425560 (8.8479
         161984
         ------
20248.8) 1716942
         1619904          20240
         -------          + 8.8479
20248.84) 9703855         ----------
          8099536         20248.8479, root. Q.E.F.
          -------         ----------
20248.847) 160431960
           141741929
           ---------
            18690031
```

Q RULE

RULE IV.

Divide the given reſolvend by three times the ſuppoſed root, and from the quotient ſubtract one-twelfth of the ſquare of the ſuppoſed root; the ſquare root of the remainder, added to half the ſuppoſed root, will give the true root required.

What is the cube root of 146708.483?

Suppoſe the root 50.

Then $50 \times 3 = 150)\ 146708.483\ (978.8565$

$50 \times 50 = 2500$; alſo $12)\ 2500\ (208.3333$

770.5232

$\sqrt{770.5232} = 27.7$

$2)\ 50\ (= 25,$ half the ſuppoſed root.

52.7, the root.

But for greater exactneſs I proceed to another operation.

Thus, $52.7 \times 3 = 158.1)\ 146708.483\ (927.94739$

$52.7 \times 52.7 = 2777.29 \ldots 12)\ 2777.29\ (231.44083$

696.50656

$\sqrt{696.50656} = 26.39141$

$2)\ 52.7\ (\quad 26.35$

$\therefore 52.74141$, the root more exact.

Extract the cube root of 2.

$1 \times 3 = \quad 3)\ 2.0\ (.66\cancel{6}$

1×1 and $12)\ 1.0\ (.08\cancel{3}$

$.58\cancel{3}$

$\sqrt{.5833}\ (.7$

$2)\ 10\ (.5$

1.2, root.

By a ſecond operation $1.2 \times 3 = 3.6)\ 2.00\ (.\cancel{5}55$

$1.2 \times 1.2 = 1.44 \qquad 12)\ 1.44\ (.12$

$\sqrt{4355}\ (.659 \qquad .43\cancel{5}$

$2)\ 1.200\ (.6..$

1.259, root.

By

By a third operation $1.259 \times 3 = 3.777)$ 2.000 (.5295207866.
$1.259 \times 1.259 = 1.585081$ . . . 12) 1.585081 (.1320900833

$\sqrt{.3974307033} = .63042113$ .3974307033

2) 1.259 (.6295

1.25992113, the root. Q. E. F.

What is the cube root of .0001357?

Supposed root .05.

$.05 \times 3 = .15)$ .0001357 (.0009456
$.05 \times 05 = 0025$, and 12) 0025 (.0002083

$\sqrt{.0007383}$ (.027   .0007383

2) .05 (.025

.052 root, which by involution I find too much.

∴ I take .051 for the supposed root.

$.051 \times 3 = .153)$ .0001357 (.00088751
$.051 \times 051 = .002601$, and 12) .002601 (00021758

.00066993

$\sqrt{00066993} = .02588$

2) .051 (.0255

.05138 root.

Then $.05138 \times 3 = .15414)$ .000135700 (.0008803685
$.05138 \times .05138 = .0026399044$
12) .0026399044 (.0002199920

.0006603765

$\sqrt{.0006603765} = .02569779$

$\frac{05138}{2} = .02569$

.05138779, the root. Q. E. F.

What is the cube root of $13\frac{2}{3}$?

In decimals $13\frac{2}{3} = 13.6$

Supposed root $2 \times 3 = 6)$ 13.6 (2.27
$2 \times 2 = 4$; also 12) 4.0 (0.33

1.94

 $\sqrt{1.94}$

$\sqrt{1.94} = 1.39$
$\frac{2}{2} = 1.00$

2.39, root.

$2.39 \times 3 = 7.17)\ 13.6\cancel{0}\ (\ 1.90609019$
$2.39 \times 2.39 = 5.7121$; and $\frac{5.7121}{12} = 0.47600833$
$\sqrt{1.43008186} = 1.1958$
$\frac{2.39}{2} = 1.195$

2.3908, root.

$2.3908 \times 3 = 7.1724)\ 13.666\cancel{0}\ (1.9054454125$
$2.3908 \times 2.3908 = 5.71592464$
$12)\ 5.71592464\ (0.4763270533$

1.4291183592

$\sqrt{1.4291183592} = 1.195457397$
$2)\ 2.3908 = 1.1954$

2.390857397, the root. Q. E. F.

The second method of extracting the cube root is that used by that great mathematician Mr. Emerson, in his treatise of arithmetic, and doubles the figures in the root at each operation.

The third is the method Mr. J. Robertson, F. R. S. uses in his mensuration, by which each operation triples the figures in the root.

But the fourth and last I take to be the easiest, as the operations are performed by easy divisions, and an extraction of the square root.

N. B. This method only doubles the figures in the root at each operation.

SECT IV.

Some Uses *of the* Cube-root.

THE cube-root is of very great use in mathematics, but I shall only exhibit a few cases.

CASE

CASE I.

To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

RULE.

Extract the cube root of the solid content of the given body, which will be the side of a cube of an equal solidity.

Suppose a chest, whose length is 4 feet 7 inches, breadth 2 feet 3 inches, and depth 1 foot 9 inches; required the side of a cube of equal solidity?

	F.	I.		
Length	4	7	= 55	inches.
Breadth	2	3	= 27	
			385	
			110	
			1485	
Depth	1	9	= 21	
			1485	
			2970	
			31185	solid inches.

30 × 3 = 90) 31185 (346.5
30 × 30 = 900.12) 900 (75

271.5

271.5 (16.4
1

26) 171
156

324) 1550

16.4
2) 30 (15

31.4, root.

For a second operation, 31.4 × 3 = 94.2) 31185 (331.05106
31.4 × 31.4 = 985.96 . . . 12) 985.96 (82.16333

248.88773

√248.88773 (15.7762
2) 31.4 (15.7

31.4762, side of the cube required.

CASE II.

Having the dimensions of any solid body, to find those of a similar solid, any number of times, greater or less than the solid given,

RULE.

Multiply the cube of each of the given dimensions by the difference between the solid given, and that required, if greater (or divide by the difference, if less) than the solid given; then extract the cube root of each product or quotient, which will be the dimensions of the solid required.

Suppose the length of a ship's keel be 125 feet, the breadth of the midship beam 25 feet, and the depth of the hold 15 feet; I demand the dimensions of another ship, of the same form, that shall carry three times the burthen?

$$125 \times 125 \times 125 \times 3 = 5859375$$
$$25 \times 25 \times 25 \times 3 = 46875$$
$$15 \times 15 \times 15 \times 3 = 10125$$

$$\left.\begin{array}{ll} \therefore \sqrt[3]{5859375} = 180.28, & \text{keel.} \\ \text{Also } \sqrt[3]{46875} = 36.05, & \text{midship beam.} \\ \text{And } \sqrt[3]{10125} = 21.6, & \text{depth in the hold.} \end{array}\right\} \text{Q. E. F.}$$

Or suppose the ship was to be but of half the burthen of that whose dimensions are given as above.

$$\frac{125 \times 125 \times 125}{2} = 976562.5$$
$$\frac{25 \times 25 \times 25}{2} = 7812.5$$
$$\frac{15 \times 15 \times 15}{2} = 1687.5$$

$$\therefore \sqrt[3]{976562.5} = 99.202$$
$$\text{Also } \sqrt[3]{7812.5} = 19.84$$
$$\text{And } \sqrt[3]{1687.5} = 11.906$$

CASE III.

Having the dimensions and capacity of a solid, to find the dimensions of a similar solid of a different capacity,

RULE.

Divide the cube of the dimensions given, multiplied into the capacity of the vessel or body required; the cube root of the quotient will be the result.

If

If a ſhip of 100 tuns be 44 feet long at the keel, of what length ſhall the keel of that ſhip be, whoſe burthen is 220 tuns?

First $44 \times 44 \times 44 \times 220 = 18740480$

$100)\,18740480\,(187404.8$

$\sqrt[3]{187404.8} = 57.22592$, the anſwer required.

CASE IV.

Between two given numbers, to find two mean proportionals,

RULE.

Multiply the leſs extreme by the cube-root of the quotient of the greater extreme, divided by the leſs; the product is the leaſt of the two mean proportionals, which multiplied by the ſaid cube root gives the greater mean ſought.

Find two mean proportionals between 7 and 15379.

$7)\,15379\,(2197$

$11 \times 3 = 33)\,2197\,(\ 66.575$

$12)\,121\,(\ 10.083$

$\sqrt{56.492} = 7.5$

5.5

$13.$ cube root.

$\because\ 7 \times 13 = 91$, firſt ⎫ mean propor. Q. E. F.
and $91 \times 13 = 1183$, ſecond ⎭

For as $7 : 91 :: 1183 : 15379$.

SECT. V.

To extract the BIQUADRATE ROOT.

RULE.

EXTRACT the ſquare-root of the given reſolvend, and the ſquare root of that firſt root will be the biquadrate root required.

Extract the biquadrate root of 334815812249134441.

```
.  .  .  .  .  .  .  .  .
334815812249134441 (182979729 ( 13752, biquadrate
1                   1                  root required.

28) 234             23) 82
    224                 69

362) 1081           265)1397
      724               1325

3649) 35758         2702)7297
      32841              5404

36587)291712        27047)189329
      256109              189329

365949)3560324                 0
       3293541

3659587)26678391
        25617109

        &c.
```

SECT. VI.

To extract the Sursolid Root.

RULE.

HAVING pointed the given reſolvend into periods of five figures, ſeek ſuch a ſurſolid number in the table of powers, (or otherwiſe) as comes neareſt to the firſt period of the reſolvend, whether greater or leſs; and call the reſpective root, either more than juſt, or leſs than juſt, as it falls out; annexing ſo many cyphers to it as there are remaining periods of whole numbers in the reſolvend.

2. Find the difference between the reſolvend and the ſurſolid number, ſo taken, by ſubtracting the leſs from the greater.

3. Find the cube of the foreſaid ſurſolid root, with its annexed cyphers, which alſo may be done by the table of powers, and multiply that cube into five, the index of the ſurſolid, and divide the difference between the reſolvend and the

the surfolid number by that product; by which it will be depressed to a square, and when pointed into periods of two figures each, call it the new resolvend.

4. Make the first root without cyphers a divisor, enquiring how often it may be found in the first period of the new resolvend; with this consideration, if the root, now a divisor, be less than just, annex twice the quotient figure to it; but if more than just, subtract twice the quotient figure from a cypher, either annexed, or supposed to be annexed, to that divisor, or root, multiplying it, so increased or diminished, with the said quotient figure; setting down the units place of the product under the pointed figure of that period, subtracting it as in division.

Extract the surfolid root of 30768282106715625

30768282106715625 (3
243

6468282106715625
3000 cubed = 27000000000
27000000000 × 5 = 135000000000, divisor
135000000000) 6468282106715625 (47913

3) 479132 (14
+ 1 × 2 = 2 32

32) 1591 3000
+ 4 × 2 = 8 1312 140

3140

By a second operation.

30768282106715625
3140 ⊚ 5 = 30524477618240000

243804492431 5625
314 ⊚ 3 = 30959144000
5

154795720000, divisor.

154795720000) 243804492431 5625 (15750

314) 15750 (5 First root 3140
5 × 2 = 10 15750 + 5

3150 0 True root 3145

Extract

Extract the surſolid root of 976379602989073960279630 2988.

```
                1024
                97637960298907396027963o2988
                ----------------------------
                47620397010926039720369701 2
 400000 ⊙ 3 = 6400000000000000000
                    5
                  -----
                  320000000000000000000
   32)47620397010 (1488137406
   400) 1488137406 (037
  — 06   1182             400000
  ----   ----            — 037
   394 ) 29613           ------
  — 14   27482           396300, root.
   ----                  ------
   3926
```

But I only take 396 for a ſecond operation, which I find by involution to be leſs than juſt.

```
          97637960298907396027963o2988
 396 ⊙ 5 = 97381381109760000000000000
           ---------------------------
           2365791881473960279630 2988
 396000 ⊙ 3 = 62099136000000000
 62099136)23657918814796c2,7963 (38970160.5871
 396000 ) 38970160.5871 (098.3889
      18      356562
   -------   -------
   39612      3322060
      16      3169088
   -------   -------
   396136     15297258
        6     11884098
   -------   --------
   3961366    341316071
        16    316909408
   --------  ---------
   39613676   34406663
              31690941
              --------
               3715722
   396000
      098.3889
   ----------
   396098.3889, the root ſought.
```

SECT.

SECT. VII.

To Extract the ROOT *of the* SQUARE CUBED; *or*, SIXTH POWER.

RULE.

EXTRACT the square root of the given resolvend; then extract the cube root of that square root, which will be the root of the sixth power required.

Or you may first extract the cube root of the resolvend, and then the square root of that cube root, and that will be the root required.

Extract the sixth power of 435728381009267809889764416

```
435728381009267809889764416 (20874107909304.
4
—
408) 3572
     3264
     ————
4167) 30883
      29169
      —————
41744) 171481
       166976
       ——————
417481) 450500
        417481
        ——————
41748207) 330199267
          292237449
          —————————
             &c.
```

I find by pointing the square root into periods of three figures each, that there will be five figures in whole numbers in the root of the sixth power.

```
2000 × 3 = 60000) 20874107909304 (347901798
              12) 400000000    ( 33333333
                                 —————————
                                 314568465
                                 —————————

314568465 (17600
           10000
           —————
```

27600, which by involution I find too much, therefore take 275. 27500

27500 × 3 = 82500) 20874107909304 (253019489
275 × 275 = 75625 . . 12) 756250000 (63020833

189998656

189998656 (13784
1   2) 275 (1375

23) 89   27534, true root of the sixth power.
69
267) 2099
1869
2748) 23086
21984
27564) 110256
110256

SECT. VIII.

To Extract the ROOT *of the Second* SURSOLID, *or* SEVENTH POWER.

RULE.

HAVING pointed the resolvend into periods of seven figures, seek out such a number of the seventh power, by the table, as comes nearest to the first period of the resolvend, whether greater or less, calling its root more than just, or less than just, annexing a proper number of cyphers.

2. Find the difference between the resolvend, and that number of the seventh power, by subtracting the less from the greater.

3. Find the sursolid, or fifth power of that root, with its annexed cyphers, by the table of powers; and multiply that sursolid number into seven, the index of the resolvend.

4. Make that product a divisor, by which the foresaid difference must be divided; so that it may be depressed to a square, and pointed as such.

5. Make the first root, without cyphers, a divisor, working with it and the new resolvend, as in the sursolid; only here

here you must increase or diminish the divisor with thrice the quotient figure.

Extract the 7th power of 34487717467307513182492153794673.

34487717467307513182492153794673

$3 \circledcirc 7 = 2187$

12617717467307513182492153794673

First root $30000 \circledcirc 5 = 24300000000000000000000$

243, &c. $\times 7 = 170100000000000000000000$

Contracted 1701) 1261771174673 (74178233

3) 74178233 (20

$+ 2 \times 3 = 6$ 72

360) 21782

30000
20

32000

Second operation.

34487717467307513182492153794673

$32 \circledcirc 7 = 34359738368$

12797909930751318249215379 4673

$32 \circledcirc 5 = 33554432$, which $\times 7 = 234881024$

234881024) 1279790993075 13 (544868

320) 544868 (017

$+ 1 \times 3 = 3$ 3203

3203) 224568

$+ 7 \times 3 = 21$ 224357

32051 211

32000
+017

32017, true root.

SECT. IX.

To Extract the ROOT *of the* BIQUADRATE SQUARED, *or* EIGHTH POWER.

RULE.

EXTRACT the square root of the given resolvend, which will reduce it to a biquadrate number, which call a new resolvend; the square root of which will be a square

square number; of which extract the square root, which root will be the result required.

Let 1121016281320476236246497942460481 be the given resolvend, whereof the root of the eighth power is to be extracted.

1121016281320476236246497942460481 (33481581224913441, biquad. resol.
9
63)221
189
664)3201
2656
6688)54562
53504
66961)105881
66961
669625)3892032
3348125
6696308)54390704
53570464
66963161) 82024076
66963161
669631622)1506091523
1339263244
6696316242)16682827962
13392632484
66963162444)329019547846
267852649776
669631624489)6116689807049
6026684620401
6696316244981) 9000518664879
6696316244981
66963162449823)23042024198984 2
20088948734946 9
669631624498264)2953075464037346
2678526497993056
6696316244982684)27454896604429004
26785264979930736
66963162449826881)66963162449862 81
66963162449862 81
0

Then $\sqrt{33481581224913441} = 182979729$
$\therefore \sqrt{182979729} = 13527$, root of the eighth power. Q. E. F.

SECT.

SECT. X.

To Extract the ROOT *of the* CUBE CUBED, *or* NINTH POWER.

RULE.

EXTRACT the cube root of the given resolvend, and the result will be a cubic resolvend; of which extract the cube-root also, which will be the root of the ninth power required.

Let 9763796029890739602796302988890 be the resolvend given, out of which the root of the ninth power is to be extracted.

$99 \times 3 = 297)\ 976379602989073960279$

99 $\qquad$ 32874734107

——

891 $\qquad$ 12) 9805 (8170833333

891

——

9805

$\sqrt{247039007740} = 4970000000$

$\frac{99}{2} = 495$

Root 9920000000

Again,

$9920 \times 3 = 29760)\ 976379602989073960279630$

$9920 \odot 2 = 9840640 \qquad (3280845440151$

$\frac{9840640}{12} = \qquad 820053333333$

$\qquad 2460792106818$

$\sqrt{24607921068180} = \qquad 4960637000$

$\frac{992}{2} = 496$

Root 9920637000

And,

$9920637 \times 9920637 = 98419038485769000000$

$\times 3$

$29761911)\ 97637960298907396027963028890$

$(32806347784558389421.99$

$12)\ 98419038485769\ (\qquad 8201586540480750000$

24604761244077639421.99

$\sqrt{24604761244077639421.99} = 4960318663.5616$

$\frac{9920637}{2} = 49603185$

Cube root required 9920637163.5616, or new resolv.

Then

Then,

$2100 \times 3 = 6300)\ 9920637163{\cdot}5616\ (1574543{\cdot}99$

$2100 \times 2100 = 4410000.\quad 12)\ 4410000\ (\ \underline{367666.66}$

$\sqrt{1206877} = 1098$

$\frac{2100}{2} = \underline{1050}$

Root $\underline{2148}$ in whole numbers.

Again,

$2148 \times 2148 = 4613904$

$2148 \times 3 = 6444)\ 9920637163{\cdot}5616\ (1539515{\cdot}3885$

$12)\ 4613904\ (\ \underline{384492}$

$\underline{1155023{\cdot}3885}$

$\sqrt{1155023{\cdot}3885} = 1074{\cdot}7201$

$\frac{2148}{2} = \underline{1074{\cdot}}$

The root of the 9th power $\underline{2148.7201}$. Q. E. F.

Thus have I endeavoured to make plain the method of extracting the root of all the powers, whose index is any number not exceeding the nine digits, which by a little consideration may be extended to still higher powers. I now conclude this chapter with the following method proposed by Mr. Halliday and others, viz.

SECT. XI.

TO multiply several figures by several, and the product to be produced in one line only.

RULE.

Multiply the units of the multiplicand by the units of the multiplier, setting down the units of the product, and carry the tens; next multiply the tens in the multiplicand by the units of the multiplier, to which add the product of the units of the multiplicand, multiplied by the tens in the multiplier, and the tens carried; then multiply the hundreds in the multiplicand by the units of the multiplier, adding the product of the tens in the multiplicand multiplied by the tens in the multiplier, and the units of the multiplicand by the hundreds in the multiplier; and so proceed till you have

have multiplied the multiplicand all through, by every figure in the multiplier.

$$\begin{array}{r} 321432 \\ 213213 \\ \hline 68533481016 \\ \hline \end{array}$$

EXPLANATION.

First, $\overline{3\times2}=6$; secondly, $\overline{3\times3}+\overline{1\times2}=11$, *i.e.* 1 and carry 1.

Thirdly, $\overline{3\times4}+\overline{2\times2}+\overline{1\times3}+1=20$, that is 0 and go 2.

Fourthly, $\overline{3\times1}+\overline{3\times2}+\overline{1\times4}+\overline{2\times3}+2=21$, *i.e.* 1 and go 2.

Fifthly, $\overline{3\times2}+\overline{1\times2}+\overline{1\times1}+\overline{3\times3}+\overline{2\times4}+2=28$, *i.e.* 8 and go 2.

Sixthly, $\overline{3\times3}+\overline{2\times2}+\overline{1\times2}+\overline{1\times3}+\overline{2\times1}+\overline{3\times4}+2=34$, *i.e.* 4 and go 3.

Seventhly, $\overline{1\times3}+\overline{2\times3}+\overline{2\times2}+\overline{1\times4}+\overline{3\times1}+3=23$, *i.e.* 3 and go 2.

Eighthly, $\overline{2\times3}+\overline{2\times4}+\overline{1\times1}+\overline{3\times2}+2=23$, *i.e.* 3 and go 2.

Ninthly, $\overline{3\times3}+\overline{2\times1}+\overline{1\times2}=15$, *i.e.* 5 and go 1.

Tenthly, $\underline{1\times3}+2\times2+1=8$, to set down.

Lastly, $2\times3=6$, which finishes the work.

$$\begin{array}{r} 35234 \\ 52424 \\ \hline 187107216 \\ \hline \end{array}$$

First, $\overline{4\times4}=\underline{16}$ that is 6 and go 1.

$\underline{3\times4}+\underline{4\times2}+\underline{1}=21$, that is, 1 and go 2.

$\underline{2\times4}+\underline{3\times2}+\underline{4\times4}+\underline{2}=32$, *i.e.* 2 and go 3.

$5\times4+2\times2+3\times4+4\times2+3=47$, *i.e.* 7 and go 4.

$\underline{3\times4}+\underline{5\times2}+\overline{2\times4}+\overline{3\times2}+\overline{4\times5}+4=60$.

$\underline{3\times2}+\underline{5\times4}+\overline{2\times2}+3\times5+6=51$.

$3\times4+5\times2+2\times5+5=37$.

$$\overline{3\times 2}+\overline{5\times 5}+3=34.$$

Lastly, $\overline{3\times 5}+3=18$.

Mr. Halliday says, that this is not only performed very expeditiously in small figures, but also in great figures may be done readily enough by any person who can add one number to another, not exceeding 81; but I for my part think it a hazardous puzzling operation, and only fit for the practice of another Jedidiah Buxton.

END OF THE FIRST BOOK.

BOOK

Arithmetical Collections AND IMPROVEMENTS.

BOOK I.

Containing PROPORTION, with its USE; also the USE of the RULES of PRACTICE, in various branches of MERCHANDIZE and TRADE.

CHAPTER I.

PROPORTION DISJUNCT; CALLED THE GOLDEN RULE; or, RULE of THREE.

PROPORTION Disjunct, or the Golden Rule, are either direct or reciprocal, called Inverse, and those are both single and compound.

SECT. I.

DIRECT PROPORTION.

DIRECT proportion is when of four numbers the first beareth the same ratio, or proportion to the second, as the third doth to the fourth; as in these:

$$5 : 35 :: 17 : 119 \text{; or, } 65 : 13 :: 20 : 4.$$

By ratio is here meant the common multiplier or divisor; and it shews the habitude or relation one number hath to another, viz. whether it be double, triple, quadruple, &c. so that proportionality is a similitude of ratios.

 That

That is, the greater or less the second term is in respect to the first, the greater or less will the fourth be in respect to the third.

Thus the ratio or common multiplier is 7 in the first four proportional numbers, viz, 35; the second term in the proportion is 7 times greater than 5, the first term; so is 119, the fourth term, 7 times greater than 17, the third term.

Also 5, is the ratio or common divisor in the second four proportional numbers; for 13, the second term in the proportion, is 5 times less than 65, the first term; so is 4, the fourth term, 5 times less than 20, the third term.

If four numbers are in direct proportion, the product of the two extremes will always be equal to the product of the two means, viz. $5 \times 119 = 35 \times 17$, each being equal to 595, and $65 \times 4 = 13 \times 120 = 260$.

If four numbers are proportional, they will also be so in alternation, inversion, composition, subtraction, conversion, and mixtly. Euclid 5. Def. 12, 13, 14, 15, 16.

That is, if $65 : 13 :: 20 : 4$ be in direct proportion.
Then $65 : 20 :: 13 : 4$ alternate.
And $13 : 65 :: 4 : 20$ inverted.
Also $65 + 13 : 13 :: 20+4 : 4$ compounded.
Or $65 + 20 : 20 :: 13+4 : 4$ alternately compound.
Again, $65 - 13 : 13 :: 20-4 : 4$ subtracted.
Or $65 - 20 : 20 :: 13-4 : 4$ alternately subtracted.
And $65 : 13+65 :: 20 : 4+20$ converted.
Lastly, $65 + 13 : 65 - 13 :: 20 + 4 : 20 - 4$ mixtly.

When three numbers are given, and a fourth proportional is required, in order to state the question right, observe the following directions, viz.

First, That always two of the three given terms are only supposed, and assign or limit the ratio or proportion; the third moves the question, and the fourth gives the answer.

Secondly, the term which moves the question, hath generally some of these words before it, viz. What will? How many? How long? How far? Or how much? &c.

Thirdly, That the first term in the supposition be of the same kind and denomination with that term which moves the question, and the term sought will be of the same kind and denomination with the second term in the supposition.

All questions thus prepared may be answered by three several rules, but the first is most commonly used.

RULE.

RULE. I.

Multiply the second and third terms together, and divide their product by the first term; the quotient will be the answer required.

RULE. II.

Divide the second term by the first, then multiply the quotient into the third term, and their product will be the answer required.

RULE III.

Divide the third term by the first, then multiply that quotient into the second term, and their product will be the answer.

1. If $3\frac{3}{4}$ yards of kersey cost 8s. 9d. what will $257\frac{1}{2}$ yards cost at that rate?

By Reduction.

	yds.	s.	d.	
As	$3\frac{3}{4}$:	8	9 ::	$257\frac{1}{2}$
	4	12		4
	15	105		1030
				105
				515
				103

12)
15) 108150 (7210 pence.
31   20) 600 10d.
15
0   30 10d.

By Decimals.

$4 \,|\, 3$
$3 \cdot 75 = 3\frac{3}{4}$ yards.

$12 \,|\, 9$
$20 \,|\, 8 \cdot 75$
£ ·4375

$4 \,|\, 2$
$257 \cdot 5$ yards.

As 3.75 : .4375 :: 257.5

257.5
21875
30625
21875
8750

3.75) 112.65625 (30.0416 = 30 l. – s. 10 d. as before.
..1562
..625
2500
250, &c.

2. If $1\frac{3}{4}$ ounce of silver plate cost 10s. $11\frac{1}{4}$ d. what will a service, weighing 327 oz. 12 pwt. 9 gr. cost at that rate?

oz. pwt. s. d. oz. pwt. gr.
As 1 15 : 10 $11\frac{1}{4}$ :: 327 12 9

20	12	20
35	131	6552
24	4	24
140	525 farth.	26208
70		13104
840 gr.		157257
		525
		786285
		314514
		786285

840) 82559925 (98285 farthings.
695
239
719
472
525

4) 98285 farthings.
12 | $24571\frac{1}{4}$ pence.
20 | 2047 $7\frac{1}{4}$
£ 102 7 $7\frac{1}{4}$, ans.

By DECIMALS.

20 | 15
oz | 1.75

4 | 1
12 | 11.25
20 | 10.9375
£ | 0.546875

$4 \times 6 = 24$ | 6) 9.225
20 | 12.375
oz. | 327.61875

As

```
As 1.75 : .546875 :: 327.61875
                        .546875
                     ___________
                      163809375
                     229333125
                    262095000
                   196571250
                  131047500
                 163809375
                 _____________
      1.75)179.166503906 25(102.380859 =
             416          102l. 7s. 7¼d. as before.
              666
              1415
               1503
                1039
                 1640
                 ____
                   65
```

3. If 2 cwt. 3 qrs. 21 lb. of sugar cost 6 l. 1 s. 8 d. what will 12 cwt. 2 qrs. cost at that rate?

By Decimals.

```
7 × 4 | 21.3        | 12 | 8          | 4 | 2
      |  3.75       | 20 | 1.6        |   |
______|______       |____|______      |___|_____
Cwt.  | 2.9375      | £  | 6.083      |   | 12.5

Cwt.      £.        Cwt.
2.9375 : 6.083 :: 12.5
           12.5
         ______
          30416
         73000
        _______
2.9375)76.0416(25.8865 = 25l. 17s. 8¾d. the answer.
        172916
         26041
          2591
           191
            15
           ___
```


By

By Reduction.

	cwt.	qrs.	lb.		l.	s.	d.		cwt.	qrs.
As	2	3	21	:	6	1	8	::	12	2
	4				20				4	
	11				121				50	
	28				12				28	
	329 lb.				1460				1400	
					1400					

$$\begin{array}{l} \qquad\qquad\quad 12) \\ 329)2344000(6212\tfrac{3}{4} \\ \qquad 700 \quad 20)517 \;-\; 8\tfrac{3}{4} \\ \qquad 420 \\ \qquad 910 \quad \pounds 25 \;\; 17 \;\; 8\tfrac{3}{4}, \text{ as before.} \\ \qquad 252 \\ \qquad\;\;\; 4 \\ \qquad 1008 \end{array}$$

By Vulgar Fractions.

cwt. qr. lb. cwt. l. s. d. l. cwt. qr. cwt.

2 3 21 $= \frac{47}{16}$, 6 1 8 $= \frac{73}{12}$; and 12 2 $= \frac{25}{2}$.

As $\frac{47}{16} : \frac{73}{12} :: \frac{25}{2}$. First, $\frac{73}{12} \times \frac{25}{2} = \frac{1825}{24}$.

Then $\frac{47}{\cancel{16}_{2}}\Big) \frac{1825}{\cancel{24}_{3}} \Big(\frac{3650}{141} =$ 25l. 17. $8\frac{2}{4}$d. answer as before.

When any one term in the proportion is an unit, the answer will sometimes be most readily obtained by practice, as in the two following examples.

4. If I give 5s. 4d. for one ounce of silver, what must I pay for $32\frac{1}{2}$ ounces at that rate?

s. d. s oz.
5 4 = $5.\dot{3}$; and $32\frac{1}{2}$ oz. = 32.5

oz. s. oz.
1 : $5.\dot{3}$ :: 32.5
$5.\dot{3}$

9) 975

$108.\dot{3}$
1625

$173.\dot{3}\dot{3}$ = 8l. 13s. 4d.

By Practice.

	s.	d.
$\frac{1}{2}$)	5	4
		4
1	1	4
		8
8	10	8
	2	8
Answer, £ 8	13	4

5. If a silver tankard, weighing 21 ounces, cost 5l. 19s. what is that an ounce?

oz. l. s. oz.
21 : 5 19 :: 1
20

21)119(5s. 8d. the answer.
14
12

168

		l.	s.	d.
3		5	19	
7		1	19	8
As before, £		–	5	8

6. If a piece of cloth cost 10l. 16s. 8d. I demand how many yards it contains, the ell English being worth 8s. 4d.?

12	4	s. d.	yd.	yd.
20	8.3			
£	$.41\dot{6}$ =	8 4	$1\frac{1}{4}$ =	1.25

12	8	l. s. d.
20	$16.\dot{6}$	
£	$10.8\dot{3}$ =	10 16 8

As $.41\dot{6}$: 1.25 :: $10.8\dot{3}$
1.25
5416
21666
108333 yds.
$.41\dot{6}$)13.541$\dot{6}$(32.5 = $32\frac{1}{2}$ yds. the answer.
41 13.541
.375) 12.1875
937
1875
....

It

It is to be obſerved, that there may be ſuperfluous terms in a queſtion, which muſt be omitted; as the 12 months in the next queſtion.

7. If 100l in 12 months gain 4l. 10s. what will 74l. 10s. gain in the ſame time, at the ſame rate of intereſt?

```
 l.     l.  s.      l.  s.
100 :  4  10 : :  74  10
  20    20          20
----  ----        ----
2000    90        1490
----  ----          90
                 -----
      2000)134100(67.0½ = 3l. 7s. -½d. ⅖, anſwer.
             ----
              100
               12
             ----
             1200
              4
             ----
             4800
             ----
              800
```

Sometimes the analogy or proportion will not bear, until ſome operations in addition, ſubtraction, multiplication, or diviſion, are performed; or, perhaps, an operation in one or more of thoſe rules may be required after the proportion, in order to find out the number ſought, as in ſome of the following examples.

8. If 19 yards of yard-wide ſtuff exactly line 14 yards of ſilk of another breadth; how many yards of the latter will line 184 pieces of the former, each piece holding 28½ yards?

```
          184
          28½
         ----
         1472
         368
           92
        -----
   19 : 5244 : : 14
           14
        -----
   19)73416(3864 yards, the anſwer.
      164
       121
        76
       ---
        0
```

9. If

9. If $24\frac{1}{2}$ lb. of raisins cost 9 s. $2\frac{1}{4}$ d. what will 18 frails cost, each weighing 3 qrs. $19\frac{1}{4}$ lb.?

4	2
4 × 7	24.5 = 6.125
4	.875
Cwt.	.21875 = $24\frac{1}{2}$ lb.

4	1
12	2.25
20	9.1875
£	.459375

4	1
28	19.25
4	3.6875
C.	.921875

.921875
18

7375000
921875

l.
.21875 : .459375 :: 16.59375
573954

6637500
829688
149344
4978
1161
83

.21875) 7.622754 (38.846875
106025
185254
10254
1504
191
16

38.846875
20

16.937500
12

11.250000
4

1.000000

Answer, 34 l, 16 s. $11\frac{1}{4}$ d.

10. The globe of the earth, under the line, is 360 degrees in circumference; each degree $69\frac{1}{2}$ miles; and this body being turned on its own axis, in the sydereal day, or 23 hours, 56 minutes; at what rate an hour are the inhabitants of Bencoolen, situate in the midst of the burning zone, carried from west to east, by this rotation?

```
                 360
                  69½
                -----
  h       '      3240
  23   56        216
  60              180
 -----          -----
 1436   :       25020  :: 60
               -------- Miles. F. P.
       1436)1501200(1045  3  9 167/718, the answer,
                6520
                 7760
                -----
                  580
                  × 8
                -----
                 4640
                 .332
                 × 40
                -----
                13280
                -----
                  354
```

11. A factor bought 72 pieces of Holland, which cost 537 l. 12 s. at 5 s. 4 d. per ell Flemish; I demand how many yards there were in all, and how many ells English in each piece?

537 l. 12 s. = 537.6 l. . . . 5 s. 4 d. = .26̸ l. and ¾ yd. = .75 yd.

```
  l      yd.     l.
 .26̸  :  .75  ::  537.6
                    75
                 -----
                 26880
                37632
             ---------|72)
         26̸)403.200 |1512 yards in all.
         2   40320   |  21 yards in a piece.
        ----  -------|  ----
        .24)  362.88
              122        1.25)21.00(16 ells English, and 1 yd.
               28               850          in a piece.
               48               ----
              ----                 1
```

12. A

12. A factor bought a certain quantity of tabby and brocade, which together cost him 126 l. 14 s. 10 d. the quantity of tabby he bought was 48 yards, at 4s. 4d. per yard, and for every two yards of tabby he had five of brocade; how many yards of brocade had he, and what did it cost him a yard?

$\frac{1}{5}$ | 48
$\frac{1}{12}$ | 9 12
 | – 16
£ | 10 8 price of the tabby.

l.	s.	d.	
126	14	10	
10	8	–	
116	16	10,	pr. of brocade.

Then as 2 : 5 :: 48
5
2)240
120 yards of brocade.

120)116 l. 6 s. 10 d.(19 s. $4\frac{1}{2}$, $\frac{11}{15}$ qr. the answer.
× 20
2326
46
× 12
552
82
× 4
328
88

13. If I sell 24 yards of Holland for 10 l. 10 s. how many ells Flemish shall I sell for 283 l. 17 s. 6 d. at that rate?

3) 24
8
32 ells Flemish = 24 yards.

As

	l.	s.	Ell F.	l.	s.	d.
As	10	10 :	32 ::	283	17	6
	20			20		
	210			5677		
	12			12		
	2520			68130		
				32		
				13626		
				20439		

2520)2180160(865$\frac{1}{7}$ ells Flemish, the answer.
1641
1296

360

14. There are two numbers, 75 is the less, to which the greater is in proportion as 8 to 5; what is their sum, and the product of their sum and difference, the difference and product of their squares, and the sum of the squares of their two quotes, the greater being divided by the less, and again the less by the greater.

As 5 : 8 :: 75 ; 120, the greater number.
75 + 120 = 195, their sum.
120 — 75 = 45, difference.
195 × 45 = 8755, product of their sum and differ.
14400 — 5625 = 8775, diff of the sq. of their sum and dif.
14400 + 5625 = 20025, sum of those squares.
75)120(1.6, quote of the greater divided by the less.
1.6 × 1.6 = 2.56, its square.
120)75.00(.625, quote of the less divided by the greater.
.625 × .625 = .390625, its square.
2.56 × .390625 = 2.950625. Q. E. F.

15. There are two numbers more, the greater is 224, bearing proportion to the other, as 8 to 7; what is the square of their sum, difference, and either quote; what is the result of the square of the sum of their difference, added to the product of their sum and difference?

8 : 7

$8 : 7 :: 224 : 196$, leſs number.

$224 + 196 = 420 \times 420 = 176400$, ſquare of their ſum.

$224 - 196 = 28 \times 28 = 784$, ſquare of their difference.

$\frac{224}{196} \times \frac{224}{196} = \frac{64}{49} = 1\frac{15}{49}$, ſq. of the quote of the gr. $\div$ leſs.

$\frac{196}{224} \times \frac{196}{224} = \frac{49}{64}$, ſq. of the quote of the leſs divided by the gr.

$420 \times 28 + 28 = 11788$, product of their ſum, and difference added to their difference.

Laſtly, $11788 \times 11788 = 138956944$. Q. E. F.

16. In a ſeries of proportional numbers, the firſt is 5, the third 8, and the product of the ſecond and third 78.4; what is the difference of the ſecond and fourth?

8) 78.4 (9.8, ſecond.

Then $5 : 9.8 :: 8 : 15.68$, fourth.

$\because 15.68 - 9.8 = 5.88$, the anſwer.

17. A may-pole 50 feet 11 inches long at noon time of the day, will caſt a ſhadow 98 feet 6 inches long; I would hereby find the breadth of a river, that running due eaſt within $20\frac{1}{2}$ feet of the foot of a ſteeple 300 feet 8 inches high, will at the ſame time throw the extremity of its ſhadow 30 feet 9 inches beyond the ſtream?

F. In. Feet. Shad. Shadow.

$50\ 11 = 50.91\dot{6} : 98.5 :: 300.6 : 581.6515$.

$20.5 + 30.75 = 51.25$.

Therefore $581.6515 - 51.25 = 530.4015 = 530$ f. 4.818 in. the anſwer required.

18. Suppoſe the ſea allowance for the common men to be 5 pounds of beef, and 3 pounds of biſcuit a day, for a meſs of four people; and that the price of the firſt barrelled be to the king $2\frac{1}{4}$ d. per pound, and of the ſecond $1\frac{1}{2}$ d; ſuch was the ſhip's company, that their fleſh coſt the government 12l. 12s. per day; pray what did they pay for their bread a week?

lb.	d.			
5 beef, value	$11\frac{1}{4} = .046875$	per day,	.328125	per week.
3 biſcuit	$4\frac{1}{2} = .01875$	or	.13125	

12l. 12s. = 12.6 l. per day, or 88.2 l. per week.

		l. worth	l. worth
beef.	biſcuit.	beef.	biſcuit.

$.328125 : .13125 :: 88.2 : 35.28 = 35$ l. 5 s. $7\frac{1}{4}$ d. anſwer.

19. In

19. In the year 1582, pope Gregory reformed the Julian kalendar; ordaining, that as the year is found to consist only of 365 days, five hours, and about 49 minutes, in order to prevent the inconveniencies of carrying the account of time too forward, by taking the solar year at 365 days and 6 hours full, which in a series of years must bring Lady-day to Michaelmas, that the Christian states for the future should drop 3 days in account every 400 years; that is to say, for each of the first three centuries in that space of time, the intercalary day in February should be omitted; but retained as formerly in the last or fourth century, beginning with the year 1600, when 10 whole days were sunk at once: by which artifice the variation of time will not, at least for a long space, be very considerable. According to this regulation, it is required to know in what year of Christ the new stile, as it is called, will be 20 days, as now it is only 11, before the old stile, which makes no such allowance?

$20 - 11 = 9$ days to be sunk.

D. Y. D.
As 3 : 400 :: 9 days : 1200 years to come.
$\therefore$ $1200 + 1700 = 2900$, the year required.

20. If the scavenger's rate, at $1\frac{1}{2}$ d. in the pound comes to 6 s. $7\frac{1}{2}$ d. where they ordinarily assess $\frac{4}{5}$ of the rent; what will the king's tax for that house be, at 4s. the pound, at the pound, rated at the full rent?

6s. $7\frac{1}{2}$d. $= \frac{53}{160}$ l. . . . $1\frac{1}{2}$ d. $= \frac{1}{160}$ l. . . . 4s. $= \frac{1}{5}$ l.
Then $\frac{4}{5}$) $\frac{53}{160}$ ($\frac{53}{128}$.
As $\frac{1}{160} : \frac{53}{128} :: \frac{1}{5} : \frac{53}{4} = 13$ l. 5s. the answer required.

21. Agreed for the carriage of $2\frac{1}{2}$ tons of goods, three miles wanting $\frac{1}{10}$, for $\frac{1}{8}$ of $\frac{3}{5}$ of a guinea; what is that per hundred for a mile?

$\frac{1}{8}$ of $\frac{3}{5}$ of $\frac{21}{20} = \frac{63}{800}$; and $2\frac{9}{10}$ miles $= \frac{29}{10}$.
Then as $\frac{5}{2}$ tons : $\frac{63}{800} :: \frac{1}{20} : \frac{63}{40000}$ l.
And $\frac{29}{10}$) $\frac{63}{40000}$ ($\frac{36}{116000}$ l. $= \frac{378}{725}$ of a farthing, or little more than $\frac{1}{2}$ a farthing, the answer required.

22. A father dying, left his son a fortune, $\frac{3}{16}$ of which he ran through in six months; $\frac{2}{3}$ of the remainder held him a twelvemonth longer, at which time he had bare 348 l. left; pray what did his father bequeath him?

$\frac{16}{16} - \frac{3}{16} = \frac{13}{16}$ remaining at the end of six months.

$\frac{2}{3}$ of $\frac{13}{16} = \frac{13}{24} = 696$ - -

For $\frac{1}{3}$ of $\frac{13}{16} = \frac{13}{48} = 348$ - - by the question.

$\frac{13}{24} : \frac{3}{16} :: \frac{696}{1} :$ 240 18 $5\frac{1}{2}$

1284 18 $5\frac{1}{2}$, the answer required.

23. A person dying, left his wife with child, and making his will, ordered, that if she went with a son, $\frac{2}{3}$ of the estate should belong to him, and the remainder to his mother; and if she went with a daughter, he appointed the mother $\frac{2}{3}$, and the girl $\frac{1}{3}$; but it happened that she was delivered both of a son and daughter, by which she lost in equity 2000 l. more than if it had been only a girl; what would her dowry have been had she only had a son?

As the son was to have twice as much as the mother, and the mother twice as much as the daughter, let the estate be divided as follows, viz. $4 + 2 + 1 = 7$, the whole estate, so that as she had both a son and a daughter, the mother must have but $\frac{2}{7}$ of the whole estate; whereas, had it been only a daughter, she would have had $\frac{2}{3}$.

$$\frac{2}{7} = \frac{6}{21}, \text{ and } \frac{2}{3} = \frac{14}{21}. \therefore \frac{14}{21} - \frac{6}{21} = \frac{8}{21} = 2000 \text{ l.}$$

$$\frac{8}{21} : \frac{2000}{1} :: \frac{1}{3} : \frac{14000}{8} = 1750 \text{ l. the answer.}$$

24. A younger brother received 2200 l. which was just $\frac{5}{12}$ of his elder brother's fortune; and $3\frac{1}{8}$ times the elder's money was half as much again as the father was worth; what was that?

$$\frac{5}{12}\Big)\frac{2200}{5}\Big(\frac{26400}{5} = 5280 \text{ l.} = \text{elder brother's fortune.}$$

$$5280 + 3\tfrac{1}{8} = 16500.$$

$$1\tfrac{1}{2} = \tfrac{3}{2} : \frac{16500}{5} :: \tfrac{1}{1} : \frac{33000}{5} = 11000 \text{ l. father's fortune.}$$

25. A person making his will, gave to one child $\frac{13}{30}$ of his estate, to another $\frac{11}{30}$; and when these legacies came to

be paid, one turned out 540l. 10s. more than the other; what did the testator die worth?

$\frac{11}{39} = \frac{330}{1170}$, and $\frac{19}{30} = \frac{741}{1170}$. Then $\frac{741}{1170} - \frac{330}{1170} = \frac{411}{1170} =$ 540l. 10s. $= \frac{1081}{2}$l. $\frac{411}{1170} : \frac{1081}{2} :: \frac{1}{1} : \frac{1264770}{822} = 1538$l. 12s. $11\frac{3}{4}$d. $+ \frac{89}{137}$q. the answer.

26. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ship be worth $\frac{1}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo, valued at 12000 l.; what did both ship and cargo stand the owner in?

$$\frac{3}{7} \text{ of } \frac{4}{5} \text{ of } \frac{7}{8} = \frac{3}{10}, \text{ and } \frac{1}{9} \text{ of } \frac{6}{7} \text{ of } \frac{11}{13} = \frac{22}{273}.$$

Then $\frac{3}{10} : \frac{22}{273} :: \frac{12000}{1} : \frac{880000}{273} = 3223$l. 8s. $10\frac{1}{4}$d. $\frac{45}{91}$.

$\therefore$ 3223l. 8s. $10\frac{1}{4}$d. $\frac{45}{91} +$ 12000 l. $=$ 15223l. 8s. $10\frac{1}{4}$d. $\frac{45}{91}$, the answer.

27. In some parishes in the country they take off 3 l. a year in 17 from the rents, in assessing the farmers; what will the landlord receive net out of a farm of 140 l. a year in those places, when the king's tax is 4 s. in the pound?

4 s. $=$ .2 l. As 17 : 3 :: 140 : 24.706 l. abatement.
Then 140 l. $-$ 24.706 l. $=$ 115.294 l.
Also 115.294 l. $\times$ .2 $=$ 23.0588 l. tax.
$\therefore$ 140 l. $-$ 23.0588 l. $=$ 116.9412 l. $=$ 116 l. 18 s. 10 d. ans.

28. If I leave Exeter at ten o'clock on Tuesday morning for London, and ride at the rate of two miles an hour without intermission; you set off from London for Exeter at six the same evening, and ride three miles an hour constantly; the question is, whereabout on the road you and I shall meet, if the distance of the two cities be 130 miles?

8 $\times$ 2 $=$ 16 miles, I had travelled before you set out.
130 $-$ 16 $=$ 114; and 2 $+$ 3 $=$ 5 miles, both go in 1 hour.
Then 5 : 1 :: 114 : $22\frac{4}{5}$ hours, they will meet.
$\therefore$ $22\frac{4}{5} \times 3 = 68\frac{2}{5}$ miles distant from London.
And $22\frac{4}{5} \times 2 + 16 = 61\frac{3}{5}$ miles distant from Exeter.

29. A sets out from London to Lincoln, at the very same time that B sets forward for London from Lincoln, distant

100 miles; at eight hours end they meet on the road, and it then appeared that A had rid $2\frac{1}{2}$ miles an hour more than B; at what rate an hour did each of them travel?

hours. miles. hour. miles.
$8 : 100 :: 1 : 12\frac{1}{2}$, both travelled.
Then $12\frac{1}{2} - 2\frac{1}{2} = 10$; and 2)10(5 miles, B rode.
And $5 + 2\frac{1}{2} = 7\frac{1}{2}$ miles, A rode an hour.

30. A reservoir for water has two cocks to supply it; by the first it may be filled alone in 44 minutes, by the second in just an hour; and it hath a discharging cock, by which it may, when full, be emptied in half an hour: now suppose these three cocks, by accident, should all of them be left open, and the water should chance to come in; what time, supposing the influx and reflux of the water to be always alike, would this cistern be in filling?

In one minute is filled by $\left\{\begin{array}{l} \text{1st } \frac{1}{44} = \frac{15}{660} \\ \text{2d } \frac{1}{60} = \frac{11}{660} \end{array}\right\}$ of the cistern.
Also $\frac{15}{660} + \frac{11}{660} = \frac{26}{660}$ of the cistern filled in a minute, both running.
In one minute runs out $\frac{1}{30} = \frac{22}{660}$ of the whole cistern.
And $\frac{26}{660} - \frac{22}{660} = \frac{4}{660} = \frac{1}{165}$ fills in one minute, all being open.
$\therefore \frac{1}{165} : 1 :: 1 : 165$ minutes $= 2\frac{3}{4}$ hours, the answer required.

31. A can do a piece of work in 10 days, B alone in 13; set them both about it together, in what time will it be finished?

A $\frac{1}{10} = \frac{13}{130}$, B $\frac{1}{13} = \frac{10}{130}$ of the work in 1 day.
Then $\frac{23}{130} : 1$ day $:: 1$ work $: \frac{130}{23} = 5\frac{15}{23}$ days, the answer.

32. B and C together can build a boat in 18 days; with the assistance of A, they can do it in 11 days; in what time will A do it by himself?

$\left.\begin{array}{l} B + C \text{ can perform } \frac{1}{18} = \frac{11}{198} \\ A + B + C \text{ can perform } \frac{1}{11} = \frac{18}{198} \end{array}\right\}$ of the work in one day.
Then $\frac{18}{198} - \frac{11}{198} = \frac{7}{198}$, A can perform of the whole work in one day.
$\therefore \frac{7}{198} : 1$ day $:: 1$ work $: \frac{198}{7} = 28\frac{2}{7}$ days, the answer.

 33. If

33. If A alone can do a piece of work in ten days, A and B together in seven days; in what time can B do it alone?

A + B can do $\frac{1}{7} = \frac{10}{70}$; A alone $\frac{1}{10} = \frac{7}{70}$ of the work in one day.

Then $\frac{10}{70} - \frac{7}{70} = \frac{3}{70}$, B's day's work.

$\because \frac{3}{70}$: 1 day :: 1 work : $\frac{70}{3} = 23\frac{1}{3}$ days, the answer required.

34. X, Y, and Z can, working together, compleat a stair-case in 12 days; Z is man enough to do it alone in 24 days, and X in 34; in what time could Y get it done by himself?

X $\frac{1}{34} = \frac{12}{408}$, Z $\frac{1}{24} = \frac{17}{408}$, and X + Y + Z do $\frac{1}{12} = \frac{34}{408}$, all working one day.

Then $\frac{12}{408} + \frac{17}{408} = \frac{29}{408}$, done in one day by X and Z working together.

And $\frac{34}{408} - \frac{29}{408} = \frac{5}{408}$, done in one day by Y alone.

$\because \frac{5}{408}$: 1 day, :: 1 work, : $\frac{408}{5} = 81\frac{3}{5}$ days, the answer.

35. Three workmen can do a piece of work in certain times, viz. A can do it in three weeks, B can do thrice the work in eight weeks, and C five times in 12 weeks; in what time can they finish it jointly?

Newton's Universal Arithmetic.

A can do $\frac{1}{3} = \frac{32}{96}$
B - - $\frac{3}{8} = \frac{36}{96}$
C - - $\frac{5}{12} = \frac{40}{96}$

their sum $\frac{108}{96} = \frac{9}{8}$ work, all working together one week.

1 week = 6 working days, and 1 day = 12 working hours.

$\because \frac{9}{8}$ work : 6 days :: 1 work : $\frac{48}{9}$ days, = 5 days 4 hours, the answer.

36. If a cardinal can pray a soul out of purgatory, by himself in an hour, a bishop in three, a priest in five, and a frier in seven; in what time can they pray out three souls, all praying together? *Palladium.*

While the cardinal prays $1 = \frac{105}{105}$
The bishop - - - $\frac{1}{3} = \frac{35}{105}$
The priest - - - - $\frac{1}{5} = \frac{21}{105}$
And the frier - - - $\frac{1}{7} = \frac{15}{105}$

their sum $\frac{176}{105}$, in an hour together.

souls. hour. souls. hours. hour. ′ ″

$\because \frac{176}{105}$: 1 :: 3 : $\frac{315}{176}$ = 1 47 23$\frac{2}{11}$, the answer.

37. I am dispatched on a commission from London to Edinburgh, distant by computation say 350 miles, and my rout is settled at 22 miles a-day; you four days after are sent after me with fresh orders, and are to travel 32 miles a day; whereabout on the road shall I be overtaken by you?

$22 \times 4 = 88$ miles you have travelled before I set out.
$32 - 22 = 10$ miles I gain each day of you.
$10 : 1 :: 88 : 8.8$ days.
Then $8.8 \times 32 = 281.6$.
$\because 350 - 281.6 = 68.4 =$ 68 miles, 3 furlongs, 8 poles, on this side Edinburgh.

38. If the sun moves every day one degree, and the moon thirteen; and at a certain time the sun be at the beginning of Cancer, and in three days after the moon in the beginning of Aries; the place of their next following conjunction is required? *Newton's Universal Arithmetic.*

$30 \times 3 = 90$ degrees, from the first of Aries to Cancer.
$90 + 3 = 93$ degrees, the sun before the moon.
$13 - 1 = 12$ degrees, the moon gains in one day.
$12 : 1 :: 93 : 7\frac{3}{4}$ days, in which time the sun will be overtaken.
$\because 7\frac{1}{4} + 3 = 10\frac{3}{4}$ degrees of Cancer, the answer.

39. If the half of fifteen be seven,
What is the fourth of eleven?

As $\frac{15}{2} : 7 :: \frac{11}{4} : \frac{77}{30} = 2\frac{17}{30}$, the answer required.

In mechanics, a lever of the second order is, when the power acts at one end, the prop fixed directly at the other, and the weight somewhere between them.

In this order of levers, their force are in a contra-proportion to their lengths.

40. If a lever be 100 inches long, what weight, lying $7\frac{1}{2}$ inches from the end, resting on a pavement, may be moved with the force of 168 lb lifting at the other end of the lever?

$100 - 7.5 = 92.5$, longest end.
inches. lb. inches. lb.
$\because 7.5 : 168 :: 92.5 : 2072$, the answer.

In a lever of the third order, the prop is planted at one end of the bar, the weight at the other end, and the moving force somewhere between.

41. A water-wheel turns a crank, working three pump-rods, fixed juſt ſix feet from the joint or pin; by which their ſeveral levers, each nine feet in length, are faſtened, for the ſake of the intended motion, at one end; the ſuckers of the pumps being worked by the other, ſhews them to be levers of the third order: now I would know what the length of the ſtroke in each of the barrels will be, if the crank be made to play juſt nine inches round its center?

$9 \times 2 = 18$ inches, the diameter of the crank.

feet. inch. feet. inch.
6 : 18 : : 9 : 27, the length of the ſtroke.

42. With what force ought that water-wheel to be driven, which, circumſtanced as in the laſt queſtion, raiſes three cubic feet of water at every revolution of the wheel, each experimentally weighing $62\frac{1}{2}$ lb. averdupoiſe; the friction of the machine rejected?

$62\frac{1}{2}$ lb. $\times 3 = 187\frac{1}{2}$ lb. = weight of 3 ſolid feet of water.
Therefore 6 : $187\frac{1}{2}$: : 9 : $281\frac{1}{4}$ lb. = force required, rejecting the friction. Q. E. F

The magnitude of ſpheres of the ſame denſity are directly in proportion to the cubes of their diameters.

43. If the diameter of the earth is 7970 miles, on the moon 2170 miles, ſuppoſing them to be exact ſpheres, of the ſame denſity, what compariſon is there between them in point of magnitude?

Cube of the earth's diameter = 506261573000.
Cube of that of the moon = 10218313000.
$\therefore$ 10218313 : 506261573 : : 1 : 49.5445. Q. E. F.
The leſs porous a body is, the greater is its denſity.

44. The compactneſs or denſity of the moon is to that of the earth, as $132\frac{1}{2}$ is to 100; what proportion then is there between the quantity of matter in the earth, and that in the moon?

The earth in the foregoing queſtion is found to be 49 5445 times bigger than the moon.

$\therefore$ 123.5

∵ $123.5 : 100 :: 49.5445 : 40.117$. Q. E. F.

That is, the earth contains 40.117 times moſt matter.

The velocity of ſound is found by experiment to be uniform; viz. about 1150 feet in one ſecond of time, if it meets with nothing to retard or obſtruct its motion.

45. If I ſee the flaſh of a piece of ordnance fired by a veſſel in diſtreſs at ſea, which happens, we will ſuppoſe, nearly at the inſtant of its going off, and hear the report a minute and three ſeconds afterwards; how far is ſhe off, reckoning for the paſſage of ſound as before?

1 minute 3 ſeconds = 63 ſeconds.

As 1 ſecond : 1150 feet :: 63 ſeconds : 72450 feet = 13 miles, 5 furlongs, 30 poles, 5 yards, the anſwer required.

46. How long after firing the warning-gun in Hyde-Park, may the ſame be heard at Highgate, taking the diſtance at $5\frac{2}{3}$ miles?

$5\frac{2}{3}$ miles = 29920 feet.

Then 1150 feet : 1 ſecond :: 29920 feet : 26 ſeconds, 1 $\frac{1}{23}$ third, the anſwer required.

47. Suppoſe a maid carrying apples to market was met by three boys, and that the firſt took half that ſhe had, but returned 10; that the ſecond took one-third that ſhe then had, but returned two; laſtly, the third took away half that ſhe had left, but returned her one; and when ſhe had got clear, ſhe had 12 apples left; what number of apples had ſhe at firſt? *Emerſon's Arithmetic.*

Firſt $12 - 1 = 11$; and $11 \times 2 = 22$, before ſhe met the laſt boy.

Alſo $22 - 2 = 20$; and $\frac{2}{3} : 20 :: \frac{3}{3} : 30$, the number ſhe had before ſhe met with the ſecond boy; and before the firſt boy returned her 10, ſhe had but 20, equal to what the boy took.

∵ $20 \times 2 = 40$ apples, at the firſt. Q. E. F.

Proof $40 \div 2 = 20$; alſo $20 + 10 = 30$, when ſhe met the ſecond boy.

Likewiſe $30 \div 3 = 10$; and $30 + 2 - 10 = 22$, when met by the laſt.

Laſtly, $22 \div 2 = 11$; and $11 + 1 = 12$ left, per queſtion.

48. A tradesman begins the world with 1000 l. and finds that he can gain 1000 l. in 5 years by land trade alone; and that he can gain 1000 l. in 8 years by sea trade alone; and likewise that he spends 1000 l. in $2\frac{1}{2}$ years by gaming; how long will his estate last if he follows all three?

Emerson's Arithmetic.

$\frac{1000}{5} = 200$ l. gain by land trade in one year.

$\frac{1000}{8} = 125$ l. gain by sea trade in one year.

325 l. his whole gain.

$\frac{1000}{2\frac{1}{2}} = 400$ l. lost by gaming in one year.

Difference 75 l. loss at the year's end.

$\therefore$ 75 l. : 1 year :: 1000 l. : $13\frac{1}{3}$ years, the answer.

49. A clock hath two hands or pointers; the first, A, goes round once in 12 hours; the second, B, once in an hour; now, if they both set forward together, in what time will they meet again? *Emerson's Arithmetic.*

As A goes only $\frac{1}{12}$ of the circumference in an hour,
And B goes the whole, or $\frac{12}{12}$;
Then $\frac{12}{12} - \frac{1}{12} = \frac{11}{12}$, B gains in an hour.

$$\therefore \frac{11}{12}\,C : 1\,h. :: 1\,C : \frac{12}{11}\,h. = 1\,h.\ 5'\ 27\tfrac{3}{11}''.$$

The velocity acquired by heavy bodies falling near the surface of the earth, is $16\frac{1}{12}$ feet in the first second; and as $16\frac{1}{12}$ feet are to the square of one second, or $1''$; so is the given distance, to the square of the seconds required.

Or by multiplying $16\frac{1}{12}$, the descent of a heavy body in one second of time, by as many of the odd numbers, beginning from unity, as there are seconds in the given time; viz. by 1 for the first, 3 for the second, 5 for the third, 7 for the fourth, &c. the sum total will give the space it hath passed.

50. Suppose a stone let go into an abyss, should be stopped at the end of the eleventh second after its delivery, what space would it have gone through?

$1^2 : 16.08\dot{3} :: 11 \times 11 = 121 : 1946.08\dot{3}$. Q. E. F.

Or,

Or,

$$16.08\dot{3}\times\left\{\begin{array}{rcl} 1 & = & 16.08\dot{3} \\ 3 & = & 48.250 \\ 5 & = & 80.41\dot{6} \\ 7 & = & 112.58\dot{3} \\ 9 & = & 144.750 \\ 11 & = & 176.91\dot{6} \\ 13 & = & 219.08\dot{3} \\ 15 & = & 241.250 \\ 17 & = & 273.41\dot{6} \\ 19 & = & 305.583 \\ 21 & = & 337.75 \end{array}\right\}\text{in the}\left\{\begin{array}{c} 1\text{ſt} \\ 2\text{d} \\ 3\text{d} \\ 4\text{th} \\ 5\text{th} \\ 6\text{th} \\ 7\text{th} \\ 8\text{th} \\ 9\text{th} \\ 10\text{th} \\ 11\text{th} \end{array}\right\}\text{ſeconds of time.}$$

$1946.08\dot{3}$, as before.

51. If a ſtone be $19\frac{1}{2}$ ſeconds in deſcending from the top of a precipice to the bottom, what is the height of the ſame?

$1^2 : 16.083 :: 19.5 \times 19.5 = 380.25 : 6115.6875.$

$\therefore$ 6) 6115.6875 (|= 1019 fathom, 1 foot, $8\frac{1}{4}$ inches. Q. E. F.

52. If a hole could be bored through the center of the earth, in what time, after the delivery of a heavy body on its ſurface, would it arrive at its center?

The ſemidiameter of the earth 3980 miles = 21014400 feet.

$16.08\dot{3} : 1^2 :: 21014400 : 1306594.82.$

$\sqrt{1306594.82} = 1135.554$ ſeconds, $= 18$ min. $55''\ 33'''$. Q. E. F.

1. If the quantities of matter in any two or more bodies, put in motion, be equal, the forces wherewith they are moved will be in proportion to their velocities.

2. If the velocities of theſe bodies be equal, their forces will be directly as the quantities of matter contained in them.

3. If both the quantities of matter and the velocities be unequal, the forces with which bodies are moved, will be in a proportion compounded of the quantities of matter they contain, and of the velocities wherewith they move.

53. There are two bodies, the one contains 25 times the matter of the other (or is 25 times heavier) but the leſſer moves with 1000 times the ſwiftneſs of the greater; in what proportion are the forces by which they are moved?

As

As $25 : 1000 :: 1 : 40$, the less is moved with a force so much greater than the other.

54. There are two bodies, one of which weighs 100 lb. the other 60; but the lesser body is impelled by a force 8 times greater than the other; the proportion of the velocities, wherewith these bodies move, is required?

$$\text{As } 60 : 100 :: 1 : 6\tfrac{2}{3} = \tfrac{5}{3}.$$

So that the velocity of the less to the greater will be $\frac{5}{3} \times \frac{8}{1} = \frac{40}{3} = 13\frac{1}{3} : 1$.

So the velocity of the less to the greater will be, as $13\frac{1}{3}$ to 1, or as 40 to 3.

55. There are two bodies, the greater contains 8 times the quantity of the matter in the less, and is moved with a force 48 times greater; the ratio of the velocity of these two bodies is required?

If the forces were equal, the velocity of the lesser would be 8 times that of the greater.

But as the force the greater is moved with is 48 times that which moves the less.

As $8 : 48 :: 1 : 6$; so the velocity of the less to that of the greater is as 1 to 6.

1. In comparing the motions of bodies, if their velocities be equal, the spaces described by them shall be in the direct proportion of the times in which they are described.

2. If the times be equal, then the spaces described will be as their velocities.

3. If the times and the velocities be unequal, the spaces will be in proportion compounded of the times and velocities.

56. There are two bodies, one of which moves 40 times swifter than the other; but the swifter body has moved but one minute, whereas the other has been in motion two hours; the ratio of the spaces described by these two bodies is required?

In two hours are 120 minutes.

As $40 : 120 :: 1 : 3$, so is the space the swifter hath moved to that of the slower.

57. Suppose one body to move 30 times swifter than another: as also the swifter to move 12 minutes, the other only

only 1; what difference will there be between the ſpaces by them deſcribed, ſuppoſing the laſt has moved 60 inches?

60 = inches = 5 feet, moved by the ſecond.
And $1:5::30\times12=360:1800$, by the firſt.
$\therefore 1800-5=1795$ feet, the anſwer.

58. There are two bodies, one whereof has deſcribed 50 miles, the other only 5; but the firſt hath moved with five times the velocity of the ſecond; what is the ratio then of the times they have been deſcribing thoſe ſpaces?

As $5:\frac{50}{5}::1:2$; ſo that the firſt body hath been in motion double the time of the ſecond.

SECT. II.

RECIPROCAL PROPORTION,

CALLED, THE

GOLDEN RULE; or, RULE of THREE INVERSE.

RECIPROCAL proportion is, when of four numbers the third (viz. that which moves the queſtion) beareth the ſame ratio or proportion to the firſt, as the ſecond does to the fourth.

Therefore the leſs the third term is, in reſpect to the firſt, the greater will the fourth be in reſpect to the ſecond.

And the greater the third term is, in reſpect to the firſt, the leſs will the fourth term be in reſpect to the ſecond.

Therefore, obſerve that in any queſtion in proportion, when MORE requires MORE, or LESS requires LESS, the terms are in direct proportion.

But if MORE requires LESS, or LESS requires MORE, then the terms will be in reciprocal proportion.

The ſame directions for ſtating the queſtion are to be obſerved here as in direct proportion.

The queſtion being truly ſtated, obſerve this general rule.

RULE.

RULE.

Multiply the firſt and ſecond terms together, and divide their product by the third term, the quotient will be the anſwer required.

1. If a penny white loaf ought to weigh 6 ounces and 12 drams averdupoiſe, when wheat is ſold at 6 s. 6 d. per buſhel; what muſt it weigh, when wheat is ſold at 4 s. the buſhel?

s. d. l. oz. dr. s. l.
6 6 = .325 - - 6 12 = 6.75, and 4 = .2.
Then as .325 : 6.75 : : .2

```
      .325
      ----
      3375
     1350
    2025
    -------          oz.        oz. dr.
 .2)2.19375 (10.96875 =  10 15½, the anſw.
```

Here it is plain, that the leſs the price of wheat, the bigger the loaf ought to be.

2. A general is beſieged in a town, in which are 1569 ſoldiers, with proviſion of victuals for three months; how many muſt depart the garriſon, that the ſame victuals may laſt the remaining ſoldiers $7\frac{1}{2}$ months?

```
              mon. ſold.   mon.
Reciprocally  3 : 1569 : : 7.5
                     3
                  ----
           7.5) 4707 (627, or 628, may ſtay.
                 207
                  570
                  ---
                   45
```

Conſequently 1569 — 627 = 942, or at the leaſt 941 muſt depart.

3. How many yards of ell-wide flannel is ſufficient to line a cloak, containing $18\frac{1}{8}$ yards of camblet, $\frac{3}{4}$ yard wide?
= $\frac{3}{4}$

```
 yd.                 yds.        yd.
3/4 = .75 - - - - 18 7/8 = 18.875, and 1 1/4 = 1.25.
Reciprocally, .75 : 18.875 : : 1.25
                          .75
                        ------
                        94375
                       132125
                      --------  yds:      yds. qr. n.
               1.25) 1415625 (11.325 = 11   1  1 1/5, the
                      165                          answer
                       406
                        312
                         625
                        -----
                         ...
                        -----
```

4. How many yards of matting, that is $2\frac{1}{2}$ feet wide, will cover a floor that is 17 feet long, and 15 feet 3 inches broad?

```
feet. in.    yds        feet   yds.     feet.    yd.
 15   3 = 5.083 - - 17 = 5 6 - - 2 1/2 = .83
             yds.
Reciprocally, 5 813 : 5.6 : : .83
                5.6
              ------
           9) 30500
              ------
               34888
              254166
             --------  yds.     yds. ft. in.
       .83) 28.9055 (34.56 = 34    1  8 2/5, the answer.
        8     28905
       -------------
       .75) 26.0150
       -------------
```

5. A borrowed of his friend B 250 l. for 7 months, promising to do him the like favour; some time after B hath an occasion for 300 l.; how long may he keep it to be made full amends for the favour?

Reciprocally,

l. mon. l.
Reciprocally, 250 : 7 : : 300
7

320) 1750 (5 months and 25 days, the anf. required.

250
× 30, days in a month.

75

6. A regiment of foldiers, confifting of 976 men, are to be new clothed, each coat to contain $2\frac{1}{2}$ yards of cloth, that is $1\frac{5}{8}$ yards wide, and lined with fhalloon $\frac{7}{8}$ yard wide; how many yards of fhalloon will line them?

976 $\quad 1\frac{5}{8} = 1.625$, and $\frac{7}{8} = .875$
$2\frac{1}{2}$

1952
488

Reciprocally, 2440 : 1.625 : : .875
1.625

1220
488
1464
244

yds. qr. nls.
.875) 3965.000 ($4531.428\cancel{5}7$ = 4531 1 $2\frac{6}{7}$, the anfwer.
4650
2750
1250
3750
2500
7500
5000
6250
125

7. If a tailor can make a coat and waiftcoat with three yards and three quarters of broad-cloth, of one yard and a half's.

half's breadth; how many yards of ſtuff, of $\frac{5}{8}$ yard's breadth, will he require to fit the ſame perſon?

yd. br.

$1\frac{1}{2} = \frac{3}{2} : 3\frac{3}{4} = \frac{15}{4} :: \frac{5}{8}$. Firſt $\frac{3}{2} \times \frac{15}{4} = \frac{45}{8}$.

$\therefore \frac{5}{8}\Big)\frac{45}{8}\Big(= 9$ yards, the anſwer required.

There being three orders of levers, or three varieties, wherein the weights, props, or moving powers, may be differently applied to the vectis, or inflexible bar, in order to effect mechanical operations in a convenient manner.

A lever of the firſt order hath the power placed at one of its ends, and the weights to be raiſed is put at the other, and the fulcrum or prop ſomewhere between them.

In this order, the power applied at one end will be reciprocally proportional to the diſtances of thoſe ends from the fulcrum, or point ſupported; or in the ſteelyards, as the diſtance of the weight from the point of ſuſpenſion.

8. What weight will a fellow be able to raiſe, who preſſes with the force of a hundred and a half on the end of an equipoiſed handſpike 100 inches long, which is to meet with a convenient prop exactly $7\frac{1}{2}$ inches above the other end of the machine?

$100 - 7.5 = 92.5$, the longeſt end of the lever from the fulcrum.

Reciprocally, 92.5 inch. : 168 lb. :: 7.5 inch. : 2072 lb. $= 18\frac{1}{2}$ cwt., the anſwer.

9. What weight, hung at 70 inches diſtance from the fulcrum of a ſteelyard, will equipoiſe a hogſhead of tobacco $9\frac{1}{2}$ cwt. freely ſuſpended at two inches diſtance on the contrary ſide?

$9\frac{1}{2}$ cwt. $=$ 1064 lb.

Reciprocally, 2 in. : 1064 lb. :: 70 in. : $30\frac{2}{5}$ lb., the anſwer.

The effects or degrees of light, heat, and attraction, are reciprocally proportional to the ſquares of their diſtances from the center whence they are propagated.

10. Suppoſe that in a room where two men, A and B, are ſitting there is a fire, from whcih A is three feet, and B ſix feet diſtant; it is required to find how much hotter it is at A's ſeat than at B's?

Reciprocally,

Reciprocally, $6 \times 6 = 36 : 1 :: 3 \times 3 = 9 : 4$; so that A's place is four times as hot as B's.

11. Supposing the earth to be 81000000 miles distant from the sun; I would know at what distance from him another body must be placed, so as to receive light and heat double to that of the earth?

$81100000 \times 81000000 = 6561000000000000.$

Recip. $1 : 6561000000000000 :: 2 : 3280500000000000.$

$\because \sqrt{3280500000000000} = 57275649$ miles, the answer.

12. Mercury, the nearest of the planets to the source of heat, light and life, in our system, the sun is about 32 million of miles from him; and Saturn, the remotest of the planets, is usually distant about 777 millions of miles; what comparison or proportion is there between the solar influences on these two bodies?

$32 \times 32 = 1024$, and $777 \times 777 = 603729$, squares of distance, cyphers omitted.

Recip. $\overset{\text{Saturn.}}{603729} : 1 :: \overset{\text{Mercury.}}{1024} : 589\frac{598}{1024}$. $\because$ The solar influence on Mercury to that of Saturn, is as $589\frac{598}{1024}$ to 1.

13. The distance between the earth and sun is accounted 81000000 of miles; the distance between Jupiter and the sun 424000000 of miles; the degrees of light and heat received by Jupiter, compared with that of the earth, is required?

$81 \times 81 = 6561$, and $424 \times 424 = 179776$, squares, of their distances, the cyphers being omitted.

Recip. $179776 : 1 :: 6561 : 27.4$; so that the sun's influence on the earth to that on the planet Jupiter, is 27.4 to 1.

14. A certain body on the surface of the earth weighs 112 lb.; the question is, whither this body must be carried, that it may weigh but 10 lb.

Recip. $\overset{\text{lb.}}{112} : \overset{\text{sq.}}{1} :: \overset{\text{r.}}{10} : \overset{\text{lb}}{1.2}$, square semidiameter.

Then $\sqrt{11.2} = 3.34664$, semidiameter of the earth from its center; or $9351\frac{1}{3}$ miles from its surface.

15. A

15. A Geographical Paradox.

There is a vaſt country in Ethiopia Superior, to whoſe inhabitants the moon doth always appear to be moſt enlightened, when ſhe is leaſt enlightened; and to be leaſt, when moſt; admitting the mean diſtance of the earth and moon's centers 240000 miles, in what proportion is this illumination?

Sun from the earth $81000000 + 240000 = 81240000$, ſun from a full moon.

$81000000 - 240000 = 80760000$ miles, the ſun from a new moon.

$\left.\begin{array}{l} 8124 \times 8124 = 65999376 \\ 8076 \times 8076 = 65221776 \end{array}\right\}$ ſquares of their different diſtances, the cyphers omitted.

Recip. $65221776 : 1 :: 65999376 : .9882$, ſo that the proportion of light and heat a new moon hath to that of a full one is,

As 1 to .9882, or as 458329 to 452929, in whole numbers.

16. If a body weighs 16 ounces upon the ſurface of the earth, what will its weight be 50 miles above it, taking the earth's at diameter 7970 Engliſh miles?

$7970 \div 2 = 3985$ miles, the earth's ſemidiameter.

$3985 \times 3985 = 15880225$, its ſquare; alſo $3985 + 50 = 4035$.

And $4035 \times 4035 = 16281225$.

Recip. $15880225 : 16 :: 16281225 : 15$ oz. $9\frac{4532803}{6512249}$ dr.

It hath been found by experiment, that a pendulum 39.2 inches long, in our latitude, vibrates 60 times in one minute; and that the length of pendulums are to one another, as the ſquare of the number of their vibrations made in the ſame ſpace of time.

17. What is the length of that pendulum which ſwings half ſeconds, or vibrates 120 times in a minute?

Recip. $3600 : 39.2 :: 14400 : 9\frac{4}{5}$ inches. Q. E. F.

18. What difference will there be in the number of vibrations made by a pendulum of 6 inches long, and another of 12 inches long, in an hour's time?

T Reciprocally,

Reciprocally, $39.2 : 3600 :: \left\{\begin{matrix} 12 : 11760 \\ 6 : 23520 \end{matrix}\right.$

$\sqrt{11760} = 108.444$ $\sqrt{23520} = 153.362$

Then $153.362 \times 60 = 9201.72$

And $108.444 \times 60 = 6506.64$

2695.08 Q. E. F.

In comparing the motions of bodies, the ratio or proportion between their velocities will be compounded of the direct ratio of the forces wherewith they are moved, and the reciprocal of the quantities of matter they contain.

19. The battering ram of Vespasian weighed, suppose 100000 pounds, and was moved, let us admit, with such a velocity, by strength of hands, as to pass through 20 feet in one second of time, and this was found sufficient to demolish the walls of Jerusalem; with what velocity must a bullet that weighs but 30lb. be moved, in order to do the same execution?

Recip. $100000 : 20 :: 30 : 66666\frac{2}{3}$ feet, in one second.

20. A body weighing 20lb. is impelled by such a force, as to send it 100 feet in a second; with what velocity would a body of 8lb. weight move, if it were impelled by the same force?

Reciprocally, $20 : 100 :: 8 : 250$ feet. Q. E. F.

SECT. III.

COMPOUND PROPORTION:

OR, THE

RULE OF FIVE.

THE rule of five is so called, from having five numbers given to find a sixth; three of which five given numbers, are only conditional, or supposed: and the other two move the question.

All

All questions in this rule include two in the rule of three, either both direct, or one indirect, and the other in reciprocal proportion; which so depend upon each other, that the answer of the first being made the middle term of the second, the fourth term of the second will be the final answer of the question.

Yet here observe, that many questions, though they may be wrought by two (or more) operations in the rule of three, cannot be answered by the rule of five.

In order to solve any question in the rule of five, observe the following directions.

Always place the three conditional terms in this order, let that number which is the principal cause of gain, loss, or action, &c. be put in the first place; that number which denotes the space of time, or distance of place, &c. be put in the second place; and that number which is the gain loss, or action, &c. be put in the third place: that done, place the other two terms, which move the question, under those of the same name.

Then if the blank or term sought fall under the third place.

RULE.

Multiply the three last terms together for a dividend, and the two first together for a divisor; the quotient arising from them will be the sixth term.

But if the blank or term sought fall under the first or second place.

RULE II.

Multiply the first, second, and last terms together for a dividend, and the other two together for a divisor; the quotient arising from them will be the sixth term.

1. If the carriage of 5 cwt. 3 qrs. weight, 150 miles, cost 3 l. 7 s. 4 d.; what must be paid for the carriage of 7 cwt. 2 qrs. 25 lb. weight, 64 miles, at the same rate?

cwt.	qr.	lb.	l.	s.	d.	d.		cwt.	qrs.	lb.	lb.
5	3	= 644	3	7	4	= 808,	and	7	2	25	= 865.

lb.		miles.		d.
644	.	150	.	808
865	.	64	.	

$808 \times 865 \times 64 = 44730880$, dividend.

$$644 \times 150 = 96600)\ 44730880(463$$

6090 — d.
2948 20) 38 7

5080 £ 1 18 7, the anf.

2. If 2 men can do $12\frac{3}{4}$ rods of ditching in $6\frac{1}{2}$ days; how many rods may be done by 18 men in 14 days?

men.	days.	rods.
2 .	6.5 .	12.75
18 .	14 .	

$12.75 \times 18 \times 14 = 3213$ dividend.
$2 \times 6.5 = 13)\ 3213\ (247\frac{2}{13}$ rods, the anfwer.

3. If a regiment of foldiers, confifting of 939, can eat up 351 quarters of wheat in 3 months; how many foldiers will eat up 1404 quarters in 2 months, at that rate?

foldiers.	months.	qrs. wheat.
939 .	3 .	351
	2 .	1404

$939 \times 3 \times 1404 = 3955068$ dividend.
$351 \times 2 = 702)\ 3955068\ (5634$
445
2386
2808

4. If 30 men can perform a piece of work in 11 days; how many will accomplifh another, four times as big, in one-fifth of the time?

men.	days	work.
30 .	11 .	1
.	$\frac{11}{5}$.	4

$30 \times 11 \times 4 = 1320$, dividend.

$$\frac{11}{5}\Big)\frac{1320}{1}\Big(\frac{6600}{11} = 600 \text{ men, the anfwer.}$$

5. If 9 men in 21 days mow 108 acres of ground; in how many days will 5 men mow 72 acres, at the fame rate of working?

men.

men.	days.	acres.
9	21	108
5	.	72

$9 \times 21 \times 72 = 13608$, dividend.

$108 \times 5 = 540)\ 13608\ (25\frac{1}{5}$ days, the anſwer.

When the terms in proportion are more than 5, as may ſometimes happen, the following rule of Mr. Emerſon's may be uſeful.

RULE.

1. Here, as in the ſingle rule of three, put that term into the ſecond place, which is of the ſame denomination with that ſought; and the terms of ſuppoſition one above another in the firſt place; alſo the terms of demand in the ſame order, one above another, in the third place; then the firſt and third of every row will be one name, and muſt be reduced to the ſame denomination, viz. the loweſt concerned.

2. Then proceed with each row, as with ſo many ſeparate queſtions in the ſingle rule of three, in order to find out the ſeveral diviſors, uſing the ſecond term in common for each of them; that is, in any row, ſay, If the firſt term gives the ſecond, does the third require more or leſs? if more, mark the leſſer extreme; if leſs, the greater for a diviſor.

3. Multiply all theſe diviſors together for a diviſor, and all the reſt of the numbes together for a dividend; the quotient is the anſwer, and of the ſame name with the ſecond term.

4. To contract the work, when the ſame numbers are concerned in both diviſor and dividend, throw them out of both; or divide any numbers by their greateſt common diviſor, and take the quotients inſtead of them.

6. If the carriage of 150 feet of wood, that weighs 3 ſtone a foot, comes to 3l. for 40 miles; how much will the carriage of 54 feet of freeſtone, that weighs 8 ſtone a foot, coſt for 25 miles?

* 150 feet.	3l. .	54 feet.
* 3 ſtone.		8 ſtone.
* 40 miles.		25 miles.

 54 ×

$$\frac{54\times 8\times 25\times 3}{150\times 3\times 40}=\frac{54\times 1\times 25\times 1}{150\times 1\times 5}=\frac{54\times 5}{150}=\frac{54}{30}=\frac{9}{5}$$

5) 9 (1 l. 16 s. the answer.

4
× 20

80

Or by an arithmetical equation further insisted upon and explained in exchange.

* $\cancel{150}$ feet.		3 l.	$\cancel{84}$ feet.
* $\cancel{3}$ stone.			$\cancel{8}$ stone.
* $\cancel{40}$ miles.			$\cancel{28}$ miles.

Divide both the divisors and dividends by their greatest common measure, cancelling as you have done with them, and setting down the quotients, till you have brought the divisor and dividend to their lowest terms.

$\cancel{8}$ $\cancel{8}$
$\cancel{8}5$ 9
∴ 5) 9 (1 l. 16 s. the answer.

4
× 20

80

7. If 248 men in $5\frac{1}{2}$ days, of 11 hours each, dig a trench of 7 degrees of hardness, and $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{3}$ deep; in how many days of 9 hours will 24 men dig a trench of 4 degrees of hardness, and $337\frac{1}{2}$ yards long, $5\frac{3}{3}$ wide, and $3\frac{1}{2}$ deep?

The application of Mr. Emerson's rule for questions of this nature, not giving the true answer in the original solution, the following method of investigation by four rule of three statings, may be thought preferable.

men.

men. days. men.

$$\text{First, } 248 : 5\tfrac{1}{2} :: 24 : \frac{248 \times 5\frac{1}{2}}{24}$$

hours. days. hours. days.

$$\text{Secondly, } 11 : \frac{248 \times 5\frac{1}{2}}{24} :: 9 : \frac{248 \times 5\frac{1}{2} \times 11}{24 \times 9}$$

both inverse.

hard. days. hard. days.

$$\text{Thirdly, } 7 : \frac{248 \times 5\frac{1}{2} \times 11}{24 \times 9} :: 5 : \frac{248 \times 5\frac{1}{2} \times 11 \times 5}{7 \times 24 \times 9}. \text{ Direct.}$$

The solidity of the trenches are $232\frac{1}{2} \times 3\frac{2}{3} \times 2\frac{1}{3}$ and $337\frac{1}{2} \times 5\frac{3}{5} \times 3\frac{1}{2}$ respectively.

work. days.

$$\text{Then it will be as } 232\tfrac{1}{2} \times 3\tfrac{2}{3} \times 2\tfrac{1}{3} : \frac{248 \times 5\frac{1}{2} \times 11 \times 5}{77 \times 24 \times 9}$$

days.

$$:: 337\tfrac{1}{2} \times 5\tfrac{3}{5} \times 3\tfrac{1}{2} : \frac{248 \times 5\frac{1}{2} \times 11 \times 5 \times 337\frac{1}{2} \times 5\frac{3}{5} \times 3\frac{1}{2}}{232\frac{1}{2} \times 3\frac{2}{3} \times 2\frac{1}{3} \times 7 \times 24 \times 9}$$

equal to 132 the answer required.

Those who want a further explanation of Mr. Emerson's method, may find it more fully treated upon in arbitration of exchange, which I had written some time before I had the perusal of Mr. Emerson's book.

* *

SECT. IV.

COMPOUND PROPORTION;

OR, THE

RULE OF THREE REPEATED.

ALL questions in the foregoing rule of five (as hath been before observed) may be resolved by two or more operations in the rule of three repeated; a few examples whereof we shall give: also several questions that cannot be

solved by the rule of five, may be answered by two or more repetitions of the rule of three: variety whereof followeth.

1. If 2 men can do $12\frac{1}{2}$ rods of ditching in $6\frac{1}{2}$ days; how many rods may be done by 18 men in 14 days?

men. rods. men.
As 2 : $12\frac{1}{2}$: : 18
$12\frac{1}{2}$

2) 225 ($112\frac{1}{2}$ rods.

Also $6\frac{1}{2}$ days $= \frac{13}{2} : 112\frac{1}{2}$ rods $= \frac{225}{2} : : \frac{14}{1}$ days.

$\frac{225}{2} \times \frac{14}{1} = \frac{1575}{1}$. And $\frac{13}{2}\Big)\frac{1575}{1}\Big(\frac{3150}{13} = 242\frac{4}{13}$ rods, the answer.

2. If a regiment of soldiers, consisting of 939, can eat up 351 quarters of wheat in 7 months; how many soldiers will eat up 1464 quarters in 5 months, at that rate?

qrs. wh. sold. qrs. wh.
Directly, 351 : 939 : : 1464 : $\frac{152744}{39}$.

Reciprocally, $\frac{7}{1} : \frac{152744}{39} : : \frac{5}{1} : 5483\frac{21}{195}$ soldiers, the answer as before in the rule of five.

3. If 9 men in 21 days mow 108 acres of ground; in how many days will 5 men mow 72 acres, at the same rate of working?

acres. days. acres. days.
Directly, 108 : 21 : : 72 : 14
men. days. men.
Reciprocally, 9 : 14 : : 5 : $25\frac{1}{5}$ days, the answer.

N. B. The first question is what is generally called by authors the double rule of three direct; and the second and third, the double of three inverse.

4. By selling 240 oranges at five for 2d. half of which cost two a penny, and the other half three a penny, I evidently lost a groat; pray how comes that about?

ora. d. ora. d. s. ora. d. ora. d. s. d.
As 2 : 1 : : 120 : 60 = 5. Again, 3 : 1 : : 120 : 40 = 3 4.
Then 5s. + 3s. 4d. = 8s. 4d. the cost.
ora. d. ora. d. s.
And 5 : 2 : : 240 : 96 = 8, consequently lost 4d.

5. If

5. If 12 apples be worth 21 pears, and 3 pears coſt a halfpenny; what will be the price of fourſcore and four apples?

pears. d. pears. d.

3 : .5 : : 21 : 3.5, price of 12 apples.

apples. d. apples d. s. d.

12 : 3.5 : : 84 : 24.5 = 2 -½, the anſwer.

6. A gay young fellow had 18200 l. left him by an old uncle, to whoſe memory he expended 3 per cent. of his whole fortune in a ſumptuous funeral and monument; 9 per cent. of the remainder he made a preſent of to his couſins, forgotten for his ſake by the old man; with $\frac{2}{7}$ of what was left he bought a fine ſeat; with $\frac{1}{8}$ of the reſidue a ſtud of horſes; he ſquandered away 550 l. upon one miſtreſs; and after he had lived after the rate of 2000 l. a year for 19 months together, he had both ruined his health, and impaired his fortune; pray at his death what was there left for his ſiſter, who was his heir at law?

100 : 3 : : 18200 : 546, funeral and monument.

18200 — 546 = 17654.

100 : 9 : : 17654 : 1588.86, couſins.

17654 — 1588.86 = 16065.14.

16065.14 × $\frac{2}{7}$ = 4590.04, ſeat.

16065.14 — 4590.04 = 11475.1

8) 11475.1 (1434.3875, horſes.

11475.1 — 1434.3875 = 10040.7125.

12 : 2000 : : 19 : 3166.$\dot{6}$, riotous living.

+ 550, miſtreſs.

10040.7125 — 3716.6 = 6324.0458$\dot{3}$.

6324.0458$\dot{3}$ l. = 6324 l. -s. 11 d. the anſwer.

7. If a ſack of coals be the allowance of 7 poor people for a week; how many poor belonged to that pariſh, which, when coals were 1 l. 16 s. per chaldron, had 41 l. to pay in 6 weeks on that account?

l. s. l. chal. l. chal.

$$1\ 16 = \frac{9}{5} : \frac{1}{1} : : \frac{41}{1} : \frac{205}{9} = 22\tfrac{7}{9} \text{ chaldron.}$$

Here 12 ſacks of 3 buſhel each are accounted 1 chaldron. Then $\frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$ chaldron burnt by 7 in 6 weeks.

$$\text{As } \frac{1}{1} : \frac{7}{1} : : \frac{205}{9} : \frac{2870}{9} = 318\tfrac{8}{9} \text{ poor, the anſwer.}$$

8. It

8. It is a rule in some parishes to assess the inhabitants in proportion to $\frac{8}{10}$ of their rent; what is the yearly rent pray of that house which pays 8 l. 10 s. to the king under this limitation, at 5 s. in the pound?

tax. rent. tax rent.
.25 : 1 :: 8.5 : 34.
.8 : 1 :: 34 : 42.5 = 42 l. 10 s. the answer.

9. A and B are on opposite sides of a wood 134 toises about, they begin to go round it both the same way at the same instant of time; A goes 11 toises in 2 minutes, and B 17 in 3: the question is, how many times will they surround the wood before the nimbler overtakes the slower?

min. toises. m.
2 : 11 :: 3 : $16\frac{1}{2}$ toises.
Then $17 - 16\frac{1}{2} = \frac{1}{2}$ toise B gains of A in going 17.
toise. toises. round.
$\therefore \frac{1}{2} : 17 :: \frac{1}{2} : 17$ rounds gone by A, $16\frac{1}{2}$ B.

10. A cistern holds 103 gallons, and being brimful, has 2 cocks to run off the water; by the first of which a pail of 3 gallons will be filled in 1 minute, by the other in 1 minute and 15 seconds; in what time will this cistern be emptied through both these apertures together, supposing the efflux of water all along the same?

First cock runs off 3 gallons $= \frac{3}{103} = \frac{15}{515}$ of the cistern in 1 minute.

As 1 min. 15 sec. $= \frac{5}{4} : \frac{3}{103} :: 1 : \frac{12}{515}$ run off by the second cock in 1 minute.

And $\frac{15}{515} + \frac{12}{515} = \frac{27}{515}$, run off by both in 1 minute.

$\therefore \frac{27}{515} : 1 :: 1 : \frac{515}{27} =$ 19 min. $4\frac{4}{9}$ seconds, the answer.

11. If, when Port wine is 17 guineas the hogshead, a company of 45 people will spend 20 l. therein, in a certain time; what is wine a pipe when 13 persons more will spend 63 l. in twice the time, drinking with equal moderation?

45 men

45 men : 20 l. : : 58 men : $25.\dot{7}$.
And $25.\dot{7} \times 2 = 51.\dot{5}$ l.'s. worth, at 17 guineas per hogſhead.
$51.\dot{5}$: 17.85 : : 63 : 21.8125 l. per hogſhead.
∴ 21.8125 l. × 2 = 43.625 l. = 43 l. 12 s. 6d. the anſwer.

12. In diſtreſs at ſea they threw out 17 hogſheads of ſugar, worth 34 l. per hogſhead, the worth of which came up to but $\frac{4}{7}$ of the indico they caſt overboard; beſides which they threw out 13 iron guns, worth 18 l. 10 s. a piece; the value of all theſe amounted to $\frac{3}{7}$ of $\frac{9}{13}$ of the ſhip and lading; pray what of this value came into port?

	l.	s.	d.
17 hogſheads of ſugar, at 34 l. per hogſhead,	578	–	–
As $\frac{4}{7} : \frac{578}{1} : : \frac{7}{7} : \frac{2023}{2}$ indico, value - -	1011	10	–
13 iron guns, at 18 l. 10 s. each, is - - - -	240	10	–
Value of the whole caſt overboard - - - - -	1830	–	–

Then 1830 l. = $\frac{3}{7}$ of $\frac{9}{13}$ of the ſhip and lading, or $\frac{27}{91} + \frac{64}{91} = \frac{91}{91}$.
∴ $\frac{27}{91} : \frac{1820}{1} : : \frac{64}{91} : \frac{39040}{9}$, arrived at port 4337 l. 15 s. $6\frac{2}{3}$d.

13. A, B and C will trench a field in 12 days; B, C and D in 14; C, D and A will do it in 15; and D, A and B in 18; in what time will it be done by all of them; and each of them ſingly?

$$\left.\begin{array}{r}\text{Firſt A, B, C}\\ \text{B, C, D}\\ \text{A, C, D}\\ \text{A, B, D}\end{array}\right\}\text{can do}\left\{\begin{array}{l}\frac{1}{12} = .0833333\\ \frac{1}{14} = .0714285\\ \frac{1}{15} = .0666666\\ \frac{1}{18} = .0555555\end{array}\right\}\begin{array}{l}\text{part of the}\\ \text{whole work.}\end{array}$$

All working three days will do .276984 part of the work.
Then, .276984 : 3 days : : 1 work : 10.8309505 days, all working.

B, C, D

B, C, D 14
— 10.83095

days.
3.16905 : 10.83095 : : 14 : 47.848 by A

A, C, D 15
— 10.83095

4.16905 : 10.83095 : : 15 : 38.969 by B

A, B, D 18
— 10.83095

7.16905 : 10.83095 : : 18 : 27.194 by C

A, B, C 12
— 10.83095

1.16905 : 10.83095 : : 12 : 111.176 by D

$\Big\}$ alone.

14. If during the tide of ebb a wherry ſets out from London, weſtward, and at the ſame inſtant another ſhould put off at Chertſey for London, taking the diſtance by water 34 miles; the ſtream forwards this, and retards the other, $2\frac{1}{2}$ miles in an hour; the boats are equally laden, the rowers equally good, and the ordinary way of working in ſtill water, would proceed at the rate of five miles an hour: the queſtion is, where in the river the two boats would meet?

It is plain from the queſtion, that he that rows

towards $\Big\}$ London goes $\Big\{$ $7\frac{1}{2}$ $\Big\}$ miles in an hour
from $\Big\}$ London goes $\Big\{$ $2\frac{1}{2}$ $\Big\}$

h m. h.
Sum 10 : 1 : : 34 : 3.4

h. miles.
∴ 1 : 2.5 : : 3.4 : $8\frac{1}{2}$ $\Big\}$ miles from $\Big\{$ London.
And 1 : 7.5 : : 3.4 : $25\frac{1}{2}$ $\Big\}$ miles from $\Big\{$ Chertſey.

15. A young hare ſtarts 5 rods before a greyhound, and is not perceived by him till ſhe has been up 34 ſeconds, ſhe ſcuds away at the rate of 12 miles an hour, and the dog in view makes after her at the rate of 20; how long will the courſe hold, and what ground will be run, beginning with the out-ſetting of the dog?

5 rods or poles = 82.5 feet - - 1 hour = 3600 ſeconds.
12 miles = 63360 feet; and 20 miles = 105600 feet.

Then

Then 3600″ : 63360 feet : 34″ : 598.4 feet.
82.5 + 598.4 = 680.9 feet, the hare had ſtart.
8 : 20 : : 680.9 : 1702¼, run by the greyhound.
105600 — 63360 = 42240.
42240 : 3600 : : 680.9 : $58\frac{1}{32}$″, run by the greyhound.

16. A lent his friend B fourſcore and eleven guineas from the 11th of December to the 10th of May following; B, on another occaſion, let A have 100 marks from September the 3d to Chriſtmas following; quere, how long ought the perſon obliged to let his friend uſe 40 l. fully, to retaliate the favour?

Firſt, 91 guineas = 95 l. 11 s. for 150 days.
And 100 marks = 66 l. 13 s. 4 d 113 days.
Reciprocally, 95.55 : 150 days : : 66.$\dot{6}$: 215 days.
215 l. — 113 = 102.
Recip. 66.$\dot{6}$ l. : 102 : : 40 : 170 days nearly. Q. E. F.

17. There are two pieces of clock-work, moving with a fly, will each of them lower a weight uniformly to the depth of 35 feet; the firſt weight, or A, deſcends $\frac{13}{10}$ inches in an hour; and when it is let down 12 feet, the ſecond, or B, is put off; and the train of wheels belonging to this machine is ſo ordered, that the weights will be, in the ſame level, 100 inches before they come to the bottom: the velocity of B's deſcent is required?

As $\frac{13}{10}$ inches : 1 hour : : 12 feet = 144 inches : $\frac{1440}{13}$.

35 feet = 420 inches — 100 inches = 320 inches, where the weights will be level.

As $\frac{13}{10}$ inches : 1 hour : : 320 inches : $\frac{3200}{14}$, time A deſcends 320 inches.

Then $\frac{3200}{13} - \frac{1440}{13} = \frac{1760}{13}$ hours, time B deſcends to a level with A.

∴ $\frac{1760}{13}$ hours : 320 inches : : 1 hour : $\frac{416}{176} = 2\frac{4}{11}$ inches, the anſwer.

18. My water tub holds 147 gallons, the pipe uſually brings in 14 gallons in 9 minutes, the tap diſcharges at a medium, 40 gallons in 31 minutes; ſuppoſing theſe both careleſsly to be left open, and the water to be turned on at two o'clock in the

the morning; the servant at five, finding the water running, shuts the tap, and is solicitous in what time the tub will be filled after this accident, in case the water continues flowing from the main?

First, 9 minutes : 14 gallons : : 31 minutes : $48\frac{2}{9}$ gallons, fills in 31 minutes.

Then $48\frac{2}{9} - 40 = 8\frac{2}{9}$ gal. in the tub at the end of 31 min.

Also 31 min. : $8\frac{2}{9}$ gal. : : $3 \times 60 = 180$: $47\frac{23}{31}$ gal. in three hours.

Further, $147 - 47\frac{23}{31} = 99\frac{8}{31}$ gal. the tub wants of being full.

And 14 gal. : 9 min. : : $99\frac{8}{31}$: 63 min. $48\frac{21}{31}$ seconds, the tub will be full.

∴ The tub will be full at 3 minutes $48\frac{21}{31}$ seconds after 6.

19. One being asked what hour of the day it was, answered, the day at this time is 16 hours long; if now $\frac{1}{2}$ of the hours past be added to $\frac{2}{3}$ of the remainder, you will have the hour desired, reckoning from sun-rising.

First, $\frac{1}{2}$ of the hours past $+ \frac{2}{3}$ of those to come $=$ hours past.

∴ $\frac{1}{2}$ hours past $= \frac{2}{3}$ of those to come.

And $\frac{2}{3} + 2 = \frac{4}{3}$ of the hours to come $=$ hours past, or time of the day; consequently, the ratio of the hours past are to those to come,

As $\frac{4}{3}$ to $\frac{3}{3}$, or as $1\frac{1}{3}$ to 1.

Then $\frac{4}{3} + \frac{3}{3} = \frac{7}{3}$, the sum of those ratios.

∴ $\frac{7}{3} : \frac{4}{3} : : 16 : 9\frac{1}{7}$, hours from sun-rising. } Q. E. F.
And $\frac{7}{3} : \frac{4}{3} : : 16 : 6\frac{6}{7}$, hours to sun-setting. }

20. A triangular bath 6 feet deep, is exactly inclosed by 3 square pavilions, and rectangular; the sum of whose plans together make just 50 poles; the area of A, the less, is to that of B, the middle one, as $4\frac{1}{2}$ to 8; and the sum of the areas of A and C, the biggest, is to that of B, as $8\frac{1}{2}$ to 4; how many wine hogsheads of water will this bath receive?

As 8B : $4\frac{1}{2}$A : : 4B : $2\frac{1}{4}$A.

$8\frac{1}{2} - 2\frac{1}{4} = 6\frac{1}{4} =$ C's proportional part.

Also A $2\frac{1}{4}$ + B 4 + C $6\frac{1}{4} = 12\frac{1}{2}$.

$$\text{As } 12.5 : 50 : : \left\{\begin{array}{l} 2.25 : 9 \\ 4 \quad : 16 \\ 6.25 : 25 \end{array}\right\} \text{poles, area of the pavilion.} \left\{\begin{array}{l} A. \\ B. \\ C. \end{array}\right.$$

And

And by inſpection the ſides of the triangle $A = 3$, $B = 4$, $C = 5$.

And as $16\frac{1}{2}$ feet make a pole, $16.5 \times 3 = 49.5$ feet, the perpendicular.

Alſo $16.5 \times 4 = 66$ feet - - - $\frac{66}{2} = 33$, half the baſe.

$49.5 \times 33 = 1633.5$ feet = area of the triangle.

And $1633.5 \times 6 = 9801$ cubic feet, ſolidity of the bath.

Alſo $9801 \times 1728 = 16936128$ cubic inches.

$\because$ 231) 16936128 ($73316\frac{1}{2} \div 63 = 1163$ hogſheads, $47\frac{1}{2}$ gallons. Q. E. F.

21. A certain man hires a labourer on this condition, that for every day he worked he ſhould receive 12 d. but for every day he was idle he ſhould be mulcted 8 d. when 390 days were paſt, neither of them were indebted to one another; how many days did he work, and how many days was he idle?

As for every day he worked he received - - 12 d.
So for every day he played he paid - - - - - 8

their ſum 20 d.

And as his idle days came to the ſame money as thoſe he worked, therefore the proportion will be contra;

$$\left.\begin{array}{l}\text{viz. } 20 : 8 :: 390 : 156 \\ \text{and } 20 : 12 :: 390 : 234\end{array}\right\} \text{days he} \left\{\begin{array}{l}\text{worked,} \\ \text{played,}\end{array}\right\} \text{Q. E. F.}$$

22. A man hired a labourer for 40 days, on condition that he ſhould have 20d. for every day he wrought, and forfeit 10d. for every day he idled, at laſt he received 41s 8d. for his labour; how many days did he work, and how many was he idle.

41 s. 8 d. = 500 pence.
20) 500 (25 days wages.

Then $40 - 25 = 15$ days more.
For every of which days he worked he had 20 d.
And for every day he played he paid - - 10

30

By contra proportion,

As $30 : 10 :: 15 : 5$
And $30 : 20 :: 15 : 10$
$\because 10 =$ days he was idle.
And $25 + 5 = 30$ days he worked.

22. There

23. There is an iſland 73 miles round, and three footmen all ſtart together, to travel the ſame way about it; A travels 5 miles a day, B 8, and C 10: when will they all come together again for the firſt time? *Emerſon's Arithmetic.*

Firſt $8 - 5 = 3$ miles, B } gains of A in one day.
Alſo $10 - 5 = 5$ miles, C }

m. d. m.
Then 3 : 1 : : 73 : $24\frac{1}{3}$ days, when A and B meet.
And 5 : 1 : : 73 : $14\frac{3}{5}$ days, when A and C meet.

But as in any multiple of theſe days they will meet again, it is evident that A and B may meet at the end of 3 times $44\frac{1}{3}$, viz. 73 days, and likewiſe A and C may meet at the end of 5 times $14\frac{3}{5}$, or 73 days hence, A. B. and C will be together for the firſt time at the end of 73 days, and again at the end of 146 days, again at the end of 219, 292, &c. &c. Q. E. F.

The following machine being accounted a leaver of the ſecond order, whoſe force is directly, and its preſſure in a contra-proportion to the length.

24 In giving directions for making an Italian chair, the ſhafts whereof were ſettled at 11 feet between the axletree, whereon the principle bearing is, and the backband, by means of which the weight is partly thrown upon the horſe. A diſpute aroſe whereabout on the ſhafts the center of the body of this machine ſhould be fixed; the coachmaker adviſed this to be done at 30 inches from the axletree; others were of opinion, that at 24 it would be a ſufficient incumbrance to the horſe. Now admitting the two paſſengers, with their baggage, ordinarily to weigh 2 cwt. a piece, and the body of the vehicle to be about 70 lb. more, pray what will the beaſt in both theſe caſes be made to bear more than his harneſs?

Firſt, $11 - 2.5 = 8.5$; alſo $11 - 2 = 9$, and 4 cwt. 70 lb. = 518 lb.
Directly, 11 : 518 : : 8.5 : $400\frac{3}{11}$, force.
Contra, 8.5 : $400\frac{3}{11}$: : 2.5 : $117\frac{8}{11}$, preſſure in the former.
Alſo 11 : 518 : : 9 : $423\frac{9}{11}$, force } in the 2d caſe.
∴ 9 : 423 : : 2 : $94\frac{2}{11}$, preſſure }
Anſwer, the beaſt bears $117\frac{8}{11}$ lb. in the former, and $94\frac{2}{11}$ lb. in the ſecond caſe.

25. If

25. If a lever 40 effective inches long will, by a certain power thrown successively thereon, in 13 hours, raise a weight 104 feet; in what time will two other levers, each 18 effective inches long, raise an equal weight 73 feet?

As 40 inches : 104 feet :: $\overline{18 \times 2}$: 93.6.
Then 93.6 feet : 13 hours :: 73 feet : 10 hours $8\frac{1}{3}$ minutes, the answer.

26. A weight of $1\frac{1}{2}$ lb. laid on the shoulders of a man, is no greater a burden to him than its absolute weight, or 24 ounces; what difference will he feel between the said weight applied near his elbow, at 12 inches from the shoulder, and in the palm of his hand, 28 inches therefrom; and how much more must his muscles then draw, to support it at right angles, that is, having his arm extended right out?

As 1 : 1.5 lb. :: 12 in. : 18 lb. weight 12 in. } from the
And 1 : 1.5 lb :: 28 in. : 42 lb. weight 28 in. } shoulder.
$\because$ 42 — 18 = 24 lb. the answer required.

27. A ball weighing four pounds upon the surface of the earth, to what height in the air must it be carried to weigh but three pounds, and how long would it be falling to the ground? *Ladies Diary.*

Taking the earth's semi-diameter at 4000 miles.
Then 4000 × 4000 = 16000000.

As the weights of bodies decrease as the square of their distance from the earth's center increases, we have,

$\because$ Recip. 4 : 16000000 :: 3 : 21333333.$\dot{3}$.
Then $\sqrt{21333333.\dot{3}}$ = 4618.8021.
Then 4618.8021 — 4000 = 618.8021 miles = 3267275 feet, or 618 miles, 6 furlongs, 16 poles, 3 yards, 1 foot, the height the ball must be carried.
Again, 16.08$\dot{3}$: 1 square second :: 3267275 : 203837.48.
$\sqrt{203837.48}$ = 451.48″ = 7′ 31″ 29‴, the time of falling.

But at that great distance from the earth, when the ball will have lost $\frac{1}{4}$ of its weight, its velocity will also be diminished.

Viz. 16.08$\dot{3}$ × .75 = 12.0625.
Then as 12.0625 : 1 :: 3267275 : 270862.
$\sqrt{270862}$ = 520″ = 8′ 40″, the time of falling.

U 28. A

28. A ball defcending by the force of gravity from the top of a tower, was obferved to fall half the way in the laft fecond of time; required the tower's height, and the whole time of defcent? *Ladies Diary*, 1755.

The fquare-roots of the diftances being as the times, viz. as the $\sqrt{1} : \sqrt{2} ::$ fo is the time of falling through the firft half, to the time of falling through the whole required height.

$\because$ As $\sqrt{2} - 1 = .4142 : \sqrt{2} = 1.4142 :: 1 : 3.414$ feconds, the time of defcent.

And $1^2 : 16.08\frac{1}{3} :: \square\ 3.4142 = 11.6574 : 187.48$ feet, the tower's height.

29. Suppofe that in every fingle revolution of the upper ftone of a water-mill, it evacuates or grinds one-eighth of a pint of meal; and fuppofing there be eight ftandards or pinions in the rounds that turn it once, and that thefe rounds are driven by a wheel of 45 teeth; alfo the mean circumference of the water-wheel on the fame axis be 40 feet, which requires one hundred and a half to move it, or put it into motion: now if a floodgate, whofe breadth is a foot, and height half a foot, and the height of the water be three feet above the furface of the hole, be let run directly againft the upper furface of the wheel, it is required to find the quantity of meal ground in an hour by the faid mill? *Gentlemen's Diary*, 1751.

Firft, $1 \times .5 \times 3 = 1.5$; alfo 62.5 lb. weight of a cubic foot of water. Then $62.5 \times 1.5 = 93.75$ lb. the inftantaneous preffure of the water.

Alfo, $16\frac{1}{12} = \frac{193}{12} : 1^2 :: 3 \text{ feet} : \frac{36}{193} =$ fquare of the time.

$$\text{And } \frac{6}{\sqrt{193}} = \text{the time.}$$

$\because \frac{6}{\sqrt{193}} : 6 :: 1 : \sqrt{193} = 13.89244$ feet, velocity per fecond.

Alfo $1\frac{1}{2}$ cwt. $= 168$ lb.

$93.75 \times 13.8924 \div 168 = 7.75248$ feet, any point of the wheel moves in one fecond.

$7.75248 : 1 :: 40 : 5.1595$ feconds, moving round.

$5.1595 : 1 :: 3600 : 697.742$ rounds, the wheel moves in an hour.

$\frac{45}{8}$

$\frac{45}{8} = 5.625$. As 1 : 5.625 :: 697.742 : 3924.799 rounds, the ſtone moves in an hour.

∵ 8)3924.799(490.6 pints = 7 buſhels, 2 pecks, 10$\frac{3}{5}$ pints, the mill grinds in an hour. Q. E. F.

30. Obſerved, that while a ſtone was deſcending to meaſure the depth of a well, a ſtring and plummet (that from the point of ſuſpenſion, or the place where it was held, to the center of oſcillation) or that part of the bob, which being divided by a circular line, ſtruck from the center aboveſaid, would divide it into two parts of equal weight meaſured juſt 18 inches, had made eight vibrations; pray what was the depth, allowing 1150 feet per ſecond, for the return of ſound to the ear?

39.2 : 3600 :: 18 : 7840.

$\sqrt{7840} = 88.54378$ vibrations in one minute.

60)88.54378(1.475729 vibrations in a ſecond.

1.475729)8.000000(5.421 ſeconds in eight vibrations.

$1^2 : 16.08\frac{1}{3} :: \square\ 5.421 = 29.387 : 472.640916$ feet.

As 1150 feet : 1 :: 472.640916 : .41099″, time the ſound was returning.

5.421 − .41099″ = 5.01, true time of the body's deſcent.

∵ 5.01 × 5.01 = 25.1001 × $16.08\frac{1}{3}$ = 403.6932 feet, the depth of the well. Q. E. F.

31. Aſpire my genius! help my rhiming muſe,
In themes I in my native country chuſe:
Whilſt others plow the waves, and tread the ſtrands
Of diſtant oceans, and of foreign lands;
To fill the mouth of fame with ſomething new
(No matter 'tis how much of it is true)
From Alps or mountains ſtories ſtrange they bring,
Of deſarts, caves, or horrid monſters ſing.
Tell how Veſuvins' ſulph'rous darts do fly,
Or Ætna's ſmoak obſcures the azure ſky;
Or magnify the hazards they have run,
Scylla's and Charybdis' pointed rocks to ſhun.
Such tales we take on truſt, from thoſe who rove,
Tho' none give rules by which the truth to prove.
But this by numbers may explained be,
By thoſe who never did the cavern ſee:
In Derbyſhire, a wonder of the Peak,
Is Eldon-hole, as poets often ſpeak;

Whose depth exactly none could e'er descry,
Tho' atheist Hobbes his utmost skill did try,
And wrote *De Mirabilibus Pecci.*
And burlesque Cotton, does strange tales rehearse,
In rustic words, and Hudibrastic verse,
How he this mighty orifice did plumb,
But could not at the bottom of it come,
With sixteen hundred yards of rope let loose;
And tells a story of a woman's goose:
Fab'lous the one, so must the other be
Erroneous too, without philosophy;
Extension of the rope might him deceive,
Or small proportion which the plumb would have
To such a length; and part in water drown'd,
When in this vast abyss, within the ground.
But I the depth have found exactly true,
By gravity, a method something new.
As heavy bodies do accelerate,
In spaces known first to our Newton great,
Four ponderous stones into the well let fall,
In measur'd time, agreed in numbers all.
A pendulum, sixty-one inches long,
By which the time I measur'd (was not wrong)
Vibrated freely (whilst that each stone fell)
Eight times; by which the depth I'd have you tell,
Allowing rightly for the approach of sound,
That your own works may not themselves confound,

Ladies Diary, 1722.

First, 39.2 : 3600 :: 61 : 2313.4426.

$\sqrt{2313.4426} = 48.09826$ vibrations in one minute.

60) 48.09826 (.8016377 vibrations in one second.

.8016377) 8.0000000 (9.097957 seconds at eight vibrations.

Also 1^2 : $16.08\frac{1}{3}$ :: $\square$ 9.97957 = 99.59182 : 1601.7584.

1150 feet : 1″ :: 1601.7584 feet : 1.39283, time of the sound's ascent.

Then 9.97957 − 1.39283 = 8.58674, true time of the stone's descent.

∴ 1^2 : $16.08\frac{1}{3}$ :: $\square$ 8.58674″ = 73.7321 : 1185.88 feet, the depth of Eldon-hole.

CHAP.

CHAPTER II.

SECT. I.

SIMPLE INTEREST.

INTEREST is a ſmall ſum of money paid for the uſe of a larger ſum, at any rate agreed upon; which according to law muſt not exceed 5 l. for the intereſt of 100 l. principal for one year.

CASE I.

The principal, rate of intereſt, and time, being given; to find the intereſt.

Firſt, When the yearly intereſt of any ſum is required:

RULE.

Multiply the principal by the rate of intereſt per cent. per annum, dividing the product by 100; which is done by cutting off the two right-hand figures, thoſe to the left being the intereſt in pounds. Then multiply the remainder (if any) by 20, cutting off, as before for ſhillings; and that remainder by 12, cutting off as before directed for pence; and find the farthings (if any) after the ſame manner; the figures to the left of thoſe cut off being the intereſt.

1. What is the intereſt of 873 l. 16 s. 8 d. for a year, at 5 per cent?

l.	s.	d.
873	16	8
		5
£ 43.69	3	4
20		
13.83 s.		
12		
10.00 d.		

By Practice.

	l.	s.	d.
5 per cent. = $\frac{1}{20}$ (	873	16	8
	£ 43	13	10

Anſwer, 43 l. 13 s. 10 d.

2. What

2. What is the intereſt of 1437 l. 17 s. for a year, at 4½ per cent.

l.	s.	d.
1427	17	–
		4½
5751	8	–
718	18	6
£ 64.70	6	6
20		
14.06 s.		
12		
.78		
4		
3.12 qrs.		

By Practice.

	l.	s.	d.
20 per cent. = $\frac{1}{5}$	1437	17	–
4 s. = $\frac{1}{5}$	287	11	4¾
$\frac{1}{8}$	57	10	3¼
	7	3	9¼
	£ 64	14	–½

Anſwer, 64 l. 14 s. – ¾ d.

3. What is the intereſt of 178 l. 16 s. for a year, at 3¾ per cent.?

l.	s.	
178	16	
	3¾	
538	8	
133	14	6 d.
£ 6.68	12	6
20		
13.72 s.		
12		
8.70 d.		
4		
2.80 qrs.		

By Practice.

	l.	s.	d.
20 per cent. = $\frac{1}{5}$	178	6	–
$\frac{1}{10}$	35	13	2¼
$\frac{1}{2}$	3	11	3¾
$\frac{1}{2}$	1	15	7¾
$\frac{1}{2}$	–	17	9¾
	–	8	10¾
	£ 6	13	8

Secondly, When the intereſt of any ſum is required for ſeveral years,

RULE

RULE.

After the yearly intereſt is found, multiply that by the number of years, the product will be the anſwer.

4. What is the intereſt of 74 l. 15 s. for five years, at $4\frac{3}{8}$ l. per cent. per annum?

By the Rule of Five.

100	. 1 .	4·375
74·75	. 5 .	

$4{\cdot}375 \times 74{\cdot}75 \times 5 = 1635{\cdot}15625$ dividend, and $100 \times 1 = 100$ diviſor.

∴ 100) 1635·15625 (16.3515625 = 16 l. 7 s. – $\frac{1}{4}$ d. the anſwer.

By the Rule.

	l.	s.	d.
$\frac{1}{4}$	74	15	
		$4\frac{3}{8}$	
	299	–	–
	18	13	9
	9	6	$10\frac{1}{2}$
	£3.27	–	$4\frac{1}{2}$
	× 20		
	5·40 s.		
	× 12		
	4.84 d.		
	4		
	3·38 qrs.		

By Practice.

	l.	s.	d.
20 per cent. = $\frac{1}{5}$	74	15	–
$\frac{1}{10}$	14	19	–
	1	9	$10\frac{3}{4}$
$\frac{1}{8}$	1	9	$10\frac{3}{4}$
$\frac{1}{2}$		3	$8\frac{3}{4}$
		1	$10\frac{1}{2}$
Intereſt for 1 year	3	5	$4\frac{3}{4}$
			5
Anſwer, £	16	6	$11\frac{3}{4}$

The ſmall defect in the practical methods, is owing to the parts of a farthing omitted.

5. What is the intereſt of 963 l. 7 s. 6 d. at $3\frac{5}{8}$ per cent. for 13 years?

	l.	s.	d.
½	963	7	6
			3⅝
	2890	2	6
	481	13	9
	120	8	5¼
	£ 34.92	4	8¼
	20		
	£ 18.44		
	12		
	5.36 s.		
	4		
	1.44 d.		

By Practice.

		l.	s.	d.
20 per cent. =	⅕	963	7	6
	$\frac{1}{10}$	192	13	6
	½	19	5	4
	½	9	12	8
	¼	4	16	4
		1	4	1
Year's intereſt	£	34	18	5
Year's int. by the firſt method	£	34	18	5¼
				× 13
Anſwer,	£	435	19	8¼

By Five Numbers.

100 . 1 . 3.625

963.375 . 13 .

963.375 × 3.625 × 13 = 45399.046875

100) 45399.046875 (453.99046875 = 453 l. 19 s. 9½ d. anſwer.

Thirdly, When the intereſt of any ſum is required for years and months.

RULE.

Firſt find the yearly intereſt, which multiply by the number of years, as before; then for the months divide the yearly intereſt by the part or parts the given months are of a year, which add together with the reſt, and their ſum will be the anſwer.

6. What is the intereſt of 56 l. 10 s. for 7 months, at 4¼ per cent. per annum?

56 l.

l.	s.	
56	10	
	$4\frac{1}{4}$	
226	–	–
14	2	6
2.40	2	6
20		
£8 02		
12		
.30		
4		
1.20 qrs.		

	l.	s.	d.	
$\frac{1}{2}$	2	8	–$\frac{1}{4}$	per annum.
$\frac{1}{6}$	1	4	–	
	–	4	–	
	£1	8	–,	the anſwer.

By Five Numbers.

P.	T.	G.
100	. 12 .	4.25
56.5	. 7 .	

$56.5 \times 4.25 \times 7 = 1680.875$, dividend.

$100 \times 12 = 1200$) 1680.875 (1.400729 = 1 l. 8 s. the anſwer.

7. What is the intereſt of 759 l. 16 s. 7 d. for 12 years, 4 months, at $5\frac{1}{8}$ l. per cent. per annum?

	l.	s.	d.
$\frac{1}{8}$	759	16	7
			$5\frac{1}{8}$
	3799	2	11
	94	19	$6\frac{3}{4}$
	£38.94	2	$5\frac{3}{4}$
	20		
	18.82 s.		
	12		
	9.89 d.		
	4		
	3.59 qrs.		

	l.	s.	d.
Year's intereſt	38	18	$9\frac{1}{4}$
			$12\frac{1}{3}$
	467	5	9
	12	19	$7\frac{1}{4}$
Anſwer,	£480	5	$4\frac{1}{4}$

By Five Numbers.

P.	T.	G.
100 .	. 1 .	. 5.125
759.8291$\dot{6}$.	12.$\dot{3}$.	

$5.125 \times 759.8291\dot{6} \times 12.\dot{3} = 48027.534232\dot{8}$

100) 48027.534 (480.27534 = 480 l. 5 s. $5\frac{1}{2}$ d. the anſwer.

8. Lent,

8. Lent, at Christmas 1760, the sum of 5000 l. at $4\frac{1}{2}$ per cent.; after which time I lent several sums at the same rate, and drew upon the borrower, as business required, viz. on Lady-day 1761, I drew for 185 guineas; on Midsummer-day, 1761, I lent 500 moidores, and drew for 700 l.; on Michaelmas-day, 1761, I lent 569 l. 17 s. I demand what cash the borrower owed me at that time?

	l.	s.	d.	
5 l. = $\frac{1}{20}$	5000 l.			principal, at $4\frac{1}{2}$ per cent.
	250			
	—25			
	225,			interest for 1 year at $4\frac{1}{2}$ per cent.
4)	225	s.		
	56	5,		interest due at Lady-day 1761.
	5000	–,		principal.
	5056	5,		amount at Lady-day,
	194	5		drawn = 185 guineas.
l. 5 = $\frac{1}{20}$	4862	–,		new principal.
$\frac{1}{2}$ = $\frac{1}{10}$	243	2		
	−24	6	$2\frac{1}{4}$	
4)	218	15	$9\frac{3}{4}$,	interest for 1 year at $4\frac{1}{2}$.
	54	13	$11\frac{1}{4}$,	interest due at Midsummer.
	4862	–	–,	last principal.
	4916	13	$11\frac{1}{4}$	
	—25	–	–	= 700 l. — 500 moidores, or 675 l.
l. 5 = $\frac{1}{20}$	4891	13	$11\frac{1}{4}$,	new principal at $4\frac{1}{2}$ per cent.
$\frac{1}{2}$ = $\frac{1}{10}$	244	11	$8\frac{1}{4}$	
	—24	9	2	
4)	220	2	$6\frac{1}{4}$,	interest for 1 year.
	55	–	$7\frac{1}{2}$,	interest due at Michaelmas.
	4891	13	$11\frac{1}{4}$,	last principal.
	569	17	–	
	£ 5516	11	$6\frac{3}{4}$,	the answer,

Fourthly,

Fourthly, When the intereſt of any ſum is required for days.

RULE.

Firſt find the intereſt for a year, then by the rule of three direct, viz. 365 : one year's intereſt : : the days the money is at intereſt : intereſt required.

9. What is the intereſt of 375 l. 15 s. for 127 days, at $4\frac{3}{4}$ per cent. per annum?

```
     l.   s.
½  375   15
         4¾
 ----------------
  1503   –    –
   187  17    6
    93  18    9
 ----------------
£ 17.84 16    3
    20
 ------
  16.86 s.
    12
 ------
  10.35 d.
     4
 ------
   1.40 qrs.
 ------
```

```
     days.    l.   s.   d.      days.
As  365   :  17   16   10¼  ::  127
                       12
           ----------------
            214    2    3
                       10
           ----------------
           2141    2    6
            124   17   11¾
           ----------------
      365)2266    –    5¾(6 l. 4 s. 1¾d. anſ.
            76
           ×20
           ----
          1520
            60
           ×12
           ----
           725
           360
           × 4
           ----
          1443
           348
           ----
```

10. What is the intereſt of 284 l. 10 s. for two years, four months, and 25 days, at $3\frac{1}{2}$ per cent. per annum?

284 l.

l. s.	days. l. s. d. days.
284 10	365 : 9 19 $1\frac{3}{4}$: : 25
$3\frac{1}{2}$	5
853 10	49 15 $8\frac{3}{4}$
142 5	5
£ 9.95 15	365)248 18 $7\frac{3}{4}$ (13s. $7\frac{1}{2}$
20	20
19.15 s.	4978
12	1328
	233
1.80 d.	× 12
4	2796
3.20 qrs.	221
	× 4
	884

l.	s.	d.	
$\frac{1}{3}$ 9	19	$1\frac{3}{4}$,	year's intereſt.
		2	
19	18	$3\frac{1}{2}$,	two years.
3	6	$4\frac{1}{2}$,	four months.
–	13	$7\frac{1}{2}$,	twenty-five days.
£ 23	18	$3\frac{1}{2}$,	the anſwer required.

Or as five per cent is ſtatute intereſt, multiply the given ſum by the number of days, and divide the product by 7300 (viz. $\frac{100 \times 365}{5}$); the quotient will give the intereſt at five per cent. but if the intereſt at a higher or lower rate is required, take aliquot parts of the quotient for a difference, which add or ſubtract accordingly.

11. What is the intereſt of 547 l. 15 s. at five per cent. for 320 days?

$$\begin{array}{r} 547.75 \\ 320 \\ \hline 109550 \\ 164325 \\ \hline \end{array}$$

7300)1752.8000(24.0109 = 24 l. – s. $2\frac{1}{2}$ d.

12. What

12. What is the intereſt of 248 l. 19 s. for 175 days, at $4\frac{1}{2}$ per cent.?

248.95
175

124475
174265
24895

$\frac{1}{10}$
73)435.6625(5.96798, at five per cent.
706 596798
496
582 5.371182 = 5 l. 7 s. 5 d. at $4\frac{1}{2}$ per cent.
715
580

13. What is the intereſt of 713 l. 17 s. 6 d. for 193 days, at $3\frac{3}{4}$ per cent.?

713.875
193

2141625
6424875
713875

4)
73) 1377.77875 (18.87368, at 5 per cent.
644 4.71842, at $1\frac{1}{2}$ per cent.
637
537 14.15526 = 14 l. 3 s. $1\frac{1}{4}$ d. the anſ.
268
497
595

11

It

It being looked upon as inaccurate in the calculation of intereſt to take aliquot parts for months, becauſe the year is divided into months conſiſting of an unequal number of days; I have therefore, to find the number of days from one time to another, inſerted the following TABLE.

Days	January	February	March	April	May	June	July	Auguſt	September	October	November	December
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	347	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

The

The Use *of the foregoing* Table.

First, To find the number of days from the end of the year to any given day in any month in the year following.

Opposite the given day in the margin look under the given month, which will shew the number of days required.

Thus, from December 31, till August 18 following, are 230.

And till October 30, are 303 days, &c.

Secondly, To find the days from any given day of any month, to the end of the year. - - - Suppose July 27.

	From 365 days in a year,
Take the number answering July 27, viz.	208
Rem.	157, days required.

Thirdly. To find the number of days between a given day in any given month, and any given day of any other month in the same year.

Suppose the days between April 5, and November 28, be required.

The number answering November 28 -	332
Subtract that answering April 5 - -	95
Rem.	237, days sought.

Fourthly, To find the number of days from any given day of any month in one year, to any given day of any month in the next year.

Suppose the number of days from the 21st of August 1758, till the 27th of May 1759, were required.

From - - - - - - - - - -	365, days in a year.
Take the No. answering Aug. 21	233
Rem.	132, to the end of the year.
Add the No. answering May 27 -	147
	279, days required.

But

But in the biſſextile, or leap-year, if any one of the given days be before the 29th of February, atd the other after, one day muſt be added on that account. Thus, if the number of days in the laſt example had been from the 21ſt of Auguſt 1759, till the 27th of May 1760, it would have been 280 days.

13. What is the intereſt of 150 l. from the 18th day of January, to the 11th of November, at $4\frac{3}{4}$ per cent.?

314 November 11.
18 January.

297
150

20)
73)745.50(6.1027
.3051

5.7976 = 5 l. 15 s. $4\frac{1}{4}$ d.

14. What is the intereſt of 384 l. 16 s. from the 7th of May to the 11 th of December, at $5\frac{1}{2}$ per cent.?

345 December 11.
127 May 7.

218
3848

30784
3848
7696

93) 838.864 (11.49128, at 5 per cent.
1.14912, at $\frac{1}{2}$ per cent.

12.6404 = 12 l. 12s. $9\frac{1}{2}$ d.

15. What is the intereſt of 537 l. 15 s. from November the 11th, 1754, till June the 5th, 1755, at $3\frac{5}{8}$ per cent.?

```
        537·75                              365
           206                              315
       -------                              ---
        322650    To the year's end          50
       107550                               156, to June 5th.
       -------   5)                         ---
73)1107.765( 15.17486, at 5 p. cent.        206 days in all.
              --------                      ---
          1/4 | 3·03497 = 1 per cent.
          1/3 |  ·75874 = 1/4 } per cent.
              |  ·37947 = 1/8 }
              | -------
              | 4·17318 = 1 3/8 per cent.
              | -------
             11.00168 = 11 l. - s. - 1/4 d. the answer.
             --------
```

Intereſt is to be calculated in the ſame manner on caſh accounts, accounts current, &c. where partial payments are made, and partial debts contracted.

16. On the 1ſt of May I lent Ralph Newlands, per bill, at one day's date, 500 l., which I received back in the following partial payments, viz. on the 13th of May 50 l.; on the 4th of June 56 l.; on the 14th of July 44 l.; on the 23d ditto 50 l.; on the 18th of Auguſt 87 l.; on the 30th ditto 13 l.; on the 21ſt of September 30 l.; on the 18th of October 30 l.; on the 29th ditto 40 l.; on the 11th of November 50 l.; on the 28th of December 50 l.; what intereſt is due, at five per cent.?

	Mr. Ralph Newlands - -	Debtor. l.	Days.	Prod.
May 1.	Lent per bill at one day's date -	500	13	6500
13.	Received in part - - -	50		
	Bal.	450	22	9900
June 4.	Received in part - - - -	56		
	Bal.	394	40	15760
July 14.	Received in part - - - -	44		
	Bal.	350	9	3150
23.	Received in part - - -	50		
	Bal.	300	26	7800
Aug. 18.	Received in part - - - - -	87		
	Bal.	213	12	2556
30.	Received in part - - -	13		
	Bal.	200	22	4400
Sept. 21.	Received in part - - - - - -	30		
	Bal.	170	27	4590
Oct. 18.	Received in part - - - -	30		
	Bal.	140	11	1540
29.	Received in part - - -	40		
	Bal.	100	13	1300
Nov. 11.	Received in part - - -	50		
	Bal.	50	47	2350
Dec. 28.	Received in full of principal -	50		
				59846

73) 598.46 (8 l. 3 s. 11½ d. interest on this account.

17. Lent John Jameson, per bill, dated 18th of January, payable one day after date, 878 l. 19 s. 10 d. which I received back in the following partial payments, viz. on the 27th of February 57 l. 15 s. 7 d.; on the 18th of March 37 l. 14 s.; on the 29th of April 34 l. 11 s.; on the 12th of May 136 l. 15 s. 7 d.; on the 19th of June 67 l. 13 s. 4 d.; on the 15th of

of July 15l. 15s. 6d.; on the 25th ditto 111l. 11s. 11d.; on the 3d of October 78l. 7s. 4d.; on the 19th of November 100l.; on the 23d ditto 100l.; and on the 30th of December received the balance of the principal: how much interest ought I to claim at five per cent.?

Cash Transactions with John Jameson.

	Extended. l.	s.	d.	Days.	Products. l.	s.	d.
Jan. 18. Lent per bill at one day's date - -	878	19	10	40	35159	13	4
Feb. 27. Received in part -	57	15	7				
Bal.	821	4	3	19	15603	-	9
Mar. 18. Received in part -	37	14					
Bal.	783	10	3	42	32907	10	6
April 29. Received in part -	34	11					
Bal.	748	19	3	13	9736	10	3
May 12. Received in part -	136	15	7				
Bal.	612	3	8	38	23262	19	4
June 19. Received in part -	67	13	4				
Bal.	544	10	4	26	14157	8	8
July 15. Received in part -	15	15	6				
Bal.	528	14	10	10	5287	8	4
Ditto 25. Received in part -	111	11	11				
Bal.	417	2	11	70	29200	4	2
Oct. 3. Received in part -	78	7	4				
Bal.	338	15	7	47	15922	12	5
Nov. 19. Received in part -	100						
Bal.	238	15	7	4	955	2	4
Ditto 23. Received in part -	100						
Bal.	138	15	7	37	5134	16	7
Dec. 30. Received in full of principal - - - -	138	15	7				
					£187327	6	8

COMPUTATION of EXAMPLE 17.

		l.	s.	d.
		878	19	10
By the Table.				
				5
25 Feb.	58	4394	19	2
18 Jan.	18			8
	40	35159	13	4
18 March	77	821	4	3
27 Feb.	58			6
	19	4927	5	6
				3
		14781	16	6
		821	4	3
		15603	0	9
29 April	119	783	10	3
18 March	77			6
	42	4701	1	6
				7
		32907	10	6
12 May	132	748	19	3
29 April	119			13
	13	9736	10	3
19 June	170	612	3	8
12 May	132			12
	38	7340	4	0
				3
		22038	12	0
		1224	7	4
		23262	19	4
15 July	196	544	10	4
19 June	170			13
	26	7078	14	4
				2
		14157	8	8
25 July	10	528	14	10
				10
		5287	8	4
3 Oct.	276	417	2	11
25 July	206			7
	70	2920	0	5
				10
		29200	4	2

		l.	s.	d.
19 Nov.	323	338	15	7
3 Oct.	276			6
	47	2032	13	6
				7
		14228	14	6
		1693	17	11
		15922	12	5
23 Nov.	4	238	15	7
				4
		955	2	4
30 Dec.	364	138	15	7
	327			6
	37	832	13	6
				6
		4996	1	0
		138	15	7
		5134	16	7

The sum of all these multiplied into their respective times are 187327 l. 6 s. 8 d. Then

```
        l.     s. d.  l.  s. d.
73) 1873.27  6  8 (25 13 2½ interest.
    413
    ----
     48.27
        20
    ------
    965.46
    235
    ------
     16.46
        12
    ------
    197.60

     51.60
         4
    ------
    206.40
```

CASE

CASE II.

The amount, rate per cent. and time given; to find the principal.

RULE.

As the amount of 100 l. at the rate and time given : is to 100 l. : : ſo is the amount given : to the principal required.

1. What principal ſum, being put out to intereſt at three per cent. per annum, will amount to 3998 l. 12 s. $10\frac{3}{4}$ d. in $3\frac{1}{4}$ years, and 54 days?

years. days.
Time 3.3979452 = $3\frac{1}{4}$ 54
3 intereſt of 100 l. for a year.

10.1938356 = intereſt of 100 l. for $3\frac{1}{4}$ years, 54 days.
+ 100

110.1938356, amount of 100 l. for the ſaid time.
As 110.1938356 : 100 : : 3998.6447916 : 3628.737275.
∴ 3628.737275 = 3628 l. 14 s. 9 d. the anſwer.
And 3998 l. 12 s. $10\frac{3}{4}$ d. — 3628 l. 14 s. 9 d. = 369 l. 18 s. $1\frac{3}{4}$ d. diſcount.

2. What is 309 l. 16 s. 10 d. due three years, one quarter, two months, 18 days hence, worth in ready money, abating or diſcounting $4\frac{3}{8}$ per cent. per annum?

3.465982, time.
573.4, rate reverſed.

1386393
103979
24262
1733

15.16367 + 100 = 115.16367.
115.16367 : 100 : : 309.8416 : 269.044627.
269.044627 l. = 269 l. – s. $10\frac{3}{4}$ d. the anſwer.
And 309 l. 16 s. 10 d. — 269 l. – s. $10\frac{3}{4}$ d. = 40 l. 15 s. $11\frac{1}{4}$ d. diſcount.

CASE III.

The amount, principal, and time given; to find the rate of intereſt.

 RULE.

RULE.

As the principal multiplied into the time : is to the whole interest : : so is 100 l. : to the rate per cent. per annum ?

1. At what rate of interest, per cent. per annum, will 3628 l. 14 s. 9 d. amount to 3998 l. 12 s. $10\frac{3}{4}$ d. in $3\frac{1}{4}$ years, and 54 days ?

3628.737275, principal.	3998.6447919, amount.
549793.3, time inverted.	3628.737275, principal.
10886211825	369.907516 interest.
1088621183	
326586355	
25401161	
3265863	
145150	
18144	

12330.249681 : 369.907516 : : 100 : 3 per cent.
369.907516 × 100 = 36990.7516.
12330.2497) 36990.7516 (3 per cent. per annum, the answer.

2. At what rate of interest, per cent. per annum, will 269 l. – s. $10\frac{3}{4}$ d. amount to 309 l. 16 s. 10 d. in $3\frac{1}{4}$ years, two months, and 18 days.

269.044627, principal.	309.841666, amount.
2895643	– 269.044627
8071339	40.797039, interest.
1076178	
161427	
13452	
2421	
215	
5	

932.5037 : 40.797039 interest : : 100 : 4.375 = $4\frac{3}{8}$ per cent. the answer.

CASE IV.

The principal, amount, and rate of interest being given, to find the time.

RULE.

RULE.

As the intereſt of the whole principal for one year, at the given rate : is to one year : : ſo is the whole : to the time required.

1. In what time will 3628 l. 14 s. 9 d. amount to 3998 l. 12 s. $10\frac{3}{4}$ d. at three per cent. per annum?

3628.7375	3998.6348, amount.
× 3	— 2628.7373, principal.
108.862125, years intereſt.	369.9075, intereſt.

108.862 : 1 : : 396.9073

108.862) 396.9073 (3.397938 = $3\frac{1}{4}$ years, 54 days.

329586	— 25
433213	147938
326586	365
106627	739690
97976	887628
	443814
8651	
7620	53.99737 = 54 days.
1021	
979	
42	
33	
9	

2. In what time will 269 l. – s. $10\frac{3}{4}$ d. amount to 309 l. 16 s. 10 d. at $4\frac{3}{8}$ per cent. per annum?

269.044627, principal.	309.841666, amount.
573.4, rate reſerved.	269.044627, principal.
10761785	40.797039, intereſt.
807134	
188330	
13452	
11.77070, year's intereſt.	

As

As 11.7707 : 1 year : : 40.797039 : 3.465982 years.

```
11.7707)40.797039(3.465982 = 3¼ years, 2 months, 18 days.
        253121    — .25     = ¼ year.
        ------      ------
        548493      215982
        470828 ⅙ =  166666  = 2 months.
        ------      ------
        776659     .049316  = 18 days.
        706242         365
        ------      ------
         70417     .246580
         58854     295896
         -----    147948
         11563    --------
         10593   18.000340
         -----   --------
           970
           942
           ---
            28
            23
            --
             5
```

QUESTIONS *in the three last* CASES *resolved by the* RULE OF FIVE.

1. What money, at $3\frac{1}{2}$ per cent. will clear 38 l. 10 s. in a year and quarter's time?

P.	T.	G.
100 .	1 .	3.5
	1.25 .	38.5

$100 \times 1 \times 38.5 = 3850$, dividend; and $3.5 \times 1.25 = 4.375$, divisor.

Then 4.375) 3850 (880 l. the answer.

2. Put out 384 l. to interest, and in $8\frac{1}{4}$ years there were 542 l. 8 s. found to be due; what rate of interest could then be implied?

542 l. 8 s. — 384 l. = 158 l. 8 s. interest.

P.	T.	G.
384 .	8.25 .	15.4
100 .	1 .	

$158.4 \times 100 \times 1 = 15840$, dividend; and $384 \times 8.25 = 3168$, divisor.

3168) 15840 (5 per cent. per annum, the answer.

3. Lent

3. Lent 109 guineas at 4 per cent. which, by the 18th of August 1760, was raised by the interest to so many moidores, bating 2 s. 6 d.; pray on what day did the bond bear date?

109 guineas = 114 l. 9 s. = 114.45, principal.
109 moidores = 147 l. 3 s. which — 2 s. 6 d. = 147 l. – s. 6 d. amount.

Then 147 l. – s. 6 d. — 114 l. 9 s. = 32 l. 11 s. 6 d. = 32.575, interest.

P.	T.	G.
100	1	4
114.45	.	32.575

$100 \times 1 \times 32.575 = 3257.5$, dividend; and $114.45 \times 4 = 457.8$.

457.8) 3257.5 (7.11555 = 7 years and 42 days.
July hath 31 days.
'Till Aug. 18

Sum 49 — 42, gives July the 7th, the answer.

4. If 100 l. in 12 years be allowed to gain 39 l. 19 s. 8 d. in what time will any other sum of money double itself by the same rate of interest?

P.	T.	G.
100	12	39.983
1		1

39.983) 1200
3998) 120

35.985) 1080.000 (30.0125 = 30 years and $4\frac{1}{2}$ days, answ.

5. In what time will the interest of 49 l. 3 s. equal the proceed of 19 l. 6 s. at use 47 days, at any rate of interest?

Reciprocally, 19.3 l. : 47 days : : 49.15 l. : 18.45 days, the answer.

6. A bond was made on the 7th of August 1713, at 6 per cent. per annum, for 1114 l. 10 s.; on the 11th of May, 1718, 140 l. was paid off, and a fresh bond entered into for the remainder, at $5\frac{1}{4}$ per cent. per annum; at the time the interest for this last was 21 l. 16 s. 8 d. there was paid off 87 l. 11 s. 9 d. The old bond being then taken up, a new one was given for the residue, which being paid off September 11, 1724, the bond-owner took no more than 1409 l.

1409 l. 16 s. 8 d. in full payment. At what rate then did he take interest per cent per annum, upon the last renewal of the bond?

To the 11th of May, 1718, are 4 years 277 days = 4.758904 1114.5 x 6 = 6687
78.66, year's interest reversed.

28553424
2855342
381712
32312

318.2279, interest for 4 years, 277 days.
140.... paid off.

178.2279, surplus,

1114.5 l. + 178.2279 = 1292.7279 l. new principal.

P.		T.		G.
100	.	1	.	5.25
1292	.	.	.	21.83

6787.8215) 2183.3333 (.3217 = 117 days, which answers to the 5th of September 1718.

Then 1292.7279 l. + 21.83 l. = 1314.5612 l.
— 87.5875 paid off.

£ 1226.9737, new princip.

From September 5, 1718, till September 11, 1724, are six years, six days = 6.01644.

And 1409.83 — 1226.9737 l. = 182.8596 l. interest.

P.		T.		G.
1226.9737	:	6.01644	.	182.8596
100	.	1	.	

∴ 7382.0136) 18285.96 (2.477096 = 2 l. 9 s. $6\frac{1}{2}$ d. the answer required.

* *

SECT. II.

INSURANCE.

INSURANCE is security given in consideration of so much per cent. paid in hand to make good ships, merchandizes, houses, &c. to the value of that for which the

the premium is received, in case of loss by storm, pirates, fire, or the like.

This, as well as brokerage and commission, is computed in the same manner as simple interest for a year.

What is the insurance of 737 l. 18 s. at $3\frac{5}{8}$ per cent.?

	l.	s.	d.
	737	18	
		$3\frac{5}{8}$	
	2213	14	
	368	19	
	92	4	9
£	26.74	17	9
	20		
	14.97 s.		
	12		
	11.73 d.		
	4		
	2.92 qrs.		

Or,

	$\frac{1}{10}$	737	18	–
2 s. =	$\frac{1}{5}$	73	15	$9\frac{1}{2}$
1 s. =	$\frac{1}{2}$	14	15	$1\frac{3}{4}$
$\frac{4}{8}$ =	$\frac{1}{2}$	7	7	$6\frac{5}{4}$
$\frac{1}{8}$ =	$\frac{1}{4}$	3	13	$9\frac{1}{4}$
		–	18	$5\frac{1}{4}$
		£26	14	11

Answer, 26 l. 14 s. $11\frac{1}{2}$ d.

What is the insurance of 874 l. 13 s. 6 d. at $13\frac{1}{2}$ per cent.?

l.	s.	d.
874	13	6
		$13\frac{1}{2}$
11370	15	6
437	6	9
118.08	2	3
20		
1.62		
12		
7.47		
4		
1.88		

Or,

		l.	s.	d.
10 per cent. =	$\frac{1}{10}$	873	13	6
2 =	$\frac{1}{5}$	87	9	4
1 =	$\frac{1}{2}$	17	9	$10\frac{1}{4}$
$\frac{1}{2}$ =	$\frac{1}{2}$	8	14	11
		4	7	$5\frac{1}{2}$
		118	1	$6\frac{3}{4}$

Answer, 118 l. 1 s. $7\frac{1}{4}$ d.

Primage

Primage is an allowance paid to mariners at their first sailing out of port for their loading the ship.

Stowage is the money paid for stowing the goods in a vessel.

Average is the quota or proportion, which each proprietor of a ship, or the goods therein, is adjudged, on a reasonable estimation, to contribute toward the losses which are sustained by some of the goods being cast overboard for the preservation of the rest, and of the ship.

What is the insurance of an East-India ship and cargo, valued at 35727 l. 17 s. 6 d. at $17\frac{7}{8}$ per cent?

	l.	s.	d.
	35727	17	6
			$17\frac{7}{8}$
10 times	357278	15	–
7 times	250095	2	6
$\frac{1}{2}$ - -	17863	18	9
$\frac{1}{4}$ - -	8931	29	$4\frac{1}{2}$
$\frac{1}{8}$ - -	4465	19	$8\frac{1}{4}$
£	6386.35	15	$3\frac{3}{4}$
	20		
	7.14 s.		
	12		
	1.83 d.		
	4		
	3.35 qrs.		

Answer, 6386 l. 7 s. 1³ d.

	l.	s.	d.
$\frac{1}{10}$	35727	17	6
10 per cent. =	3572	15	9
5 per cent. =	1786	7	$10\frac{1}{2}$
$2\frac{1}{2}$ per cent. =	893	3	$11\frac{1}{4}$
$\frac{1}{4}$ per cent. =	89	6	$4\frac{1}{2}$
$\frac{1}{8}$ per cent. =	44	13	$2\frac{1}{2}$
$11\frac{7}{8}$ per cent. =	6386	15	$3\frac{1}{2}$

* *

SECT. III.

BROKAGE.

BROKERAGE, or BROKAGE, is the fee or reward paid unto a person called a broker, for assisting a merchant or factor in buying or selling goods, &c. This business

ſineſs was formerly carried on by broken merchants or traders, from whence their name derived; and in London they are not to act without licence from the lord-mayor.

What is the brokage of 856 l. 6 s. 8 d. at 6 s. per cent?

l.	s.	d.
8.56	6	8
20		
11.26 s.		
12		
3.20 qrs.		

	l.	s.	d.
$\frac{1}{4}$	8	11	3
$\frac{2}{5}$	2	2	$9\frac{3}{4}$
	–	8	$6\frac{3}{4}$
Anſwer, £	2	11	$4\frac{1}{2}$

What is the brokage of 737 l. 13 s. at 4 s. 9 d. per cent.?

l.	s.
7.37	13
20	
7.53 s.	
12	
6.36 d.	
4	
1.44 qrs.	

	l.	s.	d.
$\frac{1}{5}$	7	7	$6\frac{1}{4}$
$\frac{1}{8}$	1	9	6
$\frac{1}{2}$		3	$8\frac{1}{4}$
		1	10
Anſwer £	1	15	$-\frac{1}{4}$

What is the brokage of 2572 l. 15 s. at $\frac{3}{8}$ per cent.?

l.	s.
25.72	15
20	
14.55 s.	
12	
6.60 d.	
4	
2.40 qrs.	

	l.	s.	d.
$\frac{1}{4}$	25	14	$6\frac{1}{2}$
$\frac{1}{2}$	6	8	$7\frac{1}{2}$
	3	4	$3\frac{3}{4}$
Anſwer, £	9	12	$11\frac{1}{4}$

SECT.

SECT. IV.

PURCHASING STOCKS.

RULE.

MULTIPLY the sum to be purchased by the excess above 100; and then proceed as before directed in computing interest, the product of which added to the given stocks, gives the purchase; or you may find it by practice, if more convenient.

What is the purchase of 987 l. 15 s. South-Sea stock, at 113⅞ per cent.?

l.	s.	d.
987	15	
	13⅞	
12840	15	–
493	17	6
246	18	9
123	9	4½
£137.05	–	7½
20		
1.00 s.		
£987	15	
+137	1	
£1124	16, the answ.	

	l.	s.	d.
1/10	987	15	–
1/5	98	15	6
1/2	19	15	1¼
1/2	9	17	6½
1/4	4	18	9¼
1/2	2	9	4½
	1	4	8¼
	£1124	15	11¾, answer.

2. 769 l. India stock, at 117⅜ per cent.?

	l.	s.	d.		l.	s.	d.
1/10	769	–	–				
1/5, 1/2	76	18	–	- - - - - -	10		
	38	9	–	- - - - - -	5		
1/2	15	7	7¼	- - - - - -	2		
	1	18	5¼	- - - - - -	–	5	–
	–	19	2½	- - - - - -	–	2	6
	£902	12	3, the answer.				

3. What

3. What is the purchase of 1537 l. 10 s. Bank stock, at $131\frac{1}{4}$ per cent.?

	l.	s.	d.		
$\frac{1}{4}$	1537	10			l.
$\frac{1}{5}$	384	7	6	- - - - - - -	25
$\frac{1}{4}$	76	17	6	- - - - - - -	5 s.
	19	4	$4\frac{1}{2}$	- - - - - - -	1 5
£	2017	19	$4\frac{1}{2}$, the answer required.		

4. 1812 l. Bank annuities at $93\frac{5}{8}$ per cent.?

First, 100
— $93\frac{5}{8}$

$6\frac{3}{8}$

	l.	s.	d.
$\frac{1}{20}$	1812		
$\frac{1}{4}$	90	12	
$\frac{1}{10}$	22	13	
	2	5	$3\frac{1}{2}$
£	115	10	$3\frac{1}{2}$

l.	s.	d.
1812	–	–
– 115	10	$3\frac{1}{2}$
£1696	9	$1\frac{1}{2}$, the answer required.

5. What is the purchase of 1727 l. Bank stock, at 119 per cent.?

l.
1727
19

£ 328.13
20

2.60 s.
12

7.20 d.

l.	s.	d.
1727	–	–
328	2	7
£2055	2	7, the answer.

By Practice.

	l.	s.	d.
$\frac{1}{10}$	1727	–	–
$\frac{1}{5}, \frac{1}{2}$	172	14	–
	86	7	–
	34	10	$9\frac{1}{2}$
	34	10	$9\frac{1}{2}$
£	2055	2	7, as before.

6 June

6. June the 23d, 1745, bought 900 l. of new South-Sea annuities, at $111\frac{3}{8}$ per cent. viz. the day before the closing the books, the brokerage whereof is always 2 s. 6d. per cent. on the capital, whether you buy or sell. The Midsummer dividend, 2 per cent. became due and payable on the 10th of August following; by which time the rebellion growing considerable in the north, the said annuities were down at $92\frac{1}{2}$ per cent. In the general alarm, sold 400l. capital at that price; but continued the remainder 'till a second, third, fourth, and fifth dividend, as before, came due: and on opening the books on the 10th of August 1747, sold out at $102\frac{5}{8}$ per cent. Now reckoning I might have made five per cent. of my money, had I kept it out of the stocks, how stood this article in point of profit and loss?

	l.	s.	d.
$100 : 111\frac{3}{8} :: 900 :$	1002	7	6
Brokerage of 900 l. at 2 s. 6 d. per cent.	1	2	6
	£1003	10	–
1745, Midsummer dividend, at 2 per cent.	18	–	–
	£985	10	–
Inter. of 1003l. for 45 days, at 5 per cent. per ann.	6	14	8
Brokerage of 400 l. at 2s. 6d. per cent.	–	10	–
	£992	14	8
Sold 400l. at $92\frac{1}{2}$ per cent.	370	–	–
	£662	14	8
Interest for half a year due the 10th of Feb. 1746	15	11	$4\frac{1}{4}$
	£638	6	$-\frac{1}{4}$
Dividend received at that time	10	–	–
	638	6	$-\frac{1}{4}$
Interest due the 10th of August, 1746	15	14	$1\frac{3}{4}$
	644	–	2
Dividend received at that time	10	–	–
	634	–	2
Interest due the 10th of February 1747	15	17	–
	649	17	2
Dividend received then	10	–	–
Carried over	639	17	2

	l.	s.	d.
Brought over -	639	17	2
Interest the 10th of August, 1747 - - - - -	15	19	11
	655	17	1
Midsummer dividend received Aug. 10, 1747	10	–	–
	645	17	1
Sold off 500 l. at $102\frac{5}{8}$ per cent. - - - - -	512	2	6
	132	14	7
Brokerage - - - - - - - - - - - - - -	–	12	6
To my damage in the whole - - - - - - £	132	2	1

SECT. V.

REBATE, OR DISCOUNT.

REBATE, or DISCOUNT, is an abatement of a sum of money due some time hence, in consideration of the prompt or present payment of the remainder.

The ready money that will satisfy the debt is called the present worth; because if it was put out to interest at the given rate per cent per annum for the time the discount is computed, it would amount to the given debt.

The true method of finding the discount of any sum, is by Case II. in simple interest; or,

RULE,

First find the interest of 100l. for the time mentioned; then, as 100 l. with the interest is : to the interest : : so is the debt or sum proposed to be discounted : to the discount required; which, subtracted from the debt, leaves the present worth, or money to be paid down.

1. What is the discount of 57 l. 18 s. due 12 months hence, at five per cent. per annum?

105 : 5 :: 57.9 | l. s. d.
5 | 57 18 –, debt.
105) 289.5 (2.75238 = 2 15 –$\frac{1}{2}$, discount.

£55 2 11$\frac{1}{2}$, present worth.

2. What is the discount of 573 l. 15s. due three years hence, at $4\frac{1}{2}$ per cent. per annum?

4.5 × 3 = 13.5 = interest of 100 l. for three years.

113.5 : 13.5 :: 573.75
13.5

286875
172125
57375

573 15 –
113.5) 7745.625 (68.24339 = 68 4 10$\frac{1}{4}$ discount.

505 10 1$\frac{3}{4}$, Q.E.F.

3. What is the discount of 725 l. 16 s. for five months, at $3\frac{7}{8}$ per cent. per annum?

M. G. M. G.
As 12 : 3.875 :: 5 : 1.614583
101.614583 : 1.614583 :: 725.8
725.8

12916666
80729166
32291666
11302083333

101.614583) 1171.864583 (
10161458) 117186458 (

91.453185) 1054.678125 (11.532435 = 11 l. 10 s. $7\frac{3}{4}$ d.
140146275
48693090
2966497
222901
39995
3259
510

Or

Or to find the present money, observe the following

RULE.

First find the interest of 100 l. for the time mentioned, as before; then say, As 100 l. with the interest added : is to 100 l. : : so is the debt or sum proposed to be discounted : to the present money.

4. What ready money will discharge a debt of 543 l. 7 s. due four months and 18 days hence, at $4\frac{5}{8}$ per cent. per annum?

4 months = .333333
18 days - = .049315

Time = .382648 × 4.625 = 1.769748, int. of 100 l.
As 101.769748 : 100 :: 543.35 : 533.9012 = 533 l. 18 s. $-\frac{1}{4}$ d.

	l.	s.	d.	
Debt - -	543	7	–	
Present worth	533	18	$-\frac{1}{4}$,	the answer.
Discount -	£ 9	8	$11\frac{3}{4}$	

5. What ready money will discharge a debt of 1377 l. 13 s. 4 d. due two years, three quarters, 25 days hence, discount $4\frac{3}{8}$ per cent. per annum?

$2\frac{3}{4}$ years = 2.75
25 days = .068493

2.818493 × 4.375 = 12.330907 = int. of 100 l.
112.330907 : 100 :: 1377.6 : 1226.4359858.
Answer, 1226.4359858 = 1226 l. 8 s. $8\frac{1}{2}$ d. present money.

6. What difference is there between the interest of 500 l. at five per cent. per annum, for 12 years, and the discount of the same sum, at the same rate, for the same time?

P.		T.		G.
100	.	1	.	5
500	.	12	.	

5 × 500 × 12 = 30000, dividend.
100 × 1 = 100) 30000 (300, the interest.
Then 12 × 5 = 60, the interest of 100 l. for 12 years.
And 100 + 60 = 160, its amount.
As 160 : 60 :: 500 : 187 l. 10 s. the discount.
∴ 300 l. — 187 l. 10 s. = 112 l. 10 s, advantage to the interest.

 SECT

SECT. VI.

EQUATION of PAYMENTS.

WHEN ſeveral debts are payable at different times, and it is mutually agreed between debtor and creditor, that all thoſe ſeveral ſums be paid at ſuch a time, that neither debtor nor creditor may be wronged thereby, this is called the equated time of payment. The rule given by Mr. Cocker and others for finding this equated time is,

RULE.

Multiply each payment by its time, and divide the ſum of all theſe products by the whole debt; the quotient was by him accounted the equated time.

A perſon dying, bequeaths to a younger ſon 1000 l. to be paid as follows; viz. 300 l. at one year's end; 300 l. more at a year and a half; and the remainder at the end of two years and a half. Now the executor agrees with the legatee, to pay the whole at one payment; how long from the death of the father muſt this payment be, ſo that neither party be wronged, or ſuffer loſs?

By the foregoing rule $300 \times 1 = 300$
$300 \times 1\frac{1}{2} = 450$
$400 \times 2\frac{1}{2} = 1000$

1000) 1750 (.75, or $1\frac{3}{4}$ year, the anſwer.

Mr. Kerſey finds fault with the foregoing method, as no intereſt is thereby implied, and thinks that a diſcount (ſtatute intereſt) ſhould be allowed for each payment; and to find the equated time, gives a rule to the following purport.

RULE.

Find the preſent worth of each payment, according to its reſpective time and rate; then add all the preſent worths together, and call their ſum the principal; laſtly, having the principal, amount, and rate of intereſt, find the time by Caſe IV. of ſimple intereſt.

Now allowing a diſcount of five per cent. the ſolution of the foregoing queſtion will be as follows:

viz.

viz. 105 : 100 :: 300 : 285.7142
107.5 : 100 :: 300 : 279.0697
112.5 : 100 :: 300 : 355.5555

920.3394
5 per cent.

£46.01697, int. for a year.

1000, amount.
920.3394, principal.

46.01697 : 1 :: 79.6606 : 1.7311 = 1 year, 8 months, 23 days, the equated time.

But the learned Mr. Alexander Malcolm justly observes, that though the debtor gains the interest of what he keeps after it was due, that he loses only the discount of what he pays before it was due, which is less than the interest; and that therefore the creditor may justly except against Mr. Cocker's method; and I apprehend, that for the same reason the debtor may have as just an exception against Mr. Kersey's.

The before-mentioned Mr. Malcolm, from an algebraic way of reasoning, founded on the principles of simple interest, raises and demonstrates a theorem, from whence is deduced the following

RULE.

Find one year's interest of the debt that is first payable, by which divide the sum of the debts (of the first and second payments) and to the quotient add the sum of the times; call this the first number found.

Then multiply each debt by its time, and divide the sum of the products by one year's interest of the first payable debt; which quotient added to the product of the two times, call the second number found.

Subtract the second number from $\frac{1}{4}$ of the square of the first number, and out of the difference extract the square root; which root being added to, or subtracted from half the first number found, the sum or difference will be the time sought.

N. B. As this rule is ambiguous, if you take the sum, if that happens to be greater than the time to the term of the last payable debt, the difference will be the time sought. Or if

if you take the difference, and that be less than the time to the term of the first payable debt, the sum will be the time sought. But if both the sum and difference happen between the two given terms, it must be examined which of them will make an equality of interest and discount.

I shall here reassume the foregoing question, allowing the interest and dividend at five per cent. per annum?

Debts $\begin{cases} 300, \text{ its interest for a year } 15\text{ l.} \\ 300 \end{cases}$ $\quad 300 \times 1 = 300$, $\quad 300 \times 1.5 = 450$

15) 600 → 40; + 2.5, sum of the times. → 42.5, first N° found.

15) 750 → 50; + 1.5 → Second N° 51.5

Then $42.5 \times 42.5 = 1806.25$, which $\div 4 = 451.5625$.
Also $451.5625 - 51.5 = 400.0625$.
$\sqrt{400.0625} = 20.0015624$; and 2) 42.5 (21.25.
$\because 21.25 - 20.0015624 = 1.248437$ years, the equated time for the two first payments.

Then to find the true equated time when the whole 1000 l. must be paid together,

Put 600 l. for the first payment, interest for a year 30 l.
400 l. second payment.

30) 1000 → 33.333333; + 3.748437 → 2) 37.08177, first number found. → 18.54088, its half.

1.248437 and 2.5 } times. → 3.748437, their sum.

Also $600\text{ l.} \times 1.2484376 = 749.06256$
$400 \times 2.5 = 1000$
30) 1749.06256 → 58.302085
$1.2484376 \times 2.5 = 3.121094$
Second number found $= 61.423179$

And

And $37.08177 \times 37.08177 = 1375.057666$

$4)\,1375.057666\,(343.764416$

$343.764416 - 61.423179 = 282.341237$

$\sqrt{282.341237} = 16.80284$

$\therefore 18.54088 - 16.80284 = 1.73804 = 1$ year, 8 mon. 26 days, the true equated time required.

2. At Michaelmas, seventeen hundred nineteen,
My writings will shew (which are yet to be seen)
That to me were three hundred and twenty pounds due,
And half of that sum, besides forty-two, viz. 202 l.
Just five years after, I then might demand,
But would fain have the whole somewhat sooner in hand.
I agree to rebate for the latter sum too,
The same rate (simple) interest our statutes allow,
But then I expect some use will accrue
From my sixteen-score pounds, that last year were due.
Now to know on what day, I should be very fond,
To receive my five hundred and twenty-two pound.

Ladies' Diary, 1720.

Debts $\begin{cases} 320 \text{ l.} \\ 202 \end{cases}$ its interest for one year is 16 l.

$16)\,522\,(32.625 + 5$ time $= 37.625$, the 1st number.

The 320 being according to the securities to be paid down.

So $202 \times 5 = 1010$

And $16)\,1010\,(63.125$, the second number.

Then $37.625 \times 37.625 = 1415.640625$

Also $4)\,1415.640625$

353.91015625

-63.125 18.8125, half the first number.

$\sqrt{295.78515625} = 17.0524$

$1.7601 = 1$ year, 277 days; which answers to July 4, 1721. Q. E. F.

As there are only a few days difference between this and the other method, and that this method will be operose, particularly when the payments are to be made at many different times, either of the former methods may do without any considerable wrong to either party; yet, in my opinion, truth is worth enquiring after.

CHAP.

CHAPTER III.

TARE AND TRET.

TARE is an allowance in merchandize made by the king to the importer, or to the buyer by the merchant, for the weight of the bag, cask, chest, wrappers, &c. in which any goods are put; several sorts of goods have their tares ascertained in a table annexed to the book of rates.

Gross weight is the whole weight of goods, with the chest, cask, bag, &c. that contains them.

Tret is an allowance, in weighable goods, of 4 lb. in 104 lb. made by the merchants in London to their tradesmen and retailers for break, waste or dust, yet himself is only allowed tare in paying custom; so that he payeth as well for the bad as the best commodity.

Cloff, clough, or draught, is a small allowance made by the king to the importer, or by the seller to the buyer, to cause the weight to hold out when goods are weighed again. The king allows 1 lb. draught for goods under 1 cwt.; 2 lb; from 1 to 2 cwt; 3 lb. from 2 to 3 cwt.; 4 lb. from 3 to 10 cwt.; 7 lb. from 10 to 18 cwt.; and 9 lb. from 11 to 30 cwt. and upwards.

Subtile, or suttle, is the weight of the goods when the tare is deducted, but not the tret.

Net weight is the remainder, when both, if both be allowed, are taken away.

ALIQUOT PARTS.

Cwt.			lb.			lb.		
10	$= \frac{1}{2}$	of a tun.	14	$= \frac{1}{8}$	of a cwt.	14	$= \frac{1}{2}$	of $\frac{1}{4}$ cwt.
5	$= \frac{1}{4}$		16	$= \frac{1}{7}$		7	$= \frac{1}{4}$	
4	$= \frac{1}{5}$		14	$= \frac{1}{4}$	of $\frac{1}{2}$ cwt.	4	$= \frac{1}{7}$	
$2\frac{1}{2}$	$= \frac{1}{8}$		8	$= \frac{1}{7}$		$3\frac{1}{2}$	$= \frac{1}{8}$	
2	$= \frac{1}{10}$		7	$= \frac{1}{8}$				

CASE I.

When the given tare is the aliquot part of an hundred, as 14 or 16 lb.

RULE

Divide the given weight by the denominator of the fraction representing the part, the quotient will be the tare.

1. What

1. What is the net weight of four barrels of figs?

		Cwt.	qrs.	lb.	
viz. N°	1 - - -	3	1	18	tare 14 lb. per cwt.
	2 - - -	4	3	24	
	3 - - -	6	–	20	
	4 - - -	5	2	26	
	8)	20	1	4,	gross.
		2	2	4,	tare.
	Cwt.	17	3	–,	net.

2. What is the net weight of five bags of cinnamon?

		Cwt.	qrs.	lb.	
viz. N°	1 - - -	1	3	24	tare 16 lb. per cwt.
	2 - - -	3	1	18	
	3 - - -	2	2	16	
	4 - - -	2	2	26	
	5 - - -	1	3	22	
	7)	12	3	22,	gross.
		1	3	11,	tare.
	Cwt.	11	–	11,	net.

CASE II.

When the tare is not an aliquot part of an hundred, but the aliquot part of $\frac{1}{2}$ or $\frac{1}{4}$ of a cwt.

RULE.

Take first the aliquot part of an hundred, and then part of that part, agreeable to the nature of the question, until you have found the true tare.

Another way of finding the tare when it is not an aliquot part of 112 lb.

RULE.

Multiply the hundreds by the tare to be allowed for 1 cwt. and for the quarters and pounds, in the gross weight take a proportional

proportional part of the said allowance, and the sum is the tare in pounds; which is either to be reduced into hundreds, and deducted from the gross weight, or the gross weight into pounds, and then deduct the tare.

What is the net weight of 19 cwt. 2 qrs. 12 lb. gross, tare 7 lb. per cwt.

	Cwt.	qrs.	lb.	
4	19	2	12,	gross.
4	4	3	17	
	1	-	$25\frac{1}{4}$,	tare.
Cwt.	18	1	$14\frac{3}{4}$,	net.

Otherways.

$19 \times 7 = 133$ lb.
$\frac{1}{2} \times 7 = 3\frac{1}{2}$
lb. $8 = \frac{1}{2}$
$4 = \frac{1}{4}$

lb. $137\frac{1}{4} = 1$ cwt. $25\frac{1}{4}$ lb. tare, as by the other method.

2. What is the net weight of 15 cwt. 2 qrs. 21 lb. tare 8 lb. per cwt.?

	Cwt.	qrs.	lb.	
7	15	2	21	
2	2	-	27	
	1	-	$13\frac{1}{2}$,	tare.
	14	2	$7\frac{1}{2}$,	net.

Or $15 \times 8 = 120$
$\frac{1}{2} \times 8 = 4$
lb. $14 = 1$
$7 = -\frac{1}{2}$

Tare $125\frac{1}{2} = 1$ cwt. $13\frac{1}{2}$ lb. as before.

3. What is the net weight or of 4 bags of hops, tare 4 lb. per cwt.?

		Cwt.	qrs.	lb.	
viz. N° 1		4	1	18	
2		3	3	24	
3		4	2	16	
4		4	3	4	
	4	17	3	6,	gross,
	7	4	1	$22\frac{1}{2}$	
			2	15,	tare,
		17	-	19,	net.

Otherways thus.

$17 \times 4 = 68$
$\frac{1}{2} \times 4 = 2$
$\frac{1}{4} \times 4 = 1$

lb. $71 = 2$ qrs. 15 lb. tare, as before.

CASE

CASE III.

When the tare is no aliquot part of an hundred weight, quarter, &c.

RULE.

Divide the given tare into aliquot parts of an hundred, quarter, &c. the sum of which will be the answer.

1. What is the net weight of 19 cwt. 3 qrs. 14 lb. of antimony, tare 6 lb. per cwt.?

	Cwt.	qrs.	lb.
7	19	3	14
2	2	3	10
	1	1	19
4	4	1	1
	1	–	$7\frac{1}{4}$ tare.
	18	3	$6\frac{3}{4}$, net.

Or thus.

$$
\begin{aligned}
19 \times 6 &= 114 \\
\tfrac{1}{2} \times 6 &= 3 \\
\tfrac{1}{4} \times 6 &= 1\tfrac{1}{2} \\
\text{lb. } 14 &= \tfrac{3}{4}
\end{aligned}
$$

$119\frac{1}{4} = 1$ cwt. $7\frac{1}{4}$ lb. tare, as before.

2. What is the net weight of 71 cwt. 3 qrs. 21 lb. of potashes, tare 10 lb. per cwt.?

	Cwt.	qrs.	lb.
7	71	3	21
4	10	1	3
	2	2	$7\frac{3}{4}$
2	12	3	$10\frac{3}{4}$
	6	1	$19\frac{1}{4}$, tare.
Cwt.	65	2	$1\frac{3}{4}$, net.

Or thus.

$$
\begin{aligned}
71 \times 10 &= 710 \\
\tfrac{1}{2} \times 10 &= 5 \\
\tfrac{1}{4} \times 10 &= 2\tfrac{1}{2} \\
14 &= 1\tfrac{1}{4} \\
7 &= \tfrac{1}{2}
\end{aligned}
$$

Tare $719\frac{1}{4} = 6$ cwt. 1 qr. $19\frac{1}{4}$ lb. as before.

3. What

3. What is the net weight of five casks of allum, tare 12 lb. per cwt.

		Cwt.	qr.	lb.
viz. N° 1	- -	2	1	27
2	- -	1	3	25
3	- -	3	-	18
4	- -	2	2	21
5	- -	3	1	15
	2	13	2	22
	7	6	3	11
	2	-	3	$25\frac{1}{2}$
		-	1	$20\frac{3}{4}$
Tare		1	1	$24\frac{1}{4}$
Net		12	-	$25\frac{3}{4}$

Otherways.

$13 \times 12 = 156$
$\frac{1}{2} \times 12 = 6$
lb. $14 = 1\frac{1}{2}$
lb. $8 = \frac{3}{4}$

Tare $164\frac{1}{4}$, as before = 1 cwt. 1 qr. $24\frac{1}{4}$ lb.

4. What is the net weight of five casks of oil, weighing as follows, tare 18 lb. per cwt.?

		Cwt.	qrs.	lb.
N° 1	- -	3	3	19
2	- -	4	1	25
3	- -	3	2	21
4	- -	5	-	17
5	- -	4	2	18
	7	21	3	16
	8	3	-	$14\frac{1}{4}$
		-	1	$15\frac{3}{4}$
Tare		3	2	2
Net		18	1	1

Otherways.

$21 \times 18 = 378$
$\frac{1}{2}$ of $18 = 9$
$\frac{1}{4}$ of $18 = 4\frac{1}{2}$
$\frac{1}{7}$ of $18 = 2\frac{1}{2}$

Tare 394 = 3 cwt. 2 qrs. 2 lb. as before.

5. What

5. What is the net weight of 27 cwt. 1 qr. 21 lb. of prunes in casks, tare 20 lb. per cwt.?

	Cwt.	qrs.	lb.		Otherways.
7	27	1	21		$27 \times 20 = 540$
					$\frac{1}{4}$ of $20 = 5$
4	3	3	19		$\frac{1}{8}$ of $20 = 2\frac{1}{2}$
	–	3	$25\frac{3}{4}$		$\frac{1}{16}$ of $20 = 1\frac{1}{4}$
	4	3	$16\frac{3}{4}$,	tare.	Tare $548\frac{3}{4} = 4$ cwt. 3 qrs. $16\frac{3}{4}$ lb. as before.
Cwt.	22	2	$4\frac{1}{4}$,	net.	

6. What is the net weight of seven fats of hogs bristles, each containing 3 qrs. 19 lb. tare 17 lb. per cwt.?

	Cwt.	qrs.	lb.		Otherways.
	–	3	19		$6 \times 17 = 102$
			$\times 7$		$\frac{1}{4} \times 17 = 4\frac{1}{4}$
					$\frac{1}{8} \times 17 = 2$
8	6	1	21		lb. $7 = 1$
7	–	3	$6\frac{1}{8}$		Tare $109\frac{1}{4} =$ 3 qrs. $25\frac{1}{4}$ lb.
2	–	–	$12\frac{3}{4}$		
	–	–	$6\frac{3}{8}$		
	–	3	$25\frac{1}{4}$,	tare.	
Cwt.	5	1	$23\frac{3}{4}$		

In many commodities the allowance for tare is not reckoned by the hundred weight, but so much of the gross; this is called invoice tare.

CASE IV.

When the tare of raw silk from Smyrna or Cyprus is to be deducted,

RULE,

For 3 cwt. and upwards allow 16 lb. tare; from 3 cwt. down to 2 cwt. 14 lb. tare; and from 2 cwt. downwards, 12 lb. tare.

Likewise in Virginia tobacco:

For all hogsheads under 3 cwt. allow 70 lb. tare; from 3 to 4 cwt. 80 lb.; from 4 to 5 cwt. 90 lb.; and from 5 cwt. upward, 100 lb. tare.

1. What

1. What is the tare of eight hogsheads of tobacco?

	Cwt.	qrs.	lb.		qrs.	lb.
viz. N° 1	2	2	18	tare	2	14
2	3	1	21		2	24
3	4	2	8		3	6
4	3	3	12		2	24
5	5	2	12		3	16
6	4	3	19		3	6
7	5	2	27		3	16
8	5	3	–		3	16
Gross	36	2	5	Tare 6	1	10

Net cwt. 30 – 23

1. What is the tare of six bales of raw silk?

	lb.		tare
viz. N° 1	325		16
2	185		12
3	274	tare	14
4	377		16
5	129		12
6	215		14

Gross 1505
Tare 84

Net 1421

CASE V.

When allowance is required for tare and tret,

RULE,

Find what is to be allowed for tare, according to the foregoing rules; which having deducted, the remainder is suttle, which divided by $\frac{104}{4} = 26$, and the quotient is what is to be allowed for tret, which deduct from the suttle, and the remainder is net.

1. What

1. What is the net weight of a puncheon of prunes, gross 13cwt. 1 qr. 21 lb. tare 14 lb. per cent. tret 4 lb. in 104?

		Cwt.	qrs.	lb.			
	8	13	1	21			
		1	2	20,	tare.		
104 ÷ 4	26	11	3	1,	suttle	=	1317 lb.
			1	22,	tret	=	50 lb.
		11	1	7,	net	=	1267 lb.

2. A merchant buys six hogsheads of tobacco, each containing 9 cwt. 1 qr. 14 lb. gross; tare 1 cwt. – qr. 18 lb. per hogshead; tret 4 lb. in 104; and cloff 3 lb. in every 336 gross; what will the net weight come to at $6\frac{1}{2}$d. per pond?

Cwt.	qr.	lb.		Cwt.	qrs.	lb.			lb.
9	1	14	× 6 =	56	1	–,	gross	=	6300
1	–	18	× 6 =	6	3	24,	tare	=	780
				49	1	4,	suttle	=	5520
				1	3	16,	tret	=	212
				47	1	16		=	5308
$\frac{336}{3}$ = 112) 6300 (= 56 lb.				–	2	–,	cloff	=	56
				46	3	16,	net	=	5252

		s.	d.
40	5252		
12	131	6	–
	10	18	10
	142	4	10, the answer.

Mr.

London, March 10, 1758.

Mr. JAMES DENTON,

Bought of JOHN SANDS, six casks of Barbadoes sugar.

		Cwt.	qrs.	lb.		qrs.	lb.	
N° 1	- weight	8	-	16	Tare	3	7	each.
2	- - -	7	3	20		× 6		
3	- - -	8	1	16				
4	- - -	8	-	12	Cwt. 4	3	14	
5	- - -	8	2	21				
6	- - -	8	3	23				
	Gross	50	-	24				
	Tare	4	3	14				
	Net	45	1	10, at 2l. 7s. 6d. per ct. 107l. 13s. 7d.				

Computation.

Cwt.	qr.	lb.		l.	s.	d.
45	1	10, at		2	7	6

	l.	s.	d.		
	2	7	6		
			5		
	11	17	6		
			9		
	106	17	6	price of	49 cwt.
$\frac{1}{7}$		11	$10\frac{1}{2}$	price of	$\frac{1}{4}$ cwt.
		1	$8\frac{1}{4}$	price of	4 lb.
$\frac{1}{2}$		1	$8\frac{1}{4}$	price of	4 lb.
		-	10	price of	2 lb.
	£107	13	7		

N. B. Below is the computation of the bill of parcels on the next page.

	lb.		s.	d.
20	2519, at		1	2
6	125	19		
	20	10	10	
	£ 146	18	10	

	lb.	d.
3	1621, at	4
20	540	4
	27	- 4

Mr.

London, April 3, 1758.

Mr. PETER MASON,

Bought of HENRY EUSTACE JOHNSON, Esq; for ready money, cotton 13 bags.

		Cwt.	qrs.	lb.			Cwt.	qrs.	lb.
viz. Nº 1	-	3	1	7	Nº 8	-	2	3	16
2	-	2	3	–	9	-	3	–	27
3	-	2	3	5	10	-	2	3	4
4	-	3	–	15	11	-	3	1	10
		11	3	27	Cwt.		12	1	1
		12	1	1					
		24	1	–, total gross.					
		—	3	13, tare allowed.					
		23	1	15, suttle = 2629 lb.					
				tret - 100					

		l.	s.	d.
net - 2519 lb.				
at 1 s. 2 d. per lb.	- -	146	18	10

More, viz.

	Cwt.	qrs.	lb.	
Nº 17 - -	2	2	7	damaged.
18 - -	3	2	8	
19 - -	3	1	26	
20 - -	3	–	10	
21 - -	2	3	12	
Gross	15	2	7	
Tare	—	2	2	
	15	–	5	suttle = 1685
				tret = 64
				net = 1621 lb.

at 4 d. per lb.	- - -	27	–	4
		£ 173	19	2

March 24, 1758.

Sir ANDAEW BUCKWORTH, and Company,

Bought of the United Eaſt-India Company, at 4 months,

Pepper, 2 lots, viz.

		Cwt.	qrs.	lb.		lb.			
Nº 15 - 10 bags, wt.		27	3	18	Tare	150			
19 - 10 ditto - -		24	1	24	- -	138	cwt.	qrs.	lb.
	Groſs	52	1	14	Tare	288	= 2	2	8
	Tare	2	2	8					
	Net	49	3	6	= 5578 lb.				

	l.	s.	d.
at $10\frac{3}{8}$ d. per lb. - -	241	2	$7\frac{1}{4}$

Redwood, 2 lots, viz.

	Ton.	cwt.
Nº 42 - 120 ſticks, wt.	10	$13\frac{3}{4}$
43 - 100 ditto - -	11	12
220 ſticks - wt.	22	$5\frac{3}{4}$

at 3l. 7s. per ton. - - - -	74	13	3

Wormſeed, 3 bales, viz.

	Cwt.	qrs.	lb.
Nº 16 - - - - - - - wt.	3	1	19
19 - - - - - - -	4	2	–
27 - - - - - - -	2	3	10
Groſs	10	3	1
Tare	1	–	15
Net	9	2	14 = 1078 lb.

at 1s. $1\frac{1}{2}$ d. per lb. - - -	60	12	9
	£ 376	8	$7\frac{3}{4}$

5578 lb.

```
     lb.      d.              cwt. qrs. lb.
 ½ | 5578, at 10⅜.      Or 49   3   6, at 10¾ d. per lb.
 ½ | 2789                                4
   |-----------                      -----
 ⅓ | 1394   6                         3  5½
 ¼ |  464  10                            7
 ½ |  116   2½                    ---------
   |   58   1¼                    1   4  2½ per qr.
1/20 |-----------                          4
   | 4822   7¾                    4  16 10 per cwt.
   |-----------                            7
 £241  2  7¾                     33  17 10
                                           7
                                237   4 10
                                  3  12  7½
                                      5  2¼
            l.  s.             £241   2  7¾, as before.
22 tun 5¾ cwt. at 3  7
                     7
                  23 9 –
                     3                     tun. cwt.
                  70 7 –  }           {  21   –
                   3 7 –  }           {   1   –
                   – 16 9 } price of  {   –   5
                   – 1  8 }           {   –   –½
                   – –  10}           {   –   –¼
                £74 13 3                 22   5¾

Cwt. qrs. lb.        s.  d.
 9    2   14, at 1   1½ per pound.
                     7
                  7 10½, price of 7 lb.
                     8
                 3 3 –
                     2
                 6 6 –, per cwt.
                     9
                56 14 –
                 3  3 –
                   15 9
              £60  12 9
```


The

The net proceeds of a hogshead of Barbadoes sugar were 4 l. 14 s. 6 d.; the custom and fees 2 l. 8 s. 6 d.; freight 22 s. 8 d.; factorage 4 s. 6 d.; the gross weight was 9 cwt. 94 lb.; tare $\frac{1}{10}$: pray how was the sugar rated in the bill of parcels?

	l.	s.	d.
Net proceeds	4	14	6
Custom, &c.	2	8	6
Freight - -	1	2	8
Factorage -	—	4	6
	8	10	2 = 8.5083 l.

Cwt.	qrs.	lb.	cwt.
9	3	10 =	9.83928577
		Tare	.98392857
		Net	8.8553572

8.8553572)8.50833333(.960812 = 19 s. $2\frac{1}{2}$ d. the answer.
53851185
719042
10613
1758

I have imported 80 jars of Lucca oil, each containing 1180 solid-inches; what came the freight to, at 4 s. 6 d. per cwt. tare 1 in 10, counting $7\frac{1}{2}$ lb. of oil to the wine gallon of 231 cubic inches?

$1180 \times 80 = 94400$ inches.
231)94400(408.658 gallons.
$408.658 \times 7.5 = 3064.935$ pounds.
$10 - 1 = 9$) 3064.935 gross.
340.548

112)3405.483(30.406 cwt.
4s. 6 d. = .225 l.
$30.406 \times .225 = 6.84135 = 6$ l. 18 s. $9\frac{3}{4}$ d. the answer.

CHAP.

CHAPTER VI.

FELLOWSHIP.

THE rule of fellowſhip is that by which the accompts of ſeveral partners trading in company are adjuſted, made up, or divided; ſo that every partner may have his juſt part of the gain (or loſs) in proportion to the money he hath in the joint ſtock, and to the time of its continuance therein.

SECT. I.

SINGLE FELLOWSHIP.

BY ſingle fellowſhip is adjuſted the accounts of ſuch partners that put all their ſeveral, and, perhaps, different ſums of money into one common ſtock at the ſame time; and therefore it is uſually called the rule of fellowſhip without time.

RULE.

As the whole ſtock : is to the whole gain or loſs : : ſo is every man's particular part of that ſtock : to his particular ſhare of the gain or loſs.

1. Three merchants, A, B, C, enter upon a joint adventure; A puts into the common ſtock 175 l. 13 s. 4 d.; B 117 l. 16 s. 8 d.: and C 98 l. 17 s. 7 d.; with this ſtock they trade, and gain 264 l.; I demand each merchant's ſhare of the gain?

A's ſtock	175 l.	13 s.	4 d.	=	175.66666
B's - -	117 l.	16 s.	8 d.	=	117.83333
C's - -	98 l.	17 s.	7 d.	=	98.87916
					392.37916

392.37916 l. : 264 l. : :

175.66666	:	118.191802	=	118	3	10	= A's	gain.
117.83333	:	79.280459	=	79	5	$7\frac{1}{4}$	= B's	
98.87916	:	66.527743	=	66	10	$6\frac{3}{4}$	= C's	
		£ 264. - - -	= whole gain.					

 But

But questions of the same kind with the foregoing, and those relating to bankruptcies, the readiest way of solution will be, by dividing the whole gain by the whole stock, or the bankrupt's whole estate by the sum of his debts; the quotient will be a common multiplier, or so much a pound as the bankrupt's estate will pay.

2. A merchant breaking, owes his creditors as follows:

	l.	s.	d.		l.
viz. To Mr. Trust -	3725	17	3	=	3725.8625
Mr. Credit -	7967	14	9	=	7967.7375
Mr. Gripe -	5674	12	6	=	5674.625
Mr. Covet -	967	10	$4\frac{1}{2}$	=	967.51875
Mr. Squeeze	734	6	$2\frac{1}{4}$	=	734.309375
Mr. Hard -	873	18	6	=	873.925
Mr. Near -	382	14	3	=	382.7125
Mr. Dunn -	125	16	$7\frac{1}{2}$	=	125.83125
Mr. Diffident	637	18	$6\frac{3}{4}$	=	637.928125
In all, £	21090	9	–	=	21090.45

His whole estate is 17500 l. what is each creditor's part of that in proportion to his debt?

whole debt. whole estate. s. d.

21090.45 : 17500 : : 1 : .829759441 = 16 $7\frac{1}{7}$ per l.

					l.	s.	d.	
3725.8625	× .829759441 =	3091.569586	=	3091	11	$4\frac{3}{4}$	T.
7967.7375		6611.305415	=	6611	6	$1\frac{1}{4}$	C.
5674.625		4708.573669	=	4708	11	$5\frac{3}{4}$	G.
967.51875		802.807817	=	802	16	$1\frac{3}{4}$	C.
734.309375		609.300137	=	609	6	–	Sq.
873.925		725.147520	=	725	2	$11\frac{1}{2}$	H.
382.7125		317.559311	=	317	11	$2\frac{1}{4}$	N.
125.83125		104.409669	=	104	8	$2\frac{1}{4}$	D.
637.928125		529.326885	=	529	6	6	D.
		17500.001802	=	17500	–	–	

In cases of bankruptcy, when there are many creditors, first find what the bankrupt's estate will pay in the pound; and then each particular part may be found by the rule of practice, very near the truth: and here note, that the small redundancy in the larger sums, in this question, is owing to 16 s. $7\frac{1}{7}$ d. being taken a small matter more than the bankrupt's

rupt's estate would allow; and the deficiency in the smaller ones to the fraction of a farthing omitted.

Trust.

	l.	s.	d.
½	3725	17	3
½	1862	18	7½
⅕	931	9	3¾
½	180	5	10¼
⅙	93	2	11
⅐	15	10	5¾
	2	4	4¼
£	3091	11	6½

Credit.

	l.	s.	d.
½	7967	14	9
½	3983	17	4½
⅕	1991	18	8¼
½	398	7	8¾
⅙	199	3	10¼
⅐	33	3	11½
	4	14	10
£	6611	6	5¼

Gripe.

	l.	s.	d.
½	5674	12	6
½	2837	6	3
⅕	1418	13	1½
½	283	14	7½
⅙	141	17	3¾
⅐	23	12	10½
	3	7	6½
£	4708	11	8¾

Covet.

	l.	s.	d.
½	967	10	4½
½	483	15	2¼
⅕	241	17	7
½	48	7	6
⅙	24	3	9
⅐	4	–	7½
	–	11	6
£	802	16	1¾

Squeeze.

	l.	s.	d.
½	734	6	2¼
½	367	3	1
⅕	183	11	6½
½	36	14	3½
⅙	18	7	1¾
⅐	3	1	2¼
	–	8	8¾
£	609	5	11¾

Hard.

	l.	s.	d.
½	873	18	6
½	436	19	3
⅕	218	9	7½
½	43	13	11
⅙	21	16	11½
⅐	3	12	9¾
	–	10	4¾
£	725	2	11½

Near.

	l.	s.	d.
½	382	14	3
½	191	7	1½
⅕	95	13	6¾
½	19	2	8½
⅙	9	11	4¼
⅐	1	11	10½
	–	4	6½
£	317	11	2

Dunn.

	l.	s.	d.
½	125	16	7½
½	62	18	3¾
⅕	31	9	1¾
½	6	5	9¾
⅙	3	2	10¾
⅐	–	10	5¾
	–	1	5¾
£	104	8	1½

Diffident.

	l.	s.	d.
½	637	18	6¾
½	318	19	3¼
⅕	159	9	7½
½	31	17	11
⅙	15	18	11½
⅐	2	13	1¾
	–	7	7
£	529	6	6

As there is no general rule for ſolving all queſtions that may occur or be propoſed in partnerſhip, or other branches of trade, the anſwer muſt be found by the ingenuity of the arithmetician; who, by this time, may be ſuppoſed to be pretty well grounded in figures.

3. A hath in ſtock 35 l. B 20 l.; they trade and gain 40 l. and agree that it ſhall be divided ſo, that A is to have 10 per cent. and B only 8; what muſt each have of the gain?

35 l. at 10 per cent. is 3.5 l. } their ſum 5.1 l.
20 l. at 8 per cent. is 1.6 l. }

		l.	s.	d.		
∵ 5.1 : 40 :: 3.5 : 27.45098	=	27	9	$-\frac{1}{4}$	= A's	share.
5.1 : 40 :: 1.6 : 12.54902	=	12	10	$11\frac{3}{4}$	= B's	

4. A, B, and C put in money together; A puts in 20 l.; B and C put in 85 l.; they gained 63 l. of which B took up 21 l.; what did A and C gain, and B and C put in?

A 20 } ſum 105.
B + C 85 }

105 : 63 :: 20 : 12, A's } gain } Which
12 + 21 = 33, and 63 − 33 = 30, C's } } were to
And 63 : 105 :: 21 : 35, B's } ſtock. } be found.
63 : 105 :: 30 : 50, C's }

5. Some others advance in trade as follow, viz. W, X, and Y raiſed 350 l. 10 s.; W, X, and Z 344 l. 10 s.; X, Y, and Z made up together 400 l.; and W, Y, and Z contributed 378 l. 4 s. In the concluſion they parted with their joint property for 450 guineas; what did they gain or loſe by their adventure?

	l.	s.	d.	
W, X, Y	350	10		each partner being mentioned three times.
W, X, Z	344	10		
X, Y, Z	400	−		
W, Y, Z	378	4		
3)	1473	4		
	491	1	4,	the whole ſtock.
20)	450 guineas.			
	22	10		
	472	10		made of their joint property.

Then 491 l. 1 s. 4 d. − 472 l. 10 s. = 18 l. 11 s. 4 d. loſs, the anſwer.

6. A,

6. A, B, and C put in trade 360 l. and gained 270 l.; of which as often as A took up 3 l. B took up 5 l.; and as often as B took up 5 l. C took up 7 l.; what did each gain and put in?

$$\begin{array}{r} 3 \\ +5 \\ +7 \\ \hline \end{array}$$

15)360(24, the common multiplier for the ſtock.

$$\left.\begin{array}{l} 3 \times 24 = 72 = \text{A's} \\ 5 \times 24 = 120 = \text{B's} \\ 7 \times 24 = 168 = \text{C's} \end{array}\right\} \text{ſtock.}$$

15) 270 (18, the common multiplier for the gain.

$$\text{and } \left.\begin{array}{l} 3 \times 18 = 54 = \text{A's} \\ 5 \times 18 = 90 = \text{B's} \\ 7 \times 18 = 126 = \text{C's} \end{array}\right\} \text{gain.}$$

7. A, B, and C put in money together; A puts in 20 l.; B 30 l.; C a ſum unknown: they gained 36 l. where of C took 16 l.; what did A and B gain, and C put in?

$$\begin{array}{r} 20 \\ +\ 30 \\ \hline \end{array} \qquad \begin{array}{r} 36 \\ -\ 16 \\ \hline \end{array}$$

$$\begin{array}{l} 50 : 20 :: 20 : 8, \text{ A's gain.} \\ 20 - 8 = 12, \text{ B's gain.} \\ 8 : 20 :: 16 : 40, \text{ C's ſtock.} \end{array}$$

8. A, B, C and D put in money together, and gained a ſum of money, of which A, B and C took 60 l.; B, C, and D took 90 l.; A, C, and D took 80 l.; and A, B, and D took up 70 l.; what diſtinct gain did each take up?

$$\begin{array}{lr} & \text{l.} \\ \text{A, B, C} & 60 \\ \text{B, C, D} & 90 \\ \text{A, C, D} & 80 \\ \text{A, B, D} & 70 \\ \hline & 3)300 \\ \hline \end{array}$$

$$\left.\begin{array}{l} \qquad\quad \text{l.} \\ 100 - 90 = 10 = \text{A's} \\ 100 - 80 = 20 = \text{B's} \\ 100 - 70 = 30 = \text{C's} \\ 100 - 60 = 40 = \text{D's} \end{array}\right\} \text{gain, the anſwer.}$$

9. A

9. A and B clear by an adventure at sea 50 guineas, with which they agreed to buy a horse and chaise; whereof they were to have the use, in proportion to the sums adventured, which was found to be A 10 : to B 7; they cleared 45 l. per cent. what money did they each send abroad?

50 guineas = 52 l. 10 s.; and 10 + 7 = 17.

45 : 100 : : 52.5 : $116.\dot{6}$ = 116 l. 13 s. 4 d. whole sum.		
17 : 10 : : $116\dot{6}$: 68.6274 = 68 l. 12 s. $6\frac{1}{2}$d.	A's	stock.
17 : 7 : : $116\dot{6}$: 48.0392 = 48 l. – s. $9\frac{1}{2}$d.	B's	

10. A father divided his fortune amongst his sons, giving A 7, as often as B 4; to C he gave as often 2, as to B 5; and yet the dividend of C came to $2166\frac{3}{8}$ l.: what was the value of the whole legacy?

		l.	s.	d.			l.	s.	d.		
C.						B.					
2	:	2166	7	6	: : 5 :		5415	18	9,	B's	share.
B.		l.	s.	d.		A.					
4	:	5415	18	9	: 7 :		9477	17	$9\frac{3}{4}$,	A's	
		and, as above,					2166	7	6,	C's	
							17060	4	$-\frac{3}{4}$,	the answer.	

11. Part 1500 acres of land; give B 72 more than A, and C 112 more than B.

B - - - - 72 } more than A.
C 72 + 112 = 184 }

sum 256

Then 1500 — 256 = 1244

3) 1244 ($414\frac{2}{3}$ = A's }
Also $414\frac{2}{3}$ + 72 = $486\frac{2}{3}$ = B's } share.
And $486\frac{2}{3}$ + 112 = $598\frac{2}{3}$ = C's }

12. Divide 1000 crowns; give A 129 more than B, and B 178 fewer than C.

1000
129 + 178 = 307

3) 693 (231 = B's.
231 + 129 = 360 = A's.
231 + 178 = 409 = C's.

13. Part

13. Part 250 l. give A 37 more than B, and let C have 28 fewer than B.

First, $37 - 28 = 9$ - - Also $250 - 9 = 241$.

$$\left.\begin{array}{r} 3)241(\ 80\frac{1}{3} = \text{B's} \\ 80\frac{1}{3} + 37 = 117\frac{1}{3} = \text{A's} \\ 80\frac{1}{3} - 98 = \ 52\frac{1}{3} = \text{C's} \end{array}\right\} \text{part.}$$

14. In an article of trade, A gains 14 s. 6 d. and his adventure was 35 s. more than B's, whose share of profit is but 8 s. 6 d.; what are the particulars of their stock?

First, 14 s. 6 d. — 8 s. 6d. = 6s. difference of their gain.

$$\begin{array}{l} \quad\ \ \text{s.} \quad \text{s.} \qquad \text{s.} \qquad \text{s.} \qquad\quad \text{l. s. d.} \\ \text{Then as } 6 : 35 :: 14.5 : 84.583 = 4\ 4\ 7 = \text{A's} \\ \qquad\qquad 6 : 35 :: 8.5 : 49.583 = 2\ 9\ 7 = \text{B's} \end{array}\Big\} \text{stock.}$$

15. Three persons entered joint trade, to which A contributed 210 l. B 312 l.; they clear 140 l. whereof 37 l. 10 s. belongs of right to C; that person's stock, and the several gains of the other two are required?

$210\text{l.} + 312 = 522\text{l.} = \text{A's stock} + \text{B's}.$

$140\text{l.} - 37.5 = 102.5\text{l.} = \text{A's gain} + \text{B's}.$

As $102.5 : 522 :: 37.5 : 190.\cancel{0}759\cancel{0}\text{l.} = 190\text{l. } 19\text{s. } \frac{6}{41}$, C's share.

$210\text{l.} + 312 + 190.\cancel{0}756\cancel{0} = 712.\cancel{0}756\cancel{0}$, whole stock.

$$\begin{array}{l} 712.\cancel{0}756\cancel{0} : \\ 140 :: \end{array}\left\{\begin{array}{l} 210 : 41.2357 = 41\text{l. } 4\text{s. } 8\frac{1}{2}\text{d.} + \text{A's} \\ 312 : 61.2643 = 61\text{l. } 5\text{s. } 3\frac{1}{2}\text{d.} - \text{B's} \end{array}\right\} \text{gain.}$$

16. A and B venturing equal sums of money, clear by joint trade 154 l.; by agreement A was to have 8 per cent. because he spent time in execution of the project; and B was only to have 5; the question is, what was allotted A for his trouble?

$$\begin{array}{l} \qquad\qquad\qquad\qquad\qquad\quad \text{l.} \quad \text{s.} \quad \text{d.} \\ \text{As } 8 + 5 = 13 : 8 :: 154 : 94\ \ 15\ \ 4\frac{8}{13},\ \text{A's} \\ \qquad\qquad\quad 13 : 5 :: 154 : 59\ \ \ 4\ \ \ 7\frac{5}{13},\ \text{B's} \end{array}\Big\} \text{gain.}$$

Answer, £ 35 10 $9\frac{3}{13}$

17. A, B, and C play a concert at hazard, and making up accompts, it appears that A and B together brought off

13 l.

13 l. 10 s.; B and C together 12 l. 12 s.; and A and C together won 11 l. 16 s. 6 d.; what did they severally get?

13 l. 10 s. + 12 l. 12 s. + 11 l. 16 s. 6 d. = 37 l. 18 s. 6 d.

2)37 18 6

				l.	s.	d.		l.	s.	d.	
18	19	3	—	12	12	–	=	6	7	3,	A's share.
18	19	3	—	11	16	6	=	7	2	9,	B's share.
18	19	3	—	13	10	–	=	5	9	3,	C's share.

18. A, B, and C, are three horses belonging to different men, and are employed as a team to draw a load of wheat from Hertford for 30 s.; A and B are deemed to do $\frac{2}{7}$ of the work; A and C $\frac{3}{8}$; and B and C $\frac{3}{10}$ of it: they are to be paid proportionably, and you know how to divide it as it should be.

$\frac{2}{7} = \frac{16}{56} = \frac{160}{560}$, and $\frac{3}{8} = \frac{21}{56}$.

$\frac{21}{56} - \frac{16}{56} = \frac{5}{56} =$ C's share more than B's.

$\frac{3}{8} = \frac{30}{80}$, and $\frac{3}{10} = \frac{24}{80}$.

$\frac{30}{80} - \frac{24}{80} = \frac{6}{80} = \frac{42}{560} =$ A's share more than B's.

$\frac{160}{560} - \frac{42}{560} = \frac{118}{560}$, $\frac{2}{1}\Big)\frac{118}{560}\Big(\frac{59}{56} =$ B's.

$\frac{59}{560} + \frac{42}{560} = \frac{101}{560} =$ A's, and $\frac{59}{560} + \frac{50}{560} = \frac{109}{560} =$ C's.

Then rejecting the common denominator,

101 + 59 + 109 = 269, sum of the numerators.

						s.		s.	d.		
269	:	30	::	101	:	11		$3\frac{45}{269}$		= A's share of the money.
269	:	30	::	59	:	6		$6\frac{258}{269}$		= B's share of the money.
269	:	30	::	109	:	12		$1\frac{235}{269}$		= C's share of the money.

£ 1 10 –, proof.

19. Three persons purchase together a West-India sloop, towards the payment whereof A advanced $\frac{1}{8}$, B $\frac{3}{7}$, and C 140 l.;

140l.; how much paid A and B, and what part of the vessel had C?

First $\frac{3}{8} = \frac{21}{56}$; and $\frac{3}{7} = \frac{24}{56}$. Then $\frac{21}{56} + \frac{24}{56} = \frac{45}{56}$, A + B's part.

$$\frac{56}{56} - \frac{45}{56} = \frac{11}{56} = \text{C's part of the vessel.}$$

$$\left.\begin{array}{l} \frac{11}{56} : \frac{140}{1} :: \frac{3}{8} : \frac{2940}{11} = 267 \quad 5 \quad 5\frac{1}{4}\ \frac{9}{11},\ \text{A} \\ \frac{11}{56} : \frac{140}{1} :: \frac{3}{7} : \frac{3360}{11} = 305 \quad 9 \quad 1 \quad \frac{4}{11},\ \text{B} \\ \text{and } 140 \quad - \quad - \end{array}\right\} \text{advanced.}$$

Whole cost, £ 712 14 $6\frac{1}{2}\ \frac{2}{11}$

20. A, B, and C have 100 l. to be divided amongst them in such manner, that two times A's share be equal to three times B's share; and four times B's share equal to five times C's.

Here it is plain, that A gets 3 l. to B's 2 l.
And that B get's 5 l. to C's 4 l.
As $5 : 4 :: 2 : 1\frac{3}{5} = 1\ 6$, to B's 2 l.
Therefore their shares are, A 3l. to B's 2l. and C's 1.6
And 3l. + 2 + 1.6 = 6.6, sum of those parts.

$$\left.\begin{array}{llll} & & & \text{l.} \quad \text{s.} \quad \text{d.} \\ \text{As } 6\ 6 : 100 :: 3 & : 45.45 & = & 45 \quad 2 \quad 1 \\ \text{Alſo } 6.6 : 100 :: 2 & : 30.30 & = & 30 \quad 6 \quad -\frac{3}{4} \\ \text{And } 6.6 : 100 :: 1.6 & : 24.24 & = & 24 \quad 4 \quad 10\frac{1}{4} \end{array}\right\} \text{fere} \left\{\begin{array}{l} = \text{A's} \\ = \text{B's} \\ = \text{C's} \end{array}\right\} \text{share.}$$

21. A and B join their stock, and vest them in brandies; A's stock was 19 l. 19 s. 8 d. more than that of B; now by selling out their commodity at 55 s. per anker, A cleared 74 l. 11 s. and B just 50 guineas; the quantity of brandy dealt for is required, and the gain upon the anker?

$$\begin{array}{ll} \text{l.} & \text{s.} \\ 74 & 11 \\ 52 & 10 \end{array} \left.\begin{array}{l} \\ \text{A} \\ \text{B} \end{array}\right\} \text{gained.}$$

£ 22 1, difference of their sums.

22l. 1s. : 19l. 19s. 8d. :: 127l. 1s. sum : 115l. 1s, 11d. cost.

$$\begin{array}{lll} 127 & 1 & = 2541 \\ 115 & 1\ 11 & = 2301\ \text{11d.} \end{array}$$

55) 4842 (88 ankers, and 2s. 11d. over.

s.
88) 127 1(1l. 8s. $7\frac{3}{4}$d. gain per anker.

22. In raising a joint stock of 400l. A advances $\frac{4}{13}$; B $\frac{12}{11}$ of $\frac{3}{8}$; C $\frac{1}{6}$ more; the difference between A's adventure and B's, and D the rest of the money; what did every one subscribe?

$$\frac{4}{13} = \frac{264}{858} = 123\text{l. 1s. 6d. }\frac{396}{858} = \text{A's}$$

$$\frac{\cancel{12}^{3}}{11} \text{ of } \frac{3}{\cancel{8}_{2}} = \frac{9}{22} = \frac{351}{858} = 163\text{l. 12s. 8d. }\frac{624}{858} = \text{B's}$$

$$\frac{351}{858} - \frac{264}{858} = \frac{87}{858}.$$

$$\frac{1}{6} \text{ or } \frac{143}{858} + \frac{87}{858} = \frac{230}{858} = 107\text{ l. 4s. 6d. }\frac{228}{858} = \text{C's}$$

$$\frac{264}{858} + \frac{351}{858} + \frac{230}{858} = \frac{845}{858}.$$

$$\frac{858}{858} - \frac{845}{858} = \frac{13}{858} = 6\text{l. 1s. 2d. }\frac{461}{858} = \text{D's}$$

share.

23. A father devised $\frac{34}{83}$ of his estate to one of his sons, and $\frac{34}{83}$ of the residur to another, and the surplus to his relict for life; the children's legacies were found to be 257l. 3s. 4d. different $= \frac{1543}{6}$; pray what money did he leave the widow the use of?

Whole estate $\frac{83}{83} - \frac{34}{83} = \frac{49}{83}$. Then $\frac{34}{83} = \frac{2822}{6889}$, and $\frac{34}{83}$ of $\frac{49}{83} = \frac{1666}{6889}$.

And $\frac{2822}{6889} - \frac{1666}{6889} = \frac{1156}{6889} = 257$ l. 3 s. 4 d. Also $\frac{2822}{6889} + \frac{1666}{6889} = \frac{4483}{6889}$.

Then $\frac{6889}{6889} - \frac{4488}{6889} = \frac{2401}{6889} =$ widow's part of the estate.

As

As $\frac{1156}{6889} : \frac{1543}{6} :: \frac{2822}{6889} : \frac{2177170}{3468} =$ 627 l. 15 s. 9 d. $1\frac{187}{289}$ eldest son.

As $\frac{1156}{6889} : \frac{1543}{6} :: \frac{1666}{6889} : \frac{1285319}{3468} =$ 370 l. 12 s. 5 d. $1\frac{187}{289}$. youngest.

As $\frac{1156}{6889} : \frac{1543}{6} :: \frac{2401}{6889} : \frac{3704743}{6936} =$ 534 l. 2 s. 8 d. (nearly) widow.

24. A father, ignorant in numbers, ordered 500 l. to be divided among his five sons, thus: Give A, says he, $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, D $\frac{1}{6}$, and E $\frac{1}{7}$; part this equitably among them, according to the father's intention.

First $\frac{1}{3} = \frac{140}{420}$, $\frac{1}{4} = \frac{105}{420}$, $\frac{1}{5} = \frac{84}{420}$, $\frac{1}{6} = \frac{70}{420}$, and $\frac{1}{7} = \frac{60}{420}$.

Then $\frac{140}{420} + \frac{105}{420} + \frac{84}{420} + \frac{70}{420} + \frac{60}{420} = \frac{459}{420}$, their sum.

	l.	s.	d.	farth.	
As $\frac{459}{420} : \frac{500}{1} :: \frac{140}{420}$:	152	10	1	$1\frac{105}{459}$	= A's.
$\frac{459}{420} : \frac{500}{1} :: \frac{105}{420}$:	114	7	6	$3\frac{423}{459}$	= B's.
$\frac{459}{420} : \frac{500}{1} :: \frac{84}{420}$:	91	10	-	$3\frac{43}{459}$	= C's.
$\frac{459}{420} : \frac{500}{1} :: \frac{70}{420}$:	76	5	-	$2\frac{282}{459}$	= D's.
$\frac{459}{420} : \frac{500}{1} :: \frac{60}{420}$:	65	7	2	$1\frac{45}{459}$	= E's
£	500	-	-		

25. A in a scuffle seized on $\frac{2}{3}$ of a parcel of sugar-plums; B catched $\frac{3}{8}$ of them in his hands; and C laid hold on $\frac{3}{10}$: D ran off with all A had left, except $\frac{1}{7}$, which E afterwards secured slily for himself: then A and C jointly set upon B, who in the conflict shed half he had, which were equally picked up by D and E, who lay perdue. B then kicked down C's hat, and to work they all went a-new for what it contained; of which a got $\frac{1}{4}$, B $\frac{1}{3}$, D $\frac{2}{7}$, and C and E equal shares

ſhares of what was left of that ſtock. D then ſtruck $\frac{3}{4}$ of what A and B laſt acquired out of their hands; they with difficulty recovered $\frac{5}{8}$ of it in equal ſhares again, but the other three carried off $\frac{1}{8}$ a-piece of the ſame. Upon this they called a truce, and agree that the $\frac{1}{3}$ of the whole left by A at firſt ſhould be equally divided amongſt them; how much of the prize, after this diſtribution, remained with each of the competitors?

Though A at the firſt ſeized $\frac{2}{3}$, he loſt all again this heat.

$$\left.\begin{array}{l} \frac{3}{8} \text{ of } \frac{2}{3} = \frac{1}{4} = \text{B's.} \\ \frac{3}{10} \text{ of } \frac{2}{3} = \frac{1}{5} = \text{C's} \end{array}\right\} \text{firſt acquiſition. Their ſum} = \frac{9}{20}.$$

$$\frac{2}{3} - \frac{9}{20} = \frac{13}{60} = \frac{91}{420}.$$

$$\left.\begin{array}{l} \frac{1}{7} \text{ of } \frac{13}{60} = \frac{13}{420} = \text{E's.} \\ \text{and } \frac{91}{420} - \frac{13}{420} = \frac{78}{420} = \text{D's.} \end{array}\right\} \text{firſt acquiſition.}$$

Thus ended the firſt heat.

$$\left.\begin{array}{l} \text{Again, } \frac{1}{2} \text{ of } \frac{1}{4} = \frac{1}{8} = \text{B's} \\ \text{Retained - - } \frac{1}{5} = \text{C's} \\ \frac{13}{70} + \frac{1}{16} -- = \frac{139}{460} = \text{D's} \\ \frac{13}{420} + \frac{1}{16} - = \frac{157}{1680} = \text{E's.} \end{array}\right\} \text{part at the end of the ſecond ſcuffle.}$$

$$\left.\begin{array}{l} \text{Proceeding, } \frac{1}{4} \text{ of } \frac{1}{5} = \frac{1}{20} = \text{A's} \\ \frac{1}{3} \text{ of } \frac{1}{5} + \frac{1}{8} = \frac{23}{120} = \text{B's} \\ \frac{2}{7} \text{ of } \frac{1}{5} + \frac{139}{560} = \frac{171}{560} = \text{D's} \end{array}\right\} \text{part after the third ſmuſs.}$$

$$\text{Then } \frac{1}{20} + \frac{1}{15} + \frac{2}{35} = \frac{73}{420}.$$

$$\left.\begin{array}{l} \frac{1}{5} - \frac{73}{420} = \frac{11}{420}, \text{ and } \frac{1}{2} \text{ of } \frac{11}{420} = \frac{11}{840} = \text{C's} \\ \frac{157}{1680} + \frac{11}{840} \text{ - - - - } = \frac{179}{1680} = \text{E's} \end{array}\right\} \text{part of the third ſmuſs.}$$

Further,

Further $\frac{1}{20}+\frac{1}{15}=\frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{60}=\frac{7}{80}$, lost by A and B.

$$\left.\begin{array}{r}
\frac{5}{16}\text{ of }\frac{7}{80}+\frac{1}{4}\text{ of }\frac{1}{20}=\frac{51}{1280}=\text{A's}\\
\frac{5}{16}\text{ of }\frac{7}{80}+\frac{1}{4}\text{ of }\frac{1}{15}\text{ of }\frac{1}{8}=\frac{649}{3840}=\text{B's}\\
\frac{1}{8}\text{ of }\frac{7}{80}+\frac{11}{840}=\frac{323}{13440}=\text{C's}\\
\frac{1}{8}\text{ of }\frac{7}{80}+\frac{171}{840}=\frac{1417}{4480}=\text{D's}\\
\frac{1}{8}\text{ of }\frac{7}{80}+\frac{170}{1680}=\frac{1579}{13440}=\text{E's}
\end{array}\right\}\text{part after the last scuffle.}$$

$$\left.\begin{array}{r}
\text{Then }\frac{51}{1280}+\frac{1}{15}=\frac{2863}{26880}=\text{A's}\\
\frac{649}{3840}+\frac{1}{15}=\frac{6335}{26880}=\text{B's}\\
\frac{323}{13440}+\frac{1}{15}=\frac{2438}{26880}=\text{C's}\\
\frac{1417}{4480}+\frac{1}{15}=\frac{10294}{26880}=\text{D's}\\
\frac{1579}{13440}+\frac{1}{15}=\frac{4950}{26880}=\text{E's}
\end{array}\right\}\text{share carried off at the last,}$$

So that if the number of sugar plumbs were 26880.

$$\left.\begin{array}{lr}
\text{A got - -} & 2863\\
\text{B - - - - -} & 6335\\
\text{C - - - -} & 2438\\
\text{D - - -} & 10294\\
\text{E - - -} & 4950
\end{array}\right\}\text{sum 26880.}$$

26. If A, having $\frac{7}{8}$ of $\frac{3}{5}$ of the half of a trading sloop and cargo worth 16131 l. 14 s. sells his brother B $\frac{3}{5}$ of $\frac{4}{5}$ of his interest therein at prime cost; what did it cost the brother, and what did his cousin P pay at the same time for $\frac{9}{11}$ of the remainder?

$\frac{7}{8}$ of $\frac{3}{5}$ of $\frac{1}{2}$ of $16131.7=4234.57125$ l. $=4234$ l. 11 s. 5d. A at first.

$\frac{3}{5}$ of $\frac{4}{5}$ of $4234.57125=2032.5942=2032$ l. 11 s. $10\frac{1}{2}$d. sold B.

$16131.7-4234.57125=11897.12875$, remainder.

$\frac{9}{11}$ of $11897.12875=9734.01443=9734$ l. – s. $3\frac{1}{2}$d. cousin P.

27. Two merchants company, A put in 20 l. and B put in 135 ducats; they gain 67 l. 10 s. of which A took 30 l. what is the value of a ducat?

67 l. 10 s. — 30 l. = 37 l. 10 s. = B's gain.

30 : 20 : : 37,5 : 25 l. B's stock = 135 ducats.

135) 25 = 500 s. (3 s. $8\frac{4}{9}$d. value of a ducat, the answer.

28. Three merchants, A, B and C, freight ships to Lisbon with sugar, to the value of 15778 l. 2 s. 6 d. sterling. A bought 250 cwt. 1 qr. 22 lb. at 2 l. 16 s. per cwt. B paid 2 l. 6 s. 8 d. per cwt. for his; but meeting with a storm at sea, the mariners were constrained, for the safety of their lives, to cast out part of the ship's lading. A's proportion ejected part was $\frac{1}{100}$th part of the ship's lading, and $3\frac{3}{4}$ times the quantity cast over board, was $3\frac{1}{7}$ times the whole freight of A and B. When they came to land, A sold his remaining part for 4 l. 4 s per cwt. and found himself a loser 10 per cent. besides charges. B advanced the remaining part of his commodity 20 per cent. and C gained 4 s. 8 d. per cwt. by the quantity he saved. Quere, What did each merchant lose by this voyage, the charge of the same amounting to 500 guineas? *By Mr. Flower.*

1 cwt. : 2 l. 16 s. $= \frac{14}{5}$: : 250 cwt. 1 qr. 22 lb. $= \frac{14025}{56}$: 701 l. 5 s. A's cost besides charges.

$\frac{100}{1} : \frac{10}{1}$: : 701 l. 5 s. $= \frac{2805}{4} : \frac{561}{8}$ = 70 l. 2 s. 6 d. A's loss besides charges.

701 l. 5 s. — 70 l. 2 s. 6 d. = 631 l. 2 s. 6 d. $= \frac{5049}{8}$, value of A's remaining part.

4 l. 4 s. $= \frac{21}{5} : 1 : : \frac{5049}{8} : \frac{8415}{56}$ = 150 cwt. 1 qr. 2 lb. A's remaining part.

250 cwt. 1 qr. 22 lb. — 150 cwt. 1 qr. 2 lb. = 100 cwt. 20 lb. $= \frac{2805}{28}$ A's ejected part.

$\frac{2805}{28} \times 100 = \frac{70125}{7}$ = 10017 cwt. 3 qrs. 12 lb. whole cargo.

$\frac{14025}{56} : \frac{2805}{28} : : \frac{70125}{7}$: 40017 cwt. 16 lb. whole ejected part $= \frac{28050}{7}$ $\frac{10}{3}$

$\frac{10}{3} : \frac{15}{4} :: \frac{28050}{7} : \frac{126225}{28} = 4508$ cwt. 4 lb. A's cargo + B's.

4508 cwt. 4 lb. — 250 cwt. 1 qr. 22 lb. = 4257 cwt. 2 qrs. 10 lb. $= \frac{238425}{56}$ B's cargo.

10017 cwt. 3 qrs. 12 lb. — 4508 cwt. 4 lb. = 5509 cwt, 3 qrs. 8 lb. $= \frac{154275}{28}$ C's cargo.

$\frac{70125}{7} : \frac{28050}{7} :: \frac{238425}{56} : \frac{47685}{28} = 1703$ cwt. 4 lb. B's ejected part.

100 cwt. 20 lb. + 1703 cwt. 4 lb. = 1803 cwt. 24 lb. A's + B's.

4007 cwt. 16 lb. — 1803 cwr. 24 lb. = 2203 cwt. 20 lb. C's ejected part.

$1 : \frac{7}{3} :: \frac{238425}{56} : \frac{556325}{56} = 9934$ l. 7 s. 6 d. B's cost, besides charges.

701 l. 5 s. + 9934 l. 7 s. 6 d. = 10635 l. 12 s. 6 d. A's + B's cost.

15778 l. 2 s. 6 d. — 10635 l. 12 s. 6 d. = 5142 l. 10 s. C's cost, besides charges.

4257 cwt. 2 qrs. 10 lb. — 1703 cwt. 4 lb. = 2554 cwt. 2 qrs. 6 lb. $= \frac{143055}{56}$, B's remainder.

5509 cwt. 3 qrs. 8 lb — 2203 cwt. 2 lb. = 3305 cwt. 3 qrs. 16 lb. $= \frac{92565}{28}$, C's remainder.

$1 : \frac{7}{3} :: \frac{143055}{56}$: 5960 l. 12 s. 6 d. value of B's remaining part at prime cost.

$\frac{100}{1} : \frac{120}{1} :: \frac{47685}{8} : \frac{57222}{8} = 7152$ l. 15 s. advanced, value of B's.

9934 l. 7 s. 6 d. — 7152 l. 15 s. = 2781 l. 12 s. 6 d. B's loss, charge excepted.

$1 : \frac{30}{7} :: \frac{92565}{28} : \frac{6171}{8} = 771$ l. 7 s. 6 d. C gained by what he saved.

$\frac{154275}{28}$: 5142 l. 10 s. $= \frac{10285}{2} :: \frac{92565}{28}$: 3085 l. 10 s. value of C's remainder.

3085 l. 10 s. + 771 l. 7 s. 6 d. = 3856 l. 17 s. 6 d. advanced, value of C's.

5142 l. 10 s. − 3856 l. 17 s. 6 d. = 1285 l. 12 s. 6 d. C's loſs, beſides charges.

$$\left.\begin{array}{l}
\frac{70125}{7} : \frac{525}{1} :: \frac{14025}{56} : \frac{105}{8} = 13\text{ l. } 2\text{ s. } 6\text{ d. A's} \\
\frac{14025}{56} : \frac{105}{8} :: \frac{238425}{56} : \frac{1785}{8} = 223\text{ l. } 2\text{ s. } 6\text{ d. B's} \\
13\text{ l. } 2\text{ s. } 6\text{ d.} + 223\text{ l. } 2\text{ s. } 6\text{ d.} = 236\text{ l. } 5\text{ s. - d.} \\
525\text{ l.} - 236\text{ l. } 5\text{ s. - d.} = 288\text{ l. } 15\text{ s. - d. C's}
\end{array}\right\}\text{part of the charges.}$$

$$\left.\begin{array}{l}
70\text{ l. } 2\text{ s. } 6\text{ d.} + 13\text{ l. } 2\text{ s. } 6\text{ d.} = 83\text{ l. } 5\text{ s. - d. A's} \\
2781\text{ l. } 12\text{ s. } 6\text{ d.} + 223\text{ l. } 2\text{ s. } 6\text{ d.} = 3004\text{ l. } 15\text{ s.} = \text{B's} \\
1285\text{ l. } 12\text{ s. } 6\text{ d.} + 288\text{ l. } 15\text{ s.} = 1574\text{ l. } 7\text{ s. } 6\text{ d.} = \text{C's}
\end{array}\right\}\text{whole loſs.}$$

29. There were at a feaſt 20 men and 30 women, and 15 ſervants, who ſpent 24 l. and for every 10 s. that a man paid, a woman paid 6 s. and a ſervant 2 s.; what did each perſon pay?

20 × 10 = 200, 30 × 6 = 180, and 15 × 2 = 30, their ſum 410.

$$\left.\begin{array}{llll}
 & \text{l.} & \text{s.} & \text{d.} \\
410 : 24 :: 200 : 11.\cancel{7}073\cancel{1} = & 11 & 14 & 1\tfrac{3}{4}\text{, men} \\
410 : 24 :: 180 : 10.\cancel{5}365\cancel{8} = & 10 & 10 & 8\tfrac{3}{4}\text{, women} \\
410 : 24 :: 30 : 1.\cancel{7}560\cancel{9} = & 1 & 15 & 1\tfrac{1}{2}\text{, ſervants}
\end{array}\right\}\text{pay.}$$

20)11 l. 14 s. $1\frac{3}{4}$ d.(11 s. $8\frac{1}{2}$ d. nearly each man.
30)10 l. 10 s. $8\frac{3}{4}$ d.(7 s. -$\frac{1}{4}$ d. $\frac{1}{6}$, each woman.
15) 1 l. 15 s. $1\frac{1}{2}$ d.(2 s. 4 d. $\frac{2}{3}$, each ſervant.

30. It is propoſed to divide 300 l. amongſt three perſons, ſo that A gets 6 l. more than $\frac{1}{2}$, B 12 l. more than $\frac{1}{3}$, and C 8 l. leſs than $\frac{2}{3}$; what is the ſhare of each?

According to the moſt obvious meaning of this queſtion, the ſolution is as follows:

$\frac{1}{2}$ of 300 l. = 150 l. $\frac{1}{3}$ of 300 l. = 100 l. and $\frac{2}{3}$ of 300 l. = 200 l.

Alſo 150 l. + 6 = 156; 100 + 12 = 112; and 200 − 8 = 192.

And 156 + 112 + 192 = 460, their ſum.

$$\left.\begin{array}{l}
\because 460 : 300 :: 156 : 101\text{ l. } 14\text{ s. } 9\text{ d. }\tfrac{9}{23} = \text{A's} \\
\text{Alſo } 460 : 300 :: 112 : 73\text{ l. - s. } 10\text{ d. }\tfrac{10}{23} = \text{B's} \\
\text{And } 460 : 300 :: 192 : 125\text{ l. } 4\text{ s. } 4\text{ d. }\tfrac{4}{23} = \text{C's}
\end{array}\right\}\text{ſhare.}$$

Others

Others taking the queſtion in a different ſenſe, ſolve it thus:

$$6 + 12 - 8 = 10;\ \text{and}\ 300 - 10 = 290.$$

Then $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$; and $\frac{2}{3} = \frac{4}{6}$; their ſum $\frac{9}{6}$.

$\frac{9}{6} : \frac{290}{1} :: \frac{3}{6} : 96\frac{6}{9} + 6 = 102$ l. 13 s. 4 d. for A.
$\frac{9}{6} : \frac{290}{1} :: \frac{2}{6} : 64\frac{4}{9} + 12 = 76$ l. 8 s. 10 d. $\frac{2}{3}$ – B.
$\frac{9}{6} : \frac{290}{1} :: \frac{4}{6} : 120\frac{8}{9} - 8 = 120$ l. 17 s. 9 d. $\frac{1}{3}$ – C.

31. It being agreed that the French King, Pope, and Pretender, are to ſhare 100000 acres in the infernal regions, in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ reſpectively; but the Pretender relinquiſhing his right, how is the territory to be divided betwixt the other two, without the help of a lawyer?

Palladium.

$\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ reduced, ſo as to have one common denominator, will be $\frac{20}{60}$, $\frac{15}{60}$, and $\frac{12}{60}$; rejecting the denominator 20, 15, and 12.

$$\text{Then } 20 + 15 + 12 = 47.$$

$$\text{As } 47 : 100000 :: \begin{cases} 20 : 42553\frac{9}{47}, \text{ French King.} \\ 15 : 31914\frac{42}{47}, \text{ Pope.} \\ 12 : 25531\frac{3}{7}, \text{ Pretender.} \end{cases}$$

But $25531\frac{43}{47}$ acres being relinquiſhed by the Pretender, muſt be divided between the French King and the Pope, as 4 to 3;

viz. $4 + 3 = 7 : 25531\frac{43}{47} :: 4 : 14589\frac{219}{329}$.
$7 : 25531\frac{43}{47} :: 3 : 10942\frac{28}{329}$.

$\therefore 42553\frac{9}{47} + 14589\frac{219}{329} = 57142\frac{282}{329}$, for the French King.

And $31914\frac{42}{47} + 10942\frac{82}{329} = 42857\frac{47}{329}$, for the Pope. Q. E. F.

32. Bought 100 quarters of malt, meal and oatmeal, together for 142 l. for every five buſhels of malt, I had three of meal; and for every eight of meal, I have ſeven of oatmeal: pray what did they coſt me ſeverally a buſhel, the malt being half as dear again as the meal, and the meal double the price of the oatmeal?

3 meal : 5 malt : : 8 meal : $13\frac{1}{3} = \frac{40}{3}$, malt.

Then $8 + 13\frac{1}{3} + 7 = 28\frac{1}{3} = \frac{85}{3}$, their ſum.

$$\frac{85}{3} : \frac{40}{3} :: \frac{100}{1} : \frac{800}{17} = \overset{\text{qrs.}}{47\frac{1}{17}} = \overset{\text{buſh.}}{376\frac{8}{17}}\ \text{malt} = \frac{6400}{17}\ \frac{85}{3}$$

qrs. bush.

$\frac{85}{3} : \frac{8}{1} :: \frac{100}{1} : \frac{480}{17} = 28\frac{4}{17} = 225\frac{15}{17}$ meal $= \frac{3840}{17}$.

$\frac{85}{3} : \frac{7}{1} :: \frac{100}{1} : \frac{420}{17} = 24\frac{12}{17} = 197\frac{11}{17}$ oatmeal $= \frac{3360}{17}$.

$$\left.\begin{array}{l} \text{And } \frac{800}{17} \times \frac{2}{3} = \frac{1200}{17} \\ \qquad\qquad\qquad \frac{480}{17} \\ \frac{480}{17} \times \frac{1}{2} = \frac{210}{17} \end{array}\right\} \text{for the price of the} \left\{\begin{array}{l} \text{malt.} \\ \text{meal.} \\ \text{oatmeal.} \end{array}\right.$$

The denominators may be omitted, and each numerator divided by 30, the quotients will still retain the same proportion;

viz. $40 + 16 + 7 = 63$.

$$\left.\begin{array}{l} \text{As } 63 : 142 :: 40 : \frac{5680}{63} = 90 \quad 3 \quad 2\frac{2}{21} \\ 63 : 142 :: 16 : \frac{2272}{63} = 36 \quad 1 \quad 3\frac{5}{21} \\ 63 : 142 :: 7 : \frac{994}{63} = 15 \quad 15 \quad 6\frac{14}{21} \end{array}\right\} \text{price of the} \left\{\begin{array}{l} \text{malt.} \\ \text{meal.} \\ \text{oatmeal.} \end{array}\right.$$

$$\left.\begin{array}{l} \frac{\overset{80}{\cancel{6400}}}{17}\Big)\frac{\overset{71}{\cancel{5680}}}{63}\Big(\frac{1207}{5040} = 4 \text{ s. } 9\frac{30}{63}\text{d. malt.} \\ \frac{\overset{120}{\cancel{3840}}}{17}\Big)\frac{\overset{71}{\cancel{2272}}}{63}\Big(\frac{1207}{7560} = 3 \text{ s. } 2\frac{20}{63}\text{d. meal.} \\ \frac{\overset{240}{\cancel{3360}}}{16}\Big)\frac{\overset{71}{\cancel{994}}}{63}\Big(\frac{1207}{15120} = 1 \text{ s. } 7\frac{10}{63}\text{d. oatmeal.} \end{array}\right\} \begin{array}{l} \text{per} \\ \text{bushel.} \\ \text{Q. E. F.} \end{array}$$

33. Three men, A, B, C, buy a ship for 310 l. 15 s. of which A paid an unknown sum; B paid $2\frac{1}{2}$ as much; and C $3\frac{1}{3}$ as much: how much did each man pay?

A $1 = \frac{6}{6}$, B $2\frac{1}{2} = \frac{5}{2} = \frac{15}{6}$, C $3\frac{1}{3} = \frac{10}{3} = \frac{20}{6}$.

Then $\frac{6}{6} + \frac{15}{6} + \frac{20}{5} = \frac{41}{6}$.

And 310 l. 15 s. $= 310\frac{3}{4} = \frac{1243}{4}$.

$\frac{41}{6}$

$$\frac{41}{\cancel{6}}\Big)\frac{1243}{\cancel{4}}\Big(\frac{3729}{82} = \overset{l.}{45}\ \overset{s.}{9}\ \overset{d.}{6\tfrac{6}{41}} = \text{A's}$$

	l.	s.	d.		
Then 45l. 9s. 6d. × $2\frac{1}{2}$ =	113	13	$9\frac{15}{41}$	= B's	part.
And 45l. 9s. 6d. × $3\frac{1}{3}$ =	151	11	$8\frac{20}{41}$	= C's	
	£ 310	15	–		

34. There were 25 coblers, 20 taylors, 18 weavers, and 12 combers, ſpent 133 ſhillings at a meeting; to which reckoning five coblers paid as much as four taylors, 12 taylors as much as nine weavers, and ſix weavers as much as eight combers; how much did each company pay, and what each man?

Per queſtion, 5 coblers, 4 taylors, 3 weavers, 4 combers } paid like ſums.

Then		× which		
1 cobler	$\frac{1}{5}$		25 = 5	parts of the reckoning
1 taylor	$\frac{1}{4}$		20 = 5	
1 weaver	$\frac{1}{3}$		18 = 6	
1 comber	$\frac{1}{4}$		12 = 3	
			19	

1 : 133 : :	5 : 35	for the	coblers.
	5 : 35		taylors.
	6 : 42		weavers.
	3 : 21		combers.

		s.	d.		
25	35	1	$4\frac{4}{5}$	for each	cobler.
20	35	1	9		taylor.
18	42	2	4		weaver.
12	21	1	9		comber.

35. Once as I walked upon the banks of Rye,
To ſee the purling ſtreams glide gently by,
And hear the pretty birds to chirp and ſing,
Making the groves with melody to ring;
I in the meads three beauteous nymphs did 'ſpy,
That for their pleaſure came as well as I;
And unto me their ſteps they did direct,
Saluting me with moſt benign reſpect;

 Saying,

Saying, Well met, we've business to impart,
Which we cannot decide without your art;
Our grannam's dead, and left a legacy,
Which is to be divided 'mongst us three:
In pounds it is two hundred twenty-nine;
Also a good mark, being sterling coin.
Then spake the eldest of the lovely three,
I'll tell you how it must divided be;
Likewise our names I unto you will tell,
Mine is Moll, the other Anne and Nell:
As oft as I five and five-ninths do take,
Anne takes four and three-sevenths her part to make;
As oft as Anne four, and one-ninth does tell,
Three and two-thirds must be took up by Nell.

L. D. 1717.

First 229l. 13s. 4d. $= 229.\cancel{6}$; $5\frac{5}{9} = 5.\cancel{5}$; $4\frac{3}{7} = 4.\cancel{4}2857\cancel{1}$.
And $4\frac{1}{9} = 4.\cancel{1}$; and $3\frac{2}{3} = 3.\cancel{6}$.
As $4.\cancel{1} : 3.\cancel{6} :: 4.\cancel{4}2857\cancel{1} : 3.949809$

Hence as often as Moll takes 5.555555
Anne takes $4.\cancel{4}2857\cancel{1}$
And Nell 3.949809

13.933935

$$\because\ \begin{matrix} 13.933935 : \\ 229.\cancel{6} :: \end{matrix} \begin{cases} 5.555555 : 91.569 = 91 \quad 11 \quad 4\frac{3}{4}, & \text{Moll.} \\ 4.\cancel{4}2857\cancel{1} : 72.995 = 72 \quad 19 \quad 10\frac{3}{4}, & \text{Anne.} \\ 3.933935 : 65.102 = 65 \quad 2 \quad -\frac{1}{7}, & \text{Nell.} \end{cases}$$

Q. E. F.

£ $229.66\cancel{6}$

SECT. II.

DOUBLE FELLOWSHIP;

OR,

FELLOWSHIP WITH TIME.

DOUBLE FELLOWSHIP is a rule whereby we compute the gain or loss of several merchants who employ different sums of money different times in partnership.

RULE.

RULE.

As the sum of the products of each man's stock and time : is to the whole gain or loss : : so is the particular product of each man's stock and time : to each man's particular gain or loss.

1. Three persons, A, B, and C, enter into partnership thus: A puts in 65l. for eight months; B puts in 78l. for 12 months; and C puts in 84l. for six months. With this they traffic, and gain 166l. 12s. I demand each man's share of the gain in proportion to his stock and time of employing it?

$$\left.\begin{array}{l}\text{A's}\\ \text{B's}\\ \text{C's}\end{array}\right\}\text{stock}\left\{\begin{array}{l}65\times 8\\ 78\times 12\\ 84\times 6\end{array}\right\}\text{times}\left\{\begin{array}{r}=520\\ =936\\ =504\\ \hline 1960\end{array}\right.$$

$$1960 : 166 : : \left\{\begin{array}{l}520 : 44\text{l. } 4\text{s. }-\text{d.} = \text{A's}\\ 936 : 79\text{l. } 11\text{s. } 2\tfrac{1}{2}\text{d.} = \text{B's}\\ 504 : 42\text{l. } 16\text{s. } 9\tfrac{1}{2}\text{d.} = \text{C's}\end{array}\right\}\text{gain.}$$

Or by finding a common multiplier; viz. 1960) 166.6 (.085.

$$\left.\begin{array}{l}\text{Then } 520\times .085 = 44.2, \text{ for A}\\ \text{Also } 936\times .085 = 79.56, \text{ for B}\\ \text{And } 504\times .085 = 42.84, \text{ for C}\end{array}\right\}\text{as before.}$$

2. Three persons, A, B and C, hired a certain pasture for 24l. in which A keeps 40 cows for four months; B keeps 30 cows for two months; and C keeps 36 cows for five months; how much of the rent ought each of them to pay?

$$\begin{array}{l}\text{A } 40\times 4 = 160\\ \text{B } 30\times 2 = \ \ 60\\ \text{C } 36\times 5 = 180\\ \hline \ \ \ \ 400\end{array}$$

$$400 : 24 \left\{\begin{array}{l}: : 160 : \ \ 9\ \ 12 = \text{A's}\\ : : \ \ 60 : \ \ 3\ \ 12 = \text{B's}\\ : : 180 : 10\ \ 16 = \text{C's}\end{array}\right\}\begin{array}{c}\text{part of}\\ \text{the}\\ \text{rent.}\end{array}$$

(the column of pounds is headed l.)

3. Six

3. Six merchants, viz. A, B, C, D, E and F, enter into partnerſhip, and compoſe a joint-ſtock in this manner;

		l.	s.					
viz.	A puts in	64	10 =	64.5	for	$4\frac{1}{2}$	months:
	B - - -	78	15 =	78.75		6	
	C - - -	100	– =	100.		$8\frac{1}{4}$	
	D - - -	80	10 =	80.5		12	
	E - - -	74	12 =	74.6		$9\frac{1}{2}$	
	F - - -	125	15 =	125.75		7	

they traffic and gain 258 l. 18 s. $4\frac{1}{2}$d. It is required to find every man's ſhare of the gain, according to his ſtock, and the time it was employed?

	l.		months.		
A's ſtock	64.5	×	4.5	=	290.25
B's - -	78.75	×	6.	=	472.5
C's - -	100.	×	8.25	=	825.
D's - -	80.5	×	12.	=	966.
E's - -	74.6	×	9.5	=	708.7
F's - -	125.75	×	7.	=	880.25
			Sum	=	4142.7

The whole gain is 258l. 18 s. $4\frac{1}{2}$d. = 258.91875.
Then 4142.7) 258.91875 (.0625, common multiplier.

				l.	s.	d.		
∵ 290.25 × .0625	=	18.140625	=	18	2	$9\frac{3}{4}$	= A's	ſhare.
472.5 × .0625	=	29.53125	=	29	10	$7\frac{1}{2}$	= B's	
825. × .0625	=	51.5625	=	51	11	3	= C's	
966. × .0625	=	60.375	=	60	7	6	= D's	
708.7 × .0625	=	44.29375	=	44	5	$10\frac{1}{2}$	= E's	
880.25 × .0625	=	55.015625	=	55	–	$3\frac{3}{4}$	= F's	
Their whole gain £				258	18	$4\frac{1}{2}$		

4. A and B in partnerſhip equally divide the gain; A's money, which was 84 l. 12 s. 6 d. lay for 19 months; and B's for no more than ſeven; the adventure of the latter is ſought.

Reciprocally, 19 mon. : 84.625 : : 7 mon. : 229.696 l. = 229 l. 13 s. 11 d. anſwer.

5. A,

5. A, B, C have a common ſtock of 1000 l. A gains 100 l. in nine months; B 80 l. in 12 months; and C 120 l. in eight months: what was each of their particular ſtocks?

9)100(11.$\cancel{1}$
12) 80(6.$\cancel{6}$
80)120(15.

			l.	s.	d.	
32.$\cancel{7}$: 1000 : : 11.$\cancel{1}$: 338.98305	=		338	19	8	A's ⎫
32.$\cancel{7}$: 1000 : : 6.$\cancel{6}$: 203.38983	=		203	7	$9\frac{1}{2}$	B's ⎬ ſtock.
32.$\cancel{7}$: 1000 : : 15. : 457.62712	=		457	12	$6\frac{1}{2}$	C's ⎭

6. A hath 200 l. more ſtock than B; but A continued his only five months, and B nine, and drew equal gains: what are their ſtocks?

m. l. m. l.

$9 - 5 = 4 : 200 : : 5 : 250 =$ A's ſtock.
$4 : 200 : : 9 : 450 =$ B's ſtock.

$700 =$ whole ſtock.

7. A and B paid equally for a horſe, February 7, 1756; A on the 10th took him a journey into the weſt, and returned on the 10th of June following; B on the 2d of Auguſt took him into Scotland, and ſtayed till November 13, and then concluded his ſervice this year. From January 17 following, A uſed him 10 days; and in ſix weeks after his return, employed him till April 30; B then rode him from May-day to Midſummer; A had him from July 14, till 14 days after St. James's tide; B, on September 30, took him into Norfolk, and came back October 19. He then was ſold for 7 l. 10 s. and they would have the Money equitably parted between them; viz. in proportion to the uſe each made of their ſteed.

	Days.	
From Feb. 10 till June 10, are - -	122	A in all
Between Jan. 17 and April 30 - -	61	208 days.
From July 14 till 14 after St. James -	24	
From Aug. 2 till Nov. 13 - - - -	104	B in all
May 1 till June 24 - - - -	55	179 days.
Sept. 30 till October 19 - - -	20	

Then 208 + 179 = 387 days, the horſe was in uſe.

	l.	s.	d.		
As 387 : 7.5 : : 179, B's time :	3	9	$4\frac{24}{43}$,	A's	ſhare of the
387 : 7.5 : : 208, A's time :	4	–	$7\frac{19}{43}$,	B's	money.

8. A

8. A for a nine months adventure received 20 l. B for one of ſeven months received 25 guineas; and C for lying out of his contribution five months has a title to 32 l. The total of their adventures multiplied into their reſpective times was 640 l. what then was the particulars?

25 guineas = 26.25 l.

Then 20 + 26.25 + 32 = 78.25, the whole gain.

$$78.25 : 640 :: \begin{cases} 20 : 163.57837 \\ 26.25 : 214.69648 \\ 32 : 261.72524 \end{cases}$$

9	163.57837	18.175	= 18	3	6	= A's		
7	214.69648	30.6709	= 30	13	5	= B's		adventure
5	261.72524	52.345	= 52	6	$10\frac{3}{4}$	= C's		

9. Ten pounds a quarter is allowed the five auditors of a fire-office; they attend about ſeven times in a quarter, and the abſentee's money is always divided equally among ſuch as do attend. A and B on theſe occaſions never miſſed; C and D are generally twice in a quarter abſent, and E only once; at the payment what had each man to receive?

5)10(2 l. each man's equal ſhare.

7)2(5 s. $8\frac{4}{7}$ d. each man for his day.

5 s. $8\frac{4}{7}$ d. × 2 = 11 s. $5\frac{1}{7}$ d. C and D to abate each for two days abſence.

2 l. — 11 s. $5\frac{1}{7}$ d. = 1 l. 8 s. $6\frac{6}{7}$ d. to C and D each for attendance.

2 l. — 5 s. $8\frac{4}{7}$ d. = 1 l. 14 s. $3\frac{3}{7}$ d. to E for his attendance.

11 s. $5\frac{1}{7}$ d. × 2 = 1 l. 2 s. $10\frac{2}{7}$ d. C's more, D's defaults.

3)1 l. 2 s. $10\frac{2}{7}$ d.(7 s. $7\frac{3}{7}$ d. A, B and E's ſhare of theſe defaults.

4)5 s. $8\frac{4}{7}$ d.(1 s. $5\frac{1}{7}$ d. A, B, C and D's ſhare of E's default.

Then 2 l. + 7 s. $7\frac{3}{7}$ d. + 1 s. $5\frac{1}{7}$ d. = 2 l. 9 s. $-\frac{4}{7}$ d. A's and B's each.

1 l. 8 s. $6\frac{6}{7}$ d. + 1 s. $5\frac{1}{7}$ d. = 1 l. 10 s. C and D each.

1 l. 14 s. $3\frac{3}{7}$ d. + 7 s. $7\frac{3}{7}$ d. = 2 l. 1 s. $10\frac{6}{7}$ d. E's ſhare.

10. A, B and C enter into partnerſhip; A puts in on the 1ſt of March 60 l. B put in on the 1ſt of May 160 yards of broad-cloth; and C puts in on the 1ſt of June 405 ducats. On the 1ſt of January following they accounted their gain, of which A and B took up 456 l. B and C took up 431 l. and C and A took up 375 l. I demand what was

was gained as well in the whole as a-part; what B valued a yard of his cloth at, and what was C's ducats a-piece?

l.

A's gain + B's = 456
B's - + C's = 431
A's - + C's = 375

2)1262 ſum, each being named twice.

631, whole gain.

Then 631 — 431 = 200 l. = A's
Alſo 631 — 375 = 256 l. = B's gain.
And 631 — 456 = 175 l. = C's

P. . T. . G.
60 . 10 . 200 $60 \times 10 \times 256 = 153600$
. . 8 . 256 $8 \times 200 = 1600$
1600)153600(96 l. value of B's cloth.
160)96 l. = 1920 s. (12 s. B's cloth per yard.

P. . T. . G.
60 . 10 . 200 $60 \times 10 \times 175 = 105000$
. . 7 . 175 $200 \times 7 = 1400$
1400)105003(75 l. value of C's ducats.
405)75 l. = 1500 s.(3s 8 $\frac{4}{9}$d. value of one ducat.
285
12

3420

180 $9(\frac{180}{405}(\frac{20}{45} = \frac{4}{9}$.

11. A clears 13 l. in ſix months; B 18 l. in five months; and C 23 l. in nine months, with a ſtock of 72 l. 10 s. what then did the general ſtock amount to?

P. . T. . G.
72.5 . 9 . 23
. . 6 . 13
$72.5 \times 9 \times 13 = 8482.5$, dividend.

$23 \times 6 = 138)\ 8482.5\ (61.46739 = 61\quad 9\quad 4 = $ A's

P.		T.		G.
72.5	.	9	:	23
.	.	5	.	18

$72.5 \times 9 \times 18 = 11745$, dividend.

$23 \times 5 = 115)11745(102.130435 = 102\quad 2\quad 7\frac{1}{4} = $ B's

$72\quad 10\quad - = $ C's

stock.

Anſwer, the whole ſtock £ $236\quad 1\quad 11\frac{1}{4}$

12. A, B and C enter partnerſhip; A puts in the 1ſt of January 100 l. and the 1ſt of May puts in 150 l. more; and on the 1ſt of September takes out 30 l. the remainder ſtays in till the year's end.

B puts in the 1ſt of January 250 l. and on the 1ſt of June 60 l. more; and on the 1ſt of November 100 l. more; which continues till the year's end.

C puts in the 1ſt of January 300 l. and the 1ſt of April takes out 200 l. and on the 1ſt of Auguſt takes out 50 l. more; the remainder ſtays in till the year's end; what muſt each have of the gain, which was 133 l.?

		l.		l.
A from 1 January	- - -	100 × 12	=	1200
1 May	- - - -	150 × 4	=	600
1 September	- -	120 × 4	=	480
				2280
B from 1 January	- - -	250 × 12	=	3000
1 June	- - - - -	60 × 7	=	420
1 November	- -	100 × 2	=	200
				3620
C from 1 January	- - -	300 × 3	=	900
1 April	- - - -	100 × 4	=	400
1 Auguſt	- - - -	50 × 5	=	250
				1550

Then $2280 + 3620 + 1550 = 7450$.

		l.	s.	d.	qrs.		
7450 : 133 ::	2280 :	40	14	$-\frac{3}{4}$	. $\frac{33}{149}$,	A's	gain.
	3620 :	64	12	6	. $\frac{72}{149}$,	B's	
	1550 :	27	13	5	. $\frac{44}{149}$,	C's	

13. A,

13. A. B and C are in company, and put in together 3822 l. A's money was in three months, B's money was in five months, and C's money was in seven months; they gained 234 l. which was so divided, as $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's gain; and $\frac{1}{3}$ of B's gain was equal to $\frac{1}{4}$ of C's gain: what did each merchant gain and put in?

$$\left.\begin{array}{l}\text{Suppose A's gain to be } 2\\ \text{Then will B's be - - } 3\\ \text{And C's - - - - - - - } 4\end{array}\right\}\text{ by the question.}$$

$$\text{Their sum} = \overline{\underline{9}}$$

$$\text{Also } \overset{\text{l}}{9} : 234 :: \left\{\begin{array}{l}2 : \overset{\text{l.}}{52} = \text{A's}\\ 3 : 78 = \text{B's}\\ 4 : 104 = \text{C's}\end{array}\right\}\text{ gain.}$$

$$\begin{array}{r}\text{And } 52 \times 3 = 156\\ 78 \times 5 = 390\\ 104 \times 7 = 728\\ \hline 1274\text{, their sum.}\end{array}$$

1274) 3822 (3, common multiplier,

$$\left.\begin{array}{l}\therefore 156 \times 3 = \overset{\text{l.}}{468} = \text{A's}\\ 390 \times 3 = 1170 = \text{B's}\\ 728 \times 3 = 2184 = \text{C's}\end{array}\right\}\text{ stock.}$$

$$\overline{\underline{3822}}\text{, whole stock.}$$

SECT. III.

FACTORAGE.

WHEN a person does not transact business himself, but commissions another to act for him, the person so commissioned is called a factor, and the business he transacts is called factorship or factorage.

1. What

1. What is the commission of 793 l. 17 s. 6 d. at $2\frac{3}{8}$ per cent.?

	l.	s.	d.
$\frac{1}{2}$	793	17	6
			$2\frac{3}{8}$
	1587	15	–
$\frac{1}{4}$	396	18	9
	99	4	$8\frac{1}{4}$
	20.83	18	$5\frac{1}{4}$
	20		
	16.78 s.		
	12		
	9.41 d. ans. 20 l. 16 s. $9\frac{1}{4}$d.		
	4		
	1.64 qr.		

Or,

	l.	s.	d.
10 per cent. or $\frac{1}{10}$	793	17	6
$\frac{1}{5}$	79	7	9
$\frac{1}{4}$	15	17	$6\frac{1}{2}$
$\frac{1}{4}$	3	19	$4\frac{1}{2}$
	–	19	10
Answer, £	20	16	9

2. What is the commission of 967 l. 13 s. 4 d. at $3\frac{7}{8}$ per cent.

	l.	s.	d.
$\frac{1}{2}$	967	13	4
			$3\frac{7}{8}$
	2903	–	–
$\frac{1}{2}$	483	16	8
$\frac{1}{2}$	241	18	4
	120	19	2
£	37.49	14	2
	20		
	9.94 s.		
	12		
	11.30 d.		
	4		
	1 20 qr.		

Or,

	l.	s.	d.
$\frac{1}{10}$	967	13	4
$\frac{1}{4}$	96	15	4
$\frac{1}{2}$	24	3	10
	12	1	11
	1	4	$2\frac{1}{4}$
£	37	9	$11\frac{1}{4}$, answer.

3. A

3. A merchant's real ſtock being 100 l. and the factor's 30 l. who received $\frac{1}{3}$ of the gain; what was his ſervices valued at?

$\frac{2}{3} : 100 :: \frac{1}{3} : 50$; therefore $50 - 30 = 20$, the anſwer.

Otherways,

$100 + 30 = 130$ l. whole ſtock. 3) 130 (43 l. 6 s. 8 d.
∴ 43 l. 6 s. 8 d. — 30 l. = 13 l. 6 s. 8 d. value of the factor's ſervice.

4. A merchant delivers to his factor 100 l. allowing him to join to it 30 l. and values his ſervice at 40 l. what ſhare of the gain ought the factor to have, the whole gain being 75 l.?

There are two ways of ſolving queſtions of this kind; but if the merchant and factor previouſly agree (as to prevent diſputes they always ſhould) the method is determined.

The moſt common method:

$30 + 40 = 70$, factor's ſtock.
100, merchant's ſtock.

	l.	s.	d.	
170 : 75 :: 70 :	30	17	$7\frac{3}{4}$, factor's	ſhare of the gain.
170 : 75 :: 100 :	44	2	$4\frac{1}{4}$, merch.	
	75	–	–	

But if the gain be made upon the real ſtock 130 l. and not upon the imaginary one 170 l. the factor ought to be gratified for his ſervice, by being allowed the profit of 40 l. of the real ſtock more than what he actually put in.

In conſideration whereof the above queſtion muſt be ſolved as follows:

$100 - 40 = 60$, merchant's ſtock.
$30 + 40 = 70$, factor's ſtock.

	l.	s.	d.	
130 : 75 :: 60 :	34	12	$3\frac{1}{4}$, merch.	ſhare.
130 : 75 :: 70 :	40	7	$8\frac{1}{4}$, factor's	
	£ 75	–	–	

5. A merchant delivers unto his factor 500 l. and allows his perſon at 200 l. when they made up their accounts they find they have gained 20 per cent. what is the factor's part?

By the firſt method.

Firſt, 500 l. + 200 = 700; and 100 : 20 :: 700 : 140.
∵ 700 : 140 :: 200 : 40, factor's part.
140 — 40 = 100 l. the merchant's part.

By the other method, the whole ſtock being but 500 l.
As 100 : 20 :: 500 : 100 l. the whole gain.
Alſo 100 : 20 :: 300 : 60 l. for the merchant.
∵ 100 : 20 :: 200 : 40 l. for the factor.

6. A merchant's real ſtock being 100 l. and the factor's ſervice valued at 20 l. who received $\frac{1}{2}$, what was the factor's real ſtock?

Firſt 100 l. — 20 = 80, which the factor muſt put in.

But by the other method, 100 l. — 20 = 80 = $\frac{1}{2}$, the real ſtock.

∵ 160 — 100 = 60, which the factor in this caſe puts in.

7. A merchant's real ſtock 100 l. and the factor being allowed $\frac{1}{4}$ of the gain for his ſervice; what real ſtock muſt he join to have $\frac{1}{3}$ of the gain?

By the firſt method.

4) 100 (25; and 100 + 25 = 125 l. imaginary ſtock.
3) 125 (41 l. 13 s. 4 d. the factor being to have $\frac{1}{3}$.
41 l. 13 s. 4 d. — 25 = 16 l. 13 s. 4 d. the factor muſt put in.

By the other method.

3) 100 (33 l. 6 s. 8 d. = $\frac{1}{4}$, imaginary ſtock.
Alſo 100 l. + 33 l. 6 s. 8 d. = 133 l. 6 s. 8 d. whole imaginary ſtock.
3) 133 l. 6 s. 8 d. (44 l. 8 s. 10$\frac{2}{3}$ d. factor's $\frac{1}{3}$.
∵ 44 l. 8 s. 10$\frac{2}{3}$ d. — 33 l. 6 s. 8 d. = 11 l. 2 s. 2$\frac{2}{3}$ d. the anſwer.

8. A merchant's real ſtock being 120 l. and the factor's 60 l. they agreed, that at the year's end the factor ſhould have

have half of both stock and gain; but they broke up at eight months end, having gained 150 l. how much ought the factor to have?

First $120+60=180$, whole stock; and $2)180(90$ l. the share of each of the stock at the year's end: so that the factor was to have 30 l. of the merchant's stock, had it continued in trade for 12 months.

But $12:30::8:20$ l. the factor's due of the merchant.

$$\left.\begin{array}{l}\text{Also } 120\text{ l.} - 20 = 100\text{ l. merchant's}\\ \text{And } 60 + 20 = 80\text{ l. factor's} \;-\; -\end{array}\right\}\text{stock at eight months.}$$

$$180:150::\left\{\begin{array}{rl}100: & 83\;\;6\;\;8,\text{ merchant's}\\ 80: & 66\;\;13\;\;4,\text{ factor's} \;-\end{array}\right\}\text{gain.}$$

(l. s. d.)

$$\left.\begin{array}{l}\therefore 100\text{ l.} + 83\;\;6\;\;8 = 183\;\;6\;\;8,\text{ merchant's}\\ \text{And } 80\text{ l.} + 66\;\;13\;\;4 = 146\;\;13\;\;4,\text{ factor's} \;-\end{array}\right\}\text{part.}$$

9. It is proposed by an elderly person in trade, desirous of a little respite, to admit a sober and industrious young fellow to share in the business, and to encourage him offers, that if his circumstances will allow him to advance 100 l. his pay should be 40 l. a year; if he shall be able to put 200 l. into the stock, he shall have 55 l. a year; and if 300 l. he shall receive 70 l. annually: in this proposal what was allowed for his attendance simply?

$$\left.\begin{array}{l}\text{First } 70\text{ l.} - 55\text{ l.} = 15\text{ l.}\\ \text{And } 55\text{ l.} - 40\text{ l.} = 15\text{ l.}\end{array}\right\}\begin{array}{l}\text{hence it is plain he proposed to}\\ \text{allow him 15 per cent.}\end{array}$$

$\therefore 40\text{ l.} - 15 = 25$ l. the answer.

SECT. IV.

LOSS AND GAIN.

BY this rule we discover what is got or lost by any parcel of goods, or how much per cent. is got or lost according to the price bought and sold at; by which we are instructed to raise or fall the price of commodities, in such proportion, that neither our gain may be exorbitant as to injure our customers, nor our loss so great as to impoverish ourselves.

 1. At

1. At what price must I sell 1 cwt. of sugar, which cost 2 l. 6 s. 8 d. to gain 10 per cent.?

	l.	s.	d.
10 per cent. is $\frac{1}{10}$)	2	6	8
		4	8
Answer,	2	11	4

2. A Manchester man buyeth yarn for 6 s. for a bundle, which not proving so good as was expected, would put it off again, so as but to lose 6 per cent. by it; what is the selling price?

	s.	d.
10 per cent. is $\frac{1}{10}$	6	–
5 per cent. is $\frac{1}{2}$	–	7.2
1 per cent. is $\frac{1}{5}$	–	3.6
	–	0.72
Loss	–	4.32

∴ 6 s. — 4.32 = 5 s. 7.68 d. the selling price.

3. If a tun of wine cost 45 l. 19 s. 10 d. how must I sell it a tun to gain 26 $\frac{1}{2}$ per cent.?

	l.	s.	d.
20 per cent. is $\frac{1}{5}$	45	19	10
5 per cent. is $\frac{1}{4}$	9	3	11$\frac{1}{2}$
1 per cent. is $\frac{1}{5}$	2	5	11$\frac{3}{4}$
$\frac{1}{2}$ per cent. is $\frac{1}{2}$	–	9	2$\frac{1}{4}$
	–	4	7
£	58	3	6$\frac{1}{2}$, the answer.

4. If I buy broad-cloth for 11 s. 6 d. a yard; how must I sell it to gain 20 per cent.?

	s.	d.
20 per cent. is $\frac{1}{5}$)	11	6
	2	3$\frac{1}{2}$
Answer,	13	9$\frac{1}{2}$

5. If

5. If a pack of yarn, weighing 240lb. cost 13 l. what must it be sold at a pound to gain $15\frac{1}{2}$ per cent.?

	l.		
10 per cent. is $\frac{1}{10}$	13		
5 per cent. is $\frac{1}{2}$	1	6	
$\frac{1}{2}$ per cent. is $\frac{1}{10}$	-	13	
	-	1	$3\frac{1}{2}$

£ 15 - $3\frac{1}{2}$, as 240 d. = 1 l. sterling, equal to the pound's weight in a pack, it will be 1 s. 3 d. a pound, and $3\frac{1}{2}$ over in the whole.

6. If I gain 2 d. in the shilling, what is my gain per cent.?

	l.		
2 d. is $\frac{1}{6}$	100		
Answer, £	16	13	4 per cent.

7. By a quantity of damaged lump-sugar lost $5\frac{3}{4}$ d. in the shilling, what did I lose per cent.?

	l.		
$\frac{1}{6}$, $\frac{1}{4}$ £	100		
$\frac{1}{4}$	25		
	16	13	4
	6	5	-
Answer, £	47	18	4 per cent.

8. If by the sale of a chest of lemons I gained 4 s. in the pound, what is my gain per cent.?

	l.
4 s. is $\frac{1}{5}$	100
Answer, £	20 per cent.

9. A grocer bought 3 cwt. 1 qr. 14 lb. weight of cloves, at the rate of 2 s. 4 d. per pound, and sold them for 52 l. 14 s. whether did he gain or lose by the bargain, and how much?

s.	d.			l.	s.	d.
2	4 per lb.		Sold for	52	14	–
	7		Cost - -	44	2	–
16	4		Gain £	8	12	–, the answer.
	4					
3 5	4					
	4					
$\frac{1}{4}$ 13 1	4, price of 1 cwt.					
	3					

	l.	s.	d.		cwt.	qrs.	lb.
	39	4	–	- - -	3	–	–
$\frac{1}{2}$	3	5	4	- - -	–	1	–
	1	12	8	- - -	–	–	14
£	44	2	–	- - -	3	1	14

10. A merchant bought 436 yards of broad-cloth for 8 s. 6 d. a yard, and sold it again for 10 s. 4 d. a yard; what did he gain by the whole?

s.	d.	
10	4,	sold at a yard.
8	6,	cost.
1	10,	gained per yard.

$\frac{1}{3}$, $\frac{1}{2}$	436	
	218	
	145	4
$\frac{1}{20}$	799	4
	39 12	4, the answ.

11. Sold goods for 50 l. 12 s. 6 d. and gained $3\frac{1}{2}$d. in the shilling, what did I gain per cent. and what the prime cost?

	l.			
3 d. is $\frac{1}{4}$	100			
$\frac{1}{2}$ d. is $\frac{1}{6}$	25			
	4	3	4	
Per cent. £	29	3	4	

		l.	s.	d.
20 per cent.	$\frac{1}{5}$	50	12	6
	$\frac{1}{5}$	10	2	6
		2	–	6
	$\frac{1}{4}$	2	–	6
	$\frac{1}{9}$	–	10	$1\frac{1}{2}$
		–	1	$8\frac{1}{4}$
Gain £		14	15	$3\frac{3}{4}$, subtract.
Prime cost £		35	17	$2\frac{1}{4}$

12. If

12. If I buy 1 cwt. of tobacco for 4 l. 13 s. 4 d. and sell it again for 11 d. a pound; what do I gain or lose, and what per cent.?

d.		s.	d.	
11 × 7 =		6	5,	price of 7 lb.
			4	
	1	5	8	
			4	
£	5	2	8	— 4 l. 13 s. 4 d. = 9 s. 4 d. gain.

As $4.\cancel{6} : 5.1\cancel{3} :: 100 : 110 \therefore$ his gain was 10 per cent.

13. A Manchester man buys 20 ton of cheese, with which he went into Ireland; it cost him 400 l. the freight and custom came to 50 l. his own expences and charges were 16 l. 13 s. 4 d. how must he sell it a pound to gain 20 l. per cent.?

	l.		
Cost - - - -	400		
Freight and custom	50		
Charges - - -	16	13	4
20 per cent. - $\frac{1}{5}$)	466	13	4
	93	6	8
	£560, selling price		

		l.
	20	560
20 {	5	28 per ton.
	4	5 12 s.
112 {	7	1 8 per cwt.
	4	- 4 s.
	4	- 1 s.

Answer, — — 3 d. p. lb.

14. A stationer sold quills at 11 s. a thousand, by which he cleared $\frac{3}{8}$ of the money; but growing scarce, raised them 13 s. 6 d. a thousand; what might he clear per cent. by the latter price?

$\frac{3}{8} \times \frac{11}{1} = \frac{33}{8} =$ 4 s. $1\frac{1}{2}$ d. gained per thousand by the first sale.

11 s. — 4 s. $1\frac{1}{2}$ d. = 6 s. $10\frac{1}{2}$ d. = $\frac{11}{32}$, cost.

13 s. 6 d. — 6 s. $10\frac{1}{2}$ d. = 6 s. $7\frac{1}{2}$ d. = $\frac{\ }{160}$, gain per 1000 by the second sale.

$\frac{11}{32} : \frac{53}{160} :: \frac{100}{1} : \frac{1060}{11} =$ 96 l. 7 s. $3\frac{3}{4}$ d. the answer.

15. Bought hose in London at 4 s. 3 d. the pair, and sold them afterwards in Dublin at 6 s. the pair; now taking the charges at an average to be 2 d. the pair; and considering I must lose 12 per cent. by remitting my money home again; what must I gain per cent. by the article of trade?

4 s. 3 d. $= \frac{51}{240} +$ 2 d. $= \frac{2}{240} =$ 4 s. 5 d. $= \frac{53}{240}$; and 6 s. $= \frac{3}{10}$ l.

As $\frac{53}{240} : \frac{3}{10} :: \frac{100}{1} : \frac{7200}{53}$. Then $100 - 12 = 88$.

Also $\frac{100}{1} : \frac{88}{1} :: \frac{7200}{53} : \frac{6336}{53} =$ 119 l. 10 s. $11\frac{17}{53}$ d.

$\therefore$ 119 l. 10 s. $11\frac{17}{53}$ d. — 100 l. = 19 l. 10 s. $11\frac{17}{53}$ d. the answer.

16. If my factor at Leghorn returns me 800 barrels of anchovies, each weighing 14 lb. net, worth $12\frac{1}{2}$ d. per pound, in lieu of 7490 pounds of Virginia tobacco; and if I find that I have gained after the rate of 17 per cent. by the said consignment, pray how was my said tobacco invoiced per pound to the factor, that is, what was the prime cost?

Barrels 800 × 14 lb. = 11200, at $12\frac{1}{2}$ d. $= \frac{1750}{3}$.

$\frac{100}{1} : \frac{17}{1} :: \frac{1750}{3} : \frac{595}{6}$

$\frac{1750}{3} - \frac{595}{6} = \frac{2905}{6}$ $\quad \therefore \frac{7490}{1} \Big) \frac{2905}{6} \Big(\frac{2905}{44940} \Big(=$ 1 s. $3\frac{1}{2}$ d. $\frac{6}{107}$.

17. If by selling hops at 3 l. 10 s. per cwt. the planter clears 30 per cent. what was his gain per cent. when the same goods sold at 4 l. and a crown?

$100 + 30 = 130 : 100 :: 3.5$ l. : $2.6923 =$ cost per cwt.

$4.25 - 2.6923 = 1.5577$ l. gain per cwt.

$2.6923 : 1.5577 :: 100 : 52.58414 =$ 52 l. 11 s. 8.19 d. the answer.

18. Sold a repeating watch for 50 guineas, and by so doing lost 17 per cent. whereas I ought in dealing to have cleared 20 per cent. then how much was it sold under the just value?

100 — 17 = 83 l. : 100 l. :: 52.5 : 63.253012, coſt.
Then 100 : 120 :: 63.253012 : 75.903614 l.
∵ 75.903614 — 52.5 = 23.403614 = 23 l. 8 s. -d. 3 f. anſwer.

19. If by remitting to Holland, at 31 s. 9 d. Flemiſh, per pound ſterling, 5 per cent. is gained: how goes the exchange, when by remittance I clear 10 per cent.?

105 l. : 31.75 :: 110 l. : 33.2619 s. = 1 l. 13 s. 3.143 d. the anſwer.

20. If by ſending pewter for Turky, and parting with it at $25\frac{2}{3}$ d. per pound, the merchant clears cent. per cent. what does he clear in Holland, where he diſpoſes of the cwt. for 8 l.

3	2.0
12	$1.\dot{6}$
20	2.138

$.1069\dot{4} \times 112 = 11.9\dot{7}$ l. ſold at per cwt. in Turky.
And as his gain was cent. per cent. it coſt him $5.9\dot{8}$ l. per cwt.
Therefore 8 l. — $5.9\dot{8}$ = 2.01 = 2 l. – s. $2\frac{2}{3}$ d. the anſwer.

21. Bought comfits to the value of 41 l. 3 s. 4 d. for 3 s. 1 d. per pound: it happened that ſo many of them were damaged in carriages, that by ſelling what remained good at 4 s. 6 d. the pound, my returns were no more than 34 l. 2 s. 6 d. pray how much of theſe goods were ſpoiled, and what did this part ſtand me in?

34 l. 2 s. 6 d. = 34.125 - - 3 s. 1 d. = $.1541\dot{6}$ - - 4 s. 6 d. = .225.
As .225 l. : 1 lb. :: 34.125 : $151.\dot{6}$, remained good.
1 : $.1541\dot{6}$ :: 151.6 : 23.381941 l. = 23 l. 7 s. $7\frac{2}{3}$ d. the goods coſt.
Then 41 l. 13 s. 4 d. — 23 l. 7 s. $7\frac{2}{3}$ d. = 18 l. 5 s. $8\frac{1}{3}$ d. the damaged coſt.

22. A had 15 pipes of Malaga wine, which he parted with to B at $4\frac{1}{2}$ per cent. profit, who ſold them to C for

38 l. 11 s. 6 d. advantage; C made them over to D for 500 l. 11 s. 6 d. and cleared thereby $6\frac{1}{2}$ per cent. what did the wine coſt A per gallon?

l. s. d.
500 16 8 = 500.8$\dot{3}$
As 100 : 6.5 : 500.8$\dot{3}$: 32.5541$\dot{6}$ = C's }
+ 38 11 6 = 38.575 = B's } profit.

500.8$\dot{3}$ — 71.1291$\dot{6}$ = 429.7041$\dot{6}$, B's coſt.
As 100 : 4.$\dot{3}$ :: 429.7041$\dot{6}$: 18.620513$\dot{8}$; A's profit.
429.76416 — 18.620513$\dot{8}$ = 411.083527$\dot{7}$, A's coſt.
126 × 15 = 1890) 411.08365$\dot{7}$ (.2175045 = 4 s. $4\frac{1}{5}$ d. per gallon, the anſwer.

23. Having bought a parcel of goods for 180 l. and ſold them again immediately for 188 l. 10 s. with four months credit; what is gained per cent. per annum?

l.		mon.		gain.
180	.	4	.	8.5
100	.	12	.	

180 × 4 = 720) 10200 (14 l. 3 s. 4 d. the anſwer.

24. Having bought 160 gallons of French brandy, at 6 s. 6 d. a gallon, there chanced to leek out 18 gallons; at what rate per gallon may I ſell the remainder, with eight months credit, ſo as to gain upon the whole prime coſt, at the rate of 12 per cent. per annum?

gal.
$\frac{1}{5}$, $\frac{1}{8}$ | 160
32
20
£ 52, prime coſt.

l.		mon.		gain.
100	.	12	.	12
152	.	8	.	

2 × 52 × 8 = 4992, dividend.

100 × 12 = 1200) 4992 (4.16 l. gain by the whole.
Alſo 52 l. + 4.16 = 56.16 l. and 160 — 18 = 142 gallons.
∴ 142) 56.16 (.3955 = 7 s. 11 d. nearly, the anſwer.

25. Having paid 14 s. a yard for 100 yards of cloth, I propoſe to gain 25 per cent. ready money; and if I ſell it upon time, to have moreover 10 per cent. per annum for the

the forbearance, what must be the price of one yard, with six months credit, to make both these gains?

$\frac{1}{5}, \frac{1}{2} \Big|$ 100
50
20

£ 70, prime cost.

25 per cent. is $\frac{1}{4}$ 70
17.5
£ 87.5

£.	T.	Gain.
100	12	10
87.5	6	

1200) 5250 (4.375 = 4 l. 7 s. 6 d.
87.5 + 4.375 = 91.875 = 91 l. 17 s. 6 d.
100) 91.875 (.91875 = 18 s. 4½ d. the answer.

26. Laid out in a lot of muslin 480 l. 12 s. upon examination of which two parts in seven proved damaged; so that I could make but 5 s. 6 d. a yard of the same; and by so doing, find I lost 48 l. 18 s. by it; at what rate per ell am I to part with the undamaged muslin, to make up my said loss?

l. s. d.

480 l. 12 s. $= \frac{2403}{5} \times \frac{2}{7} = \frac{4806}{35} = \frac{9612}{70} =$ 137 6 $3\frac{3}{7}$, cost of the damaged goods.

$-\frac{3423}{70} =$ 48 18 –, lost by the damaged goods.

$\frac{6189}{70} =$ 88 8 $3\frac{3}{7}$ made of the damaged goods.

5 s. 6 d. $= \frac{11}{40} : 1 :: \frac{6189}{70} : \frac{24756}{77} = 321\frac{39}{77}$, yards damaged.

$\frac{2}{7} : \frac{24756}{77} : 1 :: \frac{86646}{77} : 1125\frac{21}{77}$ yards in all.

$\frac{86646}{77} - \frac{24756}{77} = \frac{61890}{77} = 803\frac{59}{77}$ yards undamaged.

$\frac{2403}{5} = \frac{33642}{70} - \frac{6189}{70} = \frac{27453}{70} =$ 392 l. 3 s. $8\frac{4}{7}$ d. made of the good.

$\frac{61890}{77} : \frac{27453}{70} :: \frac{5}{4} : \frac{100661}{165030} =$ 12 s. 2 d. $1\frac{5067}{5503}$ qr. per ell. Q. E. F.

CHAPTER V.

BARTER.

WHEN merchants or tradesmen exchange one commodity for another, it is called bartering; and by the rule of proportion, the price and quantity of the goods so exchanged are determined, so that neither party shall sustain a loss by such traffic.

In solving all questions in truck, the intrinsic value of the thing received ought to tally with a like value of the thing delivered, where they deal upon a par; for if there be any difference, some one of the parties has the advantage of the other by the value of that difference.

1. How many pounds of sugar at $4\frac{1}{2}$ d. per pound must be given in barter for 60 gross of inkle, at 8 s. 8 d. per gross?

8 s. 8 d.
12
———
104
2
———
208
60
———
$4\frac{1}{2} = 9$) 12480 halfpence
———
$1386\frac{2}{3}$ pounds, the answer.
———
Or, 1386 lb. and 3 d. in money.

2. Two merchants, A and B, barter; A would exchange 5 cwt. 3 qrs. 14 lb. of pepper, at 3 l. 10 s. per cwt. with B for cotton, worth 10 d. per pound; how much cotton must B give A for his pepper?

	l.	s.	d.		l.	s.	d.	
$\frac{1}{2}$	3	10			20	11	3,	value of A's pepper.
		5			20			
	17	10			411			
	1	15			× 12			
	17	6			———			112) cwt. qr. lb.
	8	9		10)4935	)$493\frac{1}{2}$			$= 4\ 1\ 17$ cotton, the answer.
					$45\frac{1}{2}$			
£	20	11	3					

3. A and B barter; A gives 120 yards of kersey, $3\frac{1}{2}$ yards whereof cost 15 s. 9 d. for stockings at 7 s. a pair, and hats at 6 s. 6 d. each, an equal number of hats, and a pair of stockings; how many of each must B give A for his kersey?

$$\begin{array}{lllll} \text{yds.} & \text{s.} & \text{d.} & \text{l.} & \text{yds.} \\ 3.5 : & 15 & 9 = & .7875 :: & 120 \\ & & & 120 & \\ \hline & & & 94.5000 & (27\text{ l. value of the kersey.} \end{array}$$

Then 7 s. + 6 s. 6 d. = 13 s. 6 d. = .675) 27.000 (40 pair of stockings and hats, the answer.

4. Two merchants, A and B, barter; A would exchange 20 cwt. of cheese, at 1 l. 1 s. 6 d. with B for eight pieces of Irish cloth, at 3 l. 14 s. a-piece; I demand which must receive money, and how much?

	l.	s.
B's 8 pieces of cloth, at 3 l. 14 s. per piece, come to	29	12
A's 20 cwt. of cheese, at 1 l. 1 s. 6 d. per cwt. - -	21	10
So that A is debtor to B - - - £	8	2

5. Two merchants, A and B, barter; A hath 86 yards of broad-cloth, worth 9 s. 2 d. per yard, ready-money; but in barter he will have 11 s. per yard; B hath shalloon, worth 2 s. 1 d. per yard, ready money; it is required to find how many yards of shalloon B must give A for his cloth, making his gain in barter equal to that of A?

The most common method in authors of solving this question is as follows:

As 9 s. 2 d. = 110 d. : 11 s. = 132 d. :: 2 s. 1 d. = 25 d. : 30 d. = 2 s. 6 d. the advanced price of a yard of B's shalloon.

Also 86 × 11 = 946 s. = 47 l. 6 s. advanced value of the cloth.

2.5) 946.0 ($378\frac{2}{5}$ yards of shalloon, the answer.

But when the price of each quantity are raised proportionally, the quantity sought may be found by the ready money values, without having any regard to the advanced prices.

So

So that the foregoing question may be solved as follows:

s.	d.		s.	d.		
9	2		2	1	$=.1041\dot{6}$)	39.41666(
	12				1041)	3.94166(
5	10	–			.09375)	35.47500 (378.4 yards, the answer as before.
	7					

39 8 4 $= 39.41\dot{6}$, value of the cloth.

6. A has currants worth 4 d. a pound, but in truck charges 6 d. and also requires $\frac{1}{2}$ of that in ready money; B has candles at 6 s. 8 d. the dozen, and he in barter, honest man, charges but 7 s. should these persons deal together for the value of 20 l. how much will A have got of B?

6d. : 4 d. : : 20 l. : 13 l. 6 s. 8 d. worth of A's currants.

13 l. 6 s. 8 d. — 10 l. = 3 l. 6 s. 8 d.

7 s. = .35, and 6 s. 6 d. = .325 l.

	l.	s.	d.	f.
.35 l. : .325 :: 10 : $9.\dot{2}8571\dot{4}$ =	9	5	8	$2\frac{2}{7}$
	3	6	8	
A's advantage = £	5	19	–	$2\frac{2}{7}$, answ.

7. Two merchants have various kinds of goods to barter, A hath 735 yards of Indian silk, at 8 s. 6 d. per yard ready money, and in barter 10 s. also 532 canes, at 3 s. a-piece ready money, and in barter 3 s. 4 d. and 16 pieces of muslin, at 4 l. a-piece ready money, in barter 4 l. 10 s. B hath scarlet cloth, at 1 l. per yard ready money; glass manufacture, at 1 s. 8 d. per pound ready money, and a finer kind at 2 s. 4 d. per pound; how many yards of cloth, and pounds of each kind of glass, of all a like number, must B give A, advancing his goods proportionally also in barter?

	l.	s.	d.
A's Indian silk 735 yards, at 8 s. 6 d. is	312	7	6
Canes 532, at 3 s. - - - -	79	16	–
Muslin 16 pieces, at 4 l. -	64	–	–
	£ 456	3	6 = 456.175

B's

	l.	s.	d.
B's ſcarlet cloth, 1 yard - - -	1	—	—
Glaſs manufacture per lb. - - -	—	1	8
Ditto finer ſort - - - - -	—	2	4
	£ 1	4	— = 1.2

∵ 1.2) 455.175 ($380\frac{7}{48}$ of each ſort of B's things, the anſwer.

8. A and B barter; A hath 100 yards of broad-cloth, worth 12 s. a yard ready money; but in barter he will have 13 s. 6 d. and will alſo have $\frac{1}{4}$ of the barter, value in ready money; B hath ſugar at 8 d. a pound; how much ſugar ought B to deliver, and how is it to be raiſed to equal the barter?

100 yards of cloth, at 13 s. 6 d. per yard, is 67 l. 5 s.
4) 67 5

16 16 3 ready money, = 16.8125 l. and 12 s. = 6 l.
.6) 16.8125 ($28\frac{1}{48}$ yards. Alſo 8 d. = .03
Then 100 — $28\frac{1}{48}$ = $71\frac{47}{48}$ yds. which at 12 s. is 43 l. 3 s. 9 d. = 43 1875
.033) 43.1875
.03 43.1875

.03) 38.86875 ($1262\frac{1}{3}$ fere, pounds of ſugar.
Alſo 12 s. : 8 d. :: 13.5 s. : 9 d. advanced price of the ſugar.

9. A has kerſeys at 4 l. 5 s. a piece ready money; in barter they are charged by him at 5 l. 6 s. each, and half of that required down; B has flax at 3 d. a pound; how ought he to rate it in truck, not to be hurt by the extortion of A?

Firſt, 5 l. 6 s. — 4 l. 5 s. = 1 l. 1 s. gain. 2) 5 l. 6 s. (2 l. 13 s. paid.

Then 4 l. 5 s. — 2 l. 13 s. = 1 l. 12 s. = 384 pence, value of the half remaining.

And 1 l. 12 s. + 1 l. 1 s. = 2 l. 13 s. = 636 pence, made of the half remaining.

∵ 384 : 636 :: 3 : $4\frac{31}{32}$, the anſwer required.

10. A hath 40 pair of ſtockings, at 3 s. a pair ready money, or 3 s. 8 d. in barter; but he is willing to diſcount three

three per cent. of his barter price, to have $\frac{1}{4}$ of it paid in ready money; how many yards muſt he deliver with the money A requires, and what is the rate of his cloth to equal the barter?

Firſt, $100 - 3 = 97$. and 3s. 8d. $= .18\dot{3}$ l. alſo 10 s. $=$ .5 l.

$100 : 97 :: .18\dot{3} : .1778\dot{3} =$ barter price after the diſcount.

And $40 \times .1778\dot{3} = 7.11\dot{3}$ l. value of the ſtockings at the ſaid price.

$4)7.11\dot{3}(1.778\dot{3} =$ 1 l. 15 s. $6\frac{4}{5}$ d. ready money.

$.15)1.778\dot{3}(11.8\dot{5}$ pair in value.

$40 - 11.8\dot{5} = 28.1\dot{4}$ pair, at 3s. is $4.221\dot{6}$.

$.5)4.221\dot{6}(8.44\dot{3} =$ 8 yards, 1 quarter, 3 nails, nearly.

$.15 : .178\dot{3} :: .5 : .59\dot{4} =$ 11 s. $10.\dot{6}$ d. the advanced price of the broad-cloth.

11. A let B have a hogſhead of ſugar, of 18 hundred weight, worth 31 s. for 42 s. the hogſhead, $\frac{1}{3}$ of which he is to pay in caſh; B hath paper worth 14 s. the ream, which it is agreed ſhall bear no more than 15 s. 6 d. and at that rate truck for the reſt; how ſtood the account?

$$\text{18 cwt. ſugar, at } \left\{\begin{array}{rr} \text{l.} & \text{s.} \\ 2 & 2 \\ 1 & 11 \end{array}\right\} \text{ is } \begin{array}{rrl} \text{l.} & \text{s.} & \\ 37 & 16, & \text{advanced value.} \\ 27 & 18, & \text{real value,} \\ \hline \text{A £ } 9 & 18 & \text{advanced his ſugar.} \end{array}$$

3) 37 16 (12 l. 12 s. ready money, and 25 l. 4 s. in paper.

Then 15 s. 6 d. $= \frac{31}{40}$, 14 s. $= \frac{28}{40}$, and 25 l. 4 s. $= \frac{126}{5}$ £.

$$\text{And } \frac{31}{40} - \frac{28}{40} = \frac{3}{40} = \text{1 s. 6 d.}$$

As $\frac{31}{40} : \frac{3}{40} :: \frac{126}{5} : \frac{378}{155} =$ 2 l. 8 s. $9\frac{9}{31}$ d. B advanced his paper.

$\therefore$ 9 l. 18 s. $-$ 2 l. 8 s. $9\frac{9}{31}$ d. $=$ 7 l. 9 s. $2\frac{22}{31}$ d. the anſwer in A's favour.

12. A barters with B 40 lb. of cloves at 6 s. a pound, ready money, and 7 s. 6 d. in barter, but is willing to loſe 10 per cent. to have $\frac{1}{3}$ ready money, what is the ready money price of a yard of velvet, delivered by B, at 1 l. 1 s. to equal the barter, and how much was delivered?

Firſt, $100 - 10 = 90$, and 7 s. 6 d. $= .375$.

As $100 : 90 :: .375 : .3375 =$ 6 s. 9 d. $=$ A's barter price, when 10 per cent. is deducted.

40

$40 \times .3375 = 13.5 = 13$ l. 10s. value of the cloves at 6s. 9d.

21 s. $= 1.05)$ 13.5 ($12\frac{6}{7}$ yards of velvet B must deliver.

$6.75 : 6 :: 21 : 18.\dot{6} =$ 18s. 8d. the ready money, price of the velvet. Q. E. F.

I am beholden to the sagacious Alexander Malcolm, teacher of the mathematics at Aberdeen, for the foregoing as well as the 7th, 8th, and 10th questions, and some others in this work; who, after exploding the mistakes of their first proposers, points out, and fully demonstrates the above method to be true.

13. A has 50 broad-cloths, at 11 l. 10s. apiece; but in change requires 13 l. taking wool at 2s. 6d. per stone of B in return, that was really worth but 4s. 2d. a tod; the question is, how many sacks of wool will pay for the cloth, and which of the dealers has the better in the bargain?

l.	s.			l.	
13	–	× 50	=	650,	advanced value of the broad-cloth.
11	10	× 50	=	575,	real value.
				£ 75,	gain by the broad-cloth.

s.	d.			s.	d.	
2	6	× 2	=	5	–,	advanced value of the wool per tod.
				– 4	2,	real value.
			£	–	10	per tod gain, or 5d. per stone.

2s. 6d. $= .125 : 1 :: 650 : 5200$ stone.

1 sack $= 26)$ 5200 (200 sacks of wool for five pieces of cloth.

$\frac{1}{12}$	5200 stone, at 5d. per stone profit.		
		s.	d.
$\frac{1}{4}$	433	1	8
£	108	6	8, gain by the wool.

Therefore 108 l. 6s. 8d. – 75 l. = 33 l. 6s. 8d. B's clear gain by this affair.

14. A has 100 ream of paper, at 8s. ready money, which in barter he sets down at 10s. B, sensible of this, has pamphlets at 6d. apiece, ready money, which he ade-

 quately

quately charges, and insists to have over and above $\frac{1}{4}$ of the price of those he parts with in specie. What number of books is he to deliver in lieu of A's paper, what cash will make good the difference, and how much is B the gainer by this affair?

As 8 s. : 10 s. : : 6 d. : $7\frac{1}{2}$ d. advanced price of B's pamphlets.

$\frac{1}{2}$, $\frac{1}{5}$ | 100 reams of paper.

20
×20

£ 40, real } value of the paper.
£ 50, advanced }

£ 12 10 s. B to have in cash.

40, value also of B's pamphlets.
×40, sixpences in 1 l.

1600, pamphlets to be delivered.

∵ 40 l. — 12 l. 10 s. = 27 l. 10 s. what they then stood him in; so that the advantage to B was 12 l. 10 s.

15. A and B barter; A hath 140 lb. 11 oz. of plate, at 6 s. 4 d. per ounce, which in truck he rates at 7 s. 2 d. an ounce, and allows a discount on his part to have $\frac{1}{7}$ of that in ready money; B has tea worth 9 s. 6 d. the pound, which he rates at 11 s. 2 d. When they come to strike the balance, A received but 7 cwt. 2 qrs. 18 lb. of tea: pray what discount did A allow B, which of them had the advantage, and how much, in an article of trade thus circumstanced?

	oz.		s.	d.
$\frac{1}{10}$, $\frac{1}{6}$	1691	at	6	4
	281		16	8
$\frac{1}{2}$	169		2	–
	84		11	–
	£ 535		9	8, real value of A's plate.

	oz.		s.	d.
$\frac{1}{40}$, $\frac{1}{3}$	1691	at	7	2
	563		13	4
	42		5	6
	£ 605		18	10, its advanced value.

605 l. 18 s. 10 d. — 535 l. 9 s. 8 d. = 70 l. 9 s. 2 d.

$\frac{1}{7}$) 605

	l.	s.	d.			
$\frac{1}{7}$)	605	18	10			
—	86	11	$3\frac{1}{7}$,	prompt payment.		

			cwt.	qrs.	lb.	s.	d.	
519	7	$6\frac{6}{7}$	7	2	18, at	9	6	per lb.
							8	

cwt.	qr.	lb.	s.	d.					
7	2	18, at	11	2		3	16	–	
				8				7	
		4	9	4		26	12	–	
				7				2	
		31	5	4		53	4	–	
				2				7	
		$\frac{1}{7}$ 62	10	8		372	8	–	
				7		26	12	–	
						7	12	–	
		437	14	8		–	19	–	
		31	5	4					
		8	18	8		407	11	–,	real value of B's tea.
		1	2	4					
		479	1	–,	advanced value of B's tea.				

£519 7 $6\frac{6}{7}$ — 479 l. 1 s. = 40 l. 6 s. $6\frac{6}{7}$ d. discount allowed by A.

	£519	7	$6\frac{6}{7}$	
—	470	11	–	
	111	16	$6\frac{6}{7}$	
—	70	9	2,	A's advantage by the rise of his plate.
	£41	7	$4\frac{6}{7}$,	B's whole advantage. Q. E. F.

16. A, with intention to clear 30 guineas on a bargain with B, rates hops at 16 d. per lb. that ſtood him in 10 d. B, appriſed of that, ſets down malt, which coſt 20 s. a quarter at an adequate price; how much malt did they contract for?

As 10 d. : 16 d. : : 20 s. : 32 s. advanced value of the malt.

32 — 20 = 12, B's gain per quarter.

Guineas 30 × 21 = 630 ſhillings.

12) 630 ($52\frac{1}{2}$ quarters = 420 buſhels, the anſwer.

17. A and B truck; A has 14 cwt. 81 lb. of Farnham hops, at 2 l. 19 s. per cwt. but in barter insists on three guineas; B has wine worth 6 s. per gallon, which he raises in proportion to A's demand. On the balance A received but a hogshead and a half of wine, pray what had he in ready money?

cwt.	lb.		l.	s.	d.	cwt.	qrs.	lb.
14	81	at	3	3				
				7				
			22	1				
				2				
			44	2	–	14	–	–
			1	11	6	–	2	–
			–	9	–	–	–	16
			–	4	6	–	–	8
			–	–	$6\frac{1}{4}$	–	–	1
			£46	7	$6\frac{3}{4}$			

As 2.95 : 3.15 :: .3 : .320339, advanced value of one gallon of wine.

$1\frac{1}{2}$ hogshead of wine = 94.5 gal. × .320339 = 30.272.

30.272 l. = 30 l. 5 s. $5\frac{1}{4}$d. advanced value of the wine.

∴ 46 l. 7 s. $6\frac{3}{4}$ d. — 30 l. 5 s. $5\frac{1}{2}$ d = 16 l. 2 s. $1\frac{1}{2}$ d. Q. E. F.

18. A, in order to put off 720 ells of damaged Holland, worth 5 s. an ell, at 6 s. 8 d, proposes, in case he has half the value in money, to give B thereon a discount of 10 per cent. the rest A is to take out in saffron; which B, apprised of the whole management, rates in justice at 30 s. the pound: pray what was it really worth in ready money; and what quantity of saffron was he to deliver on the change?

$\frac{1}{3}$, $\frac{1}{5}$ | 720 ells.

144, real } value of the Holland.
$\frac{1}{10}$ | 240, advanced } value of the Holland.

24, discount.

$\frac{1}{2}$ | 216, remains.

108, paid in ready money.

∴ 216

∴ 216 : 144 :: 30 s. : 20 s. per pound, real value of the saffron.

And 1.5 l. : 1 lb. :: 108 : 72 lb. the quantity delivered. Q. E. F.

CHAPTER VI.

SECT. I.

EXCHANGE.

EXCHANGING the coins of one country into those of another, is like the business of bartering commodities (that is) it consists of finding what sum of one country coin will be equal in value to any proposed sum of another country's coin; and in order to perform that, it will be necessary to have a true account at all times of the just value of those foreign coins which are to be exchanged, as they are compared in value of our English coin; for the par of exchange (as the merchants call it) differs almost every day from London to other countries; that is, it rises and falls, according as money is plenty or scarce, or according to the time allowed for payment of money in exchange.

If our purchases and payments in foreign countries exactly balance their purchases and payments in ours, there will be just enough of bills on the one to clear accounts with the other; so that in this case the exchange on both sides will be at par; that is, one who gives money in one country, will receive as much in the other in weight and standard.

If a nation supplies us with more than it takes from us, or if we pay that nation more money than it pays to us, there will be a balance against us, which we must necessarily pay; in order to which, the demand for the money of that nation, or its bills of exchange, becomes greater among us than the quantity to supply that demand, which raises the value of their money or bills, and lowers ours; or in other words, puts the price of their money above par, and ours below it, which constitutes what we call the course of exchange. From hence we may naturally infer,

I. That the course of exchange betwixt two nations is a herald, which proclaims publickly the state of commerce and

money-negotiation betwixt them, and which of the two is indebted to the other.

II. That the nation which is indebted hath the disadvantage in commerce and money-transactions; and that the one which hath the balance in its favour hath in every respect the advantage.

III. That the balance of trade naturally imports specie, and renders money at home more valuable abroad; whereas, on the other hand, when the balance is against a nation, their specie is exported, and becomes thereby less valued.

The English standard for gold coin is 22 carats of fine gold and two carats or $\frac{1}{12}$ of alloy.

In the royal mint a pound of standard gold is divided into $44\frac{1}{2}$ parts, each a guinea, at which rate a guinea will weigh 5 penny-weights, 9·4382 grains.

The English standard for silver is 11 ozs. 2 pwts. of fine silver, and 18 pwts of alloy, in the pound $= \frac{3}{40}$.

The pound weight standard silver is divided into 62 parts, each a shilling; so that a shilling will weigh 3 pwts. 20.9 gr.

A TABLE *of the proportion of the value in several nations of the world between gold and silver, taken from Postlethwayt's Universal Dictionary of Trade and Commerce.*

	gold.		silver.
In Japan one ounce of gold is	1	is equal in value to	8
China	1		10
Mogul empire	1		12
France	1		15
Spain and Portugal	1		16
But as they required a premium of six per cent. on payments in silver, it reduces it to	1		$15\frac{1}{25}$
England			$15\frac{1}{5}$

Explanation and use of the following TABLE *of coins* (viz.)

One pound troy	contains	12 ounces.
one ounce		20 pennyweights.
one pennyweight		24 grains.
one grain		20 mites.

The

The first column expresses the fineness of the assayed piece; the B. signifying better, and the W. worse than the English standard.

The second column is the absolute weight of the piece.

The third column its standard weight, or its quantity of standard metal.

The fourth column its value in English money.

Ex. gr. in the second article of silver coin, the new Seville piece of eight is $1\frac{1}{2}$ pwt. in the pound worse than English standard weight; 13 penny-weights, 21 grains, 15 mites of sterling silver, is in value 43 pence, and 11 decimal parts of a penny.

And in the first article of gold coin, the old Louis d'ors is half a grain worse than English standard; its weight is four penny-weights, 7 grains, 8 mites of English standard gold, and its value 16 shillings, $9\frac{3}{10}$ pence.

The par of exchange between English and Dutch money is easily found, thus: as by Sir Isaac's table, the ducatoon of Holland is worth intrinsically 65.59 d. English; which is received at the Bank at 60 stivers, or three guilders, and consequently is equal to 10 shillings Flemish; therefore, by the rule of three, as 65.59 d. English is to 10 s. Flemish; so is 240 d. in a pound English, to a fourth number, which will be found to be 36.59 s. Flemish; and so much Bank money at Amsterdam should be received for one pound, or 240 pence sterling.

Sir Isaac Newton's TABLE of the assays, weights and values of most silver and gold coins, actually made at the Mint, by order of the privy-council, with a calculation of the real or intrinsic par of exchange.

FOREIGN SILVER COINS.

	Assay.	weight.		stand. wt.			value.
	dwt.	dr.	grs.	dt.	gr.	mi.	d.
The piaster of Spain, or Seville piece of eight reals, now ten	W. 1	17	12	17	10	2	54.
The new Seville piece of eight	W. 1½	14	—	13	21	15	43.11
The Mexico piece of eight	W. 1	17	10 5/9	17	8	14	53.83
The pillar piece of eight	Stand.	17	9	17	9	—	53.87
The Peru piece of eight, coarser, but of uncertain alloy.	—	—	—	—	—	—	—
The old ecu of France, or piece of 60 sols Turnois	W. 1	17	12	17	10	2	54
The new ecu, or piece of five livres, or 100 sols	W. 1½	19	14½	19	11	12	60.39
N. B. The ecu of France should be two pennyweights worse by law.	—	—	—	—	—	—	—
The crusado of Portugal, or ducat, worth 400 reas, raised to 480	W. 2	11	4	11	1	13	34.31
The patacks, or patagons, of Portugal, worth 500 reas, marked and raised to 600	—	—	—	—	—	—	—
The ducatoon of Flanders, or piece of 60 sols or patars	B. 4½	20	22	21	8	2	66.15
The patagoon of Flanders, or cross dollar, or piece of 48 patars	W. 12	18	1	17	1	13	52.91
The ducatoon of Holland, or piece of 63 stivers	B. 3	20	21	21	3	15	65.59
The patagoon leg dollar, or rix-dollar of Holland, or piece of 50 stivers	W. 14	18	—	16	20	17	52.28
The three guilder pieces of Holland, or piece of 60 stivers	W. 2	20	8	20	3	12	62.46
The guilder, florin, or piece of 20 stivers	W. 2	6	18½	6	17	1	20.08
The ten shilling piece of Zeland, or piece of 60 stivers	W. 2	20	6	20	1	13	62.21
The lion dollar of Holland, or ⅔ of the ducatoon	W. 44	17	14	14	2	7	43.07

The ducatoon of Cologn	B. 3	20	18	21	–	15	65.02
The rix-dollar, or patagon of Cologn	W. 13	18	–	16	22	14	52.53
The rix-dollar, or patagon of the bishop of Liege	W. 12	17	$22\frac{1}{2}$	16	22	5	55.48
The rix dollar of Ments	W. $6\frac{1}{2}$	18	8	17	19	18	55.27
The rix-dollar of Francfort	W. 9	18	8	17	14	4	54.53
The rix-dollar of the elector Palatine of the Rhine and Bavaria before 1620	–	18	5	—	—	—	—
The rix-dollar of Nuremberg	W. 6	18	10	17	22	1	55.55
The old rix dollar of Lunemberg	W. 10	18	11	17	15	2	54.65
The old rix-dollar of Hanover	W. 8	18	12	17	20	2	55.03
The double gulden of the elector of Hanover	W. 7	18	18	18	3	16	56.29
The gulden of the elector of Hanover, or piece of $\frac{2}{3}$	B. $17\frac{1}{2}$	8	10	9	1	18	28.14
The half-gulden of the elector of Hanover, or piece of $\frac{1}{3}$	B. $17\frac{1}{2}$	4	5	4	12	19	14.07
The gulden of the duke of Zell; or piece of 16 gutz grosh	W. 43	11	2	8	22	10	27.07
The gulden of the bishop of Hildesheim, or piece of 24 manen grosh, now raised to 26	W. $40\frac{1}{2}$	11	22	9	17	17	30.21
The rixdollar of Magdeburg	W. 10	18	12	17	16	1	54.27
The gulden, or guilder of Magdeburg	W. 44	11	14	9	6	–	28.67
The old rix-dollar of the elector of Brandenburg	W. 9	18	13	17	19	1	55.17
The old gulden of Brandenburg, now raised from 24 to 26 manen grosh	W. 43	12	4	9	19	9	30.41
The gulden of Brandenburgh, or piece of $\frac{2}{3}$	W. 43	11	3	8	23	6	27.81
The half-gulden of Brandenburg, or piece of $\frac{1}{3}$	W. 43	5	13	4	11	14	13.09
The gulden of the elector of Saxony, or piece of $\frac{2}{3}$	W. 41	11	3	9	1	14	28.12
The old blank dollar of Hamburgh	W. 8	18	9	17	17	4	54.92

The

The old rix-dollar of Lubec	W. 8½	18	16	17	22	17	55.54
The four-mark piece of Denmark, of coarser alloy	W.61	14	8	10	9	10	32.23
The four-mark piece of Denmark, of finer alloy	W.21	11	13½	10	11	5	32 45
The eight-mark piece of Sweden	Stand.	20	—	20	—	—	62.
The four-mark piece of Sweden	W.58	13	12	9	23	7	30.92
The two-mark piece of Sweden	W.—	6	19	—	—	—	— —
The old dollar of Dantzick	W.10½	18	9	17	12	4	54.27
The old rix-dollar of Thorn, near Dantzick	W.12	18	8½	17	8	15	53.85
The rix-dollars of Sigismund III. and Vladislaus IV. kings of Poland	W.10	18	9	17	13	14	54.04
The rix-dollar of the late emperor Leopold	W.10½	18	9	17	12	4	54.27
The rix-dollar of his predecessor Ferdinand III.	W.10½	18	9	71	12	4	54.27
The rix-dollar of Ferdinand archduke of Austria	W.10½	18	5	17	8	7	53.78
The rix-dollar of Basil	W. 7½	18	18½	18	3	6	56.24
The rix-dollar of Zune	W.13	18	1	16	23	13	52 65
The old ducat of Venice, with the words *Ducatus Venetus* upon it; a piece of six old livres, afterwards raised to six livres four sols de Picoli	W.23½	14	15	13	1	17	40.50
The half ducat	W.23½	7	7½	6	12	18	20.25
The new ducat, with the number 124 upon it, signifying 124 sols, or six livres four sols de Picoli	—	18	2	—	—	—	— —
The half thereof	—	9	1	—	—	—	— —
The crusado croisat, or St. Mark of Venice, with number 140 upon it, signifying 140 sols, or seven livres de Picoli	—	20	6	—	—	—	— —
The half crusado of the same form	—	10	3	—	—	—	— —
The quarter crusado of the same form	—	5	1½	—	—	—	— —

Another coin of Venice - - - - - - - - - - - - - - - -	W.	46	17	10	13	19	8	42.08
The piece of two jules - - - - - - - - - - - - - - - -	B.	6	3	15	3	17	7	11.05
The ducat de banco of Naples, or piece of 5 tarians, or 10 carlins, or 100 gr.	W.	3	14	— $\frac{1}{4}$	13	1	—	40.43
The half ducat - - - - - - - - - - - - - - - - - -	W.	3	7	— $\frac{1}{8}$	6	12	10	20.21
The tarin, or fifth part of the ducat - - - - - - - - - - -	W.	3	2	19	2	14	12	8.09
The carlin, or tenth part of the ducat - - - - - - - - - -	W.	3	1	$9\frac{1}{2}$	1	7	6	4.04
The escudi ecu, or crown of Rome, or piece of 10 Julios, or 100 bayoches		—	20	$14\frac{1}{2}$	—	—	—	— —
The teston of Rome, or piece of three Julios - - - - - - - -	W.	1	5	$21\frac{1}{2}$	5	20	17	18.32
The ducat of Florence and Leghorn, or piece of 7 livres, or $10\frac{1}{2}$ Julios	B.	8	20	3	20	20	6	64.62
The Julio of Rome - - - - - - - - - - - - - - - - -		—	2	5	—	—	—	— —
The piaster ecu, or crown of Ferdinand II. duke of Tuscany - - -	W.	1	17	12	17	10	2	54
The piaster ecu, or crown of Cosmus III. duke of Tuscany, whose monies are about four per cent. lighter than those of his father; this piece is $8\frac{1}{2}$ Julios - - - - - - - - - - - - - -	W.	1	16	18	16	16	4	51.69
The croisat of Genoa, or piece of $7\frac{1}{2}$ lires - - - - - - - - -	B.	7	24	15	25	9	11	78.74
The ecu d'argent of Genoa, or piece of 7 livres 12 sols - - - -		—	—	—	—	—	—	— —
The piaster ecu, or crown of Milan - - - - - - - - - - - -		—	17	21	—	—	—	— —
The Philip of Milan, or piece of 7 livres - - - - - - - - - -		—	20	20	—	—	—	— —
The livre, or 20 sols piece of Savoy - - - - - - - - - - -		—	3	22	—	—	—	— —
The 10 sols piece of Savoy - - - - - - - - - - - - - -		—	1	23	—	—	—	— —
GOLD. A roupee -	B.	$16\frac{1}{2}$	7	10	7	23	4	24.07
A gout gulden, or florin d'or, a Dutch coin of 28 stivers - - - -	W.	75	12	19	8	11	5	26.26
Another gout gulden - - - - - - - - - - - - - - - - - -	W.	48	11	—	8	14	18	26.72
Another -	W.	48	2	—	9	9	15	29.15

GOLD COINS UNWORN.

	Assay.		weight.		stand. wt.			value.	
	car.	gr.	dt.	grs.	dt.	gr.	mi.	s.	d.
The old Louis d'or - - - - - - - -	W.	- - $\frac{1}{2}$	4	8	4	7	8	16	9.3
The half and quarter in proportion - - - -	W.	- - $\frac{1}{2}$	2	4	2	3	14	8	5.
The new Louis d'or - - - - - - - -	W.	- - $\frac{1}{2}$	5	$5\frac{2}{3}$	5	3	18	20	0.6
The half and quarter in proportion - - - - -	W.	- - $\frac{1}{2}$	2	$14\frac{7}{10}$	2	13	19	10	0.3
The old Spanish double doublon - - - - -	W.	- - $\frac{1}{2}$	17	8	17	5	12	67	1.4
The old Spanish double pistole - - - - -	W.	- - $\frac{1}{2}$	8	16	8	14	16	33	6.7
The old Spanish pistole - - - - - - -	W.	- - $\frac{1}{2}$	4	8	4	7	8	16	9.3
The new Seville double pistole - - - - - -	W.	- -	8	$16\frac{1}{3}$	—	—	—	—	—
The new Seville pistole - - - - - - -	—	—	4	8 [illegible]	—	—	—	—	—
The half and quarter in proportion - - - -	—	—	—	—	—	—	—	—	—
The doppia mœda, or double mœda of Portugal, new coined -	W.	- - $\frac{1}{4}$	6	22	6	21	12	26	10.4
The doppia mœda as they come into England - - - -	W.	- - $\frac{1}{4}$	6	$21\frac{3}{4}$	6	21	7	26	9.9
The mœda of Portugal - - - - - - - -	W.	- - $\frac{1}{4}$	3	11	3	10	16	13	5.1
The half mœda - - - - - - - - -	W.	- - $\frac{1}{4}$	1	$17\frac{1}{2}$	1	17	8	6	8.5
The Hungary ducat - - - - - - - -	B.	1 2	2	$5\frac{2}{3}$	2	9	7	9	3.6
The ducat of Holland coined *ad legem imperii* - - - - -	B.	1 2	2	$5\frac{1}{2}$	2	9	3	9	3.2
The ducat of Campen in Holland - - - - - - -	B.	1 2	2	$5\frac{1}{2}$	2	9	3	9	3.2
The ducat of the bishop of Bamberg - - - - - -	B.	1 2	2	$5\frac{1}{2}$	2	9	3	9	3.2
The double ducat of the duke of Hanover - - -	B.	1 2	4	$10\frac{1}{2}$	4	17	9	18	4.8
The ducat of the duke of Hanover - - - - - -	B.	1 2	2	$5\frac{1}{1}$	2	8	18	9	2.7
The ducat of Brandenburg - - - - - - -	B.	1 2	2	$5\frac{1}{2}$	2	9	3	9	3.2
The ducat of Sweden - - - - - - - -	B.	1 2	2	$5\frac{1}{2}$	2	9	3	9	3.2

The ducat of Denmark	B.	1	2	2	$5\frac{1}{2}$	2	9	3	9	3.2
The ducat of Poland	B.	1	2	2	5	2	8	12	9	2.1
The ducat of Tranſilvania	B.	1	$1\frac{1}{2}$	2	$4\frac{3}{4}$	2	7	6	8	11.6
The ſequin, chequin, or zacheen of Venice	B.	1	$2\frac{1}{2}$	2	$4\frac{3}{4}$	2	10	7	9	5.7
The old Italian piſtole	B.	1	$3\frac{1}{2}$	4	$6\frac{3}{4}$	4	6	11	16	7.6
The double piſtole of pope Urban, 1634	W.	—	$-\frac{1}{4}$	8	$14\frac{1}{2}$	—	—	—	—	—
The half piſtole of Innocent II. 1685	—	—	—	2	4	—	—	—	—	—
A double piſtole of Placentia	—	—	—	8	10	—	—	—	—	—
A double piſtole of Genoa, 1621	—	—	—	8	16	—	—	—	—	—
A double piſtole of Milan	—	—	—	8	$13\frac{1}{2}$	—	—	—	—	—
A ſingle piſtole of Milan	—	—	—	4	$6\frac{3}{4}$	—	—	—	—	—
A piſtole of Savoy, 1675	—	—	—	4	$8\frac{1}{2}$	—	—	—	—	—
Double ducat of Caſtile, Genoa, Portugal, Florence, Hungary, and Venice	B.	1	$2\frac{1}{2}$	4	11	4	18	18	18	7.7
Single ducats of the ſame places	B.	1	$2\frac{1}{2}$	2	$5\frac{1}{2}$	2	9	9	9	3.8
Double ducats of ſeveral forms in Germany	B.	1	1	4	11	4	17	1	18	4
Single ducats of the ſame places	B.	1	1	2	5	2	8	$5\frac{1}{2}$	9	2
Double ducats of Genoa	B.	1	2	4	11	4	18	6	18	6.5
Single ducats of Genoa, Beſançon, and Zurich	B.	1	2	2	$5\frac{1}{2}$	2	9	3	9	3.2
Piſtole of Rome, Milan, Venice, Florence, Savoy, Genoa, Orange, Trevou, and Beſançon	W.	—	$-\frac{1}{4}$	4	6	4	5	17	16	6.7
A Barbary ducat, with Arabic letters on both ſides, in ſquare tablets, without any effigies or eſcutcheon	W.	2	$1\frac{1}{2}$	2	16	2	9	6	9	3.5

ENGLAND,

ENGLAND *with* HOLLAND, FLANDERS, *and* GERMANY.

The bank of Amſterdam is the moſt conſiderable in Europe; and as buſineſs therein is negotiated by transfers, millions may be paid in a day, without the intervention of any caſh; which is of the greateſt conſequence imaginable in expediting trade; and is productive of ſo great ſecurity, that bank payments is reckoned from 3 to 6 per cent. better than payments in caſh, although a premium is alſo allowed the bank for every depoſite.

The Hollanders keep their accounts in florins or guilders, ſtivers and pennings, or in pounds, ſhillings, and pence Flemiſh, divided as the pound ſterling.

8 pennings	make one	grot.
2 grots	make one	ſtiver.
6 ſtivers	make one	ſhilling.
20 ſtivers	make one	florin or guilder.
$2\frac{1}{2}$ florins	make one	rix-dollar.
6 florins	make one	pound Flemiſh.
5 guilders	make one	ducat.

Exchange is made with London from 30 to 38 ſhillings Flemiſh, for one pound ſterling.

CASE I.

Given the ſum due in one country coin, and that payable in another country coin, to find the rate of exchange.

RULE.

As the ſum due : is to that paid or payable : : ſo is an unit of the firſt : to the value of an unit of a ſecond.

1. A merchant at Amſterdam paid 150 guilders for 13 l. 15 s. received by his correſpondent at London: what is the value of a guilder?

As 150 guil. : 13.75 l. : : 1 guil. : 1s. 10d the anſwer.

2. If I receive in London 678 l. 15 s. $9\frac{3}{4}$d. for 1173 l. 14 s. 10 d. Flemiſh, due at Rotterdam; what is the rate of exchange?

As 678.790625 l. ſter. : 1173.7416 l. Flem. : : 1 : 1 l. 14 s. 7d. Flemiſh, for 1 l. ſterling.

N. B.

N. B. That the country in whose money the course of exchange is reckoned, has always the greatest advantage the lower the course of exchange runs.

CASE II.

To reduce Flemish pounds, shillings and pence into guilders and stivers,

RULE.

Bring them into pence Flemish, then divide by 40 (because 40 pence make one guilder) and the quotient will be guilders; and if any thing remain, divide it by 2 (because two pence make one stiver) and the quotient will be stivers.

3. In 1173 l. 14 s. 10 d. Flemish; how many guilders?

1173 l. 14 s. 10 d. = 281698 pence.
40) 281698 (7042 guilders, 9 stivers.

By PRACTICE.

		stiv.		
1173 × 6	=	7038	-	6 guilders being 1 l. Flemish.
10 s.	=	3	-	$\frac{1}{2}$ of 6 guilders. or 1 l.
4 s.	=	1	4	$\frac{1}{5}$ of 1 l.
- 10 d.	=	-	5	$\frac{1}{2}$ of 10 d.
Guilders		7042	9	stivers.

4. In 7042 guilders and 9 stivers, how many Flemish pounds?

guil. stiv.
6) 7042 9

1173 - , remains 5 guilders, 9 stivers.

		s.	d.
3 guilders	=	10	-
1 guilder	=	3	4
9 stivers	=	1	6
£ 1173		14	10, the answer.

CASE

CASE III.

To reduce sterling into Flemish money.

RULE.

As one pound sterling : is to the given rate of exchange :: so is the given sterling : to the Flemish sought.

5. If I pay in London 678 l. 15 s. $9\frac{3}{4}$ d. what may I draw my bill for on Amsterdam, exchange 1 l. 14 s. 7 d. per pound sterling?

As 1 l. : 1.729161. Flem. :: 678.790625 : 1173.7416 = 1173 l. 14 s. 10 d. Flemish.

By Practice.

	l.	s.	d.	
$\frac{1}{5}$, $\frac{1}{2}$	678	15	$9\frac{3}{4}$	
	339	7	$10\frac{3}{4}$	
$\frac{1}{8}$	135	15	$1\frac{1}{4}$	
$\frac{1}{6}$	16	19	$4\frac{1}{2}$	
	2	16	$6\frac{3}{4}$	
	1173	14	$9\frac{1}{2}$	Flemish money, the answer.

CASE IV.

To reduce Flemish money into sterling.

RULE.

As the given rate of exchange : is to one pound sterling :: so is the given Flemish : to the sterling required.

6. Change

6. Change 1173 l. 14 s. 10 d. Flemish into sterling exchange, at 34 s. 7 d. per pound sterling.

```
   s.  d.           l.  s.  d.
   34  7 : 1 :: 1173  14  10
 × 12               20
 ----             -----
  415             23474
                     12
                  -----     l.  s.  d.
         415)281698(678  15  9½
              3269
               3648
              -----
                328
                 20
              -----
               6560
               2410
              -----
                335
                 12
              -----
               4020
              -----
                285
                  4
              -----
               1140
              -----
                310
              -----
```

7. In 1036 l. Flemish, exchange at 34 s. $4\frac{1}{2}$ d. how much sterling?

```
s.  d.           l.
34  4½ : 1 :: 1036
12                 6
---             ----
412             6216 gilders.
  2               20
---            -----
825            12430 stivers
                   4
              ------  l.   s.  d.
825) 497280 (602  15  3¼ sterling.
      2280
      ----
       630
        20
     -----
     12600
      4350
     -----
       225
        12
      ----
      2700
       225
         4
       ---
       900
```

8. In 5875 florins 17 stivers banco, how many pounds sterling, exchange at 32 s. 10 d.?

```
s.  d.   l.    flor.  st.
32  10 : 1 :: 5875  17
12               20
---          ------
394          117517 stivers.
                  2
394) 235034 (596 l. 10 s. 8 d.
      3803
      2574
      ----
       210
        20
      ----
      4200
       260
        12
      ----
      3120
```

Agio signifies the difference of the value of current money and bank notes in Holland, Venice, &c. which in Holland is from 3 to 6 per cent. in favour of the notes; also the reward given for the changing one coin, or species of money for another.

CASE V.

To turn current money into banco.

RULE.

As 100 with the agio added to it : is to 100 :: so is any given sum current : to its value in banco.

9. In 3758 florins, 15 stivers current, agio $5\frac{5}{8}$ per cent. how many pounds sterling, exchange at 35 s. 11 d.?

As 20 stivers make 1 florin ∴ $\frac{5}{8}$ florin = $12\frac{1}{2}$ stivers.

```
             flor. stiv.    flor.     flor. stiv.
Then as 104  12½   :  100  ::  3785  15
              20                 20
             ----              ------
             2092              75175
                2                  2
             ----              ------
             4185 pence        150350
                              --------  flor.  stiv.
                     4185) 15035000 (3592  12 banco.
                            24800
                             38750
                              10850
                             ------
                               2480
                                 20
                             ------
                              49600
                               7750
                             ------
```

 Also,

```
             s.  d.  l.     flor.  ftiv.
Alfo, as 35  11 : 1 ::  3592   12
        ———             ———
        431             78152
                            2
                        ———    l.  s.  d.
                431 )143704(333  8  4¾, anfwer.
                      1440
                       1474
                      ———
                        181
                         20
                      ———
                       3620
                        172
                      ———
                         12
                      ———
                       2064
                      ———
                        340
                          4
                      ———
                       1360
                      ———
                         67
                      ———
```

CASE VI.

To turn banco into current money.

As 100 : is to 100 with the agio added :: fo is any given banco : to its value current.

10. In 456 l. 8 s. fterling, how many rix-dollars current, agio 4⅝, exchange 36 s. 1½ d.?

```
        l.    s.
 ½ | 456    8
 ½ | 228    4
 ⅕ | 114    2
 ⅛ |  22   16   4¾
   |   2   17   -½
     ———————  ———
     824    7   5¼
          × 6
   ————————————
    4646    4   7½
       2
   ————
 5)6892
   ————
Rix-dollars  1978  24 ftivers banco.
```

As

R. D. st.

As 100 : $104\frac{5}{8}$ :: 1978 24

50 8 50

5000 837 98924

837

692468

296772

791392

5.000) 82799.388

8) 16559.8776

R. D. st.

2069.9847 = 2069 49 current.

11. If by remitting to Holland, at 31 s. 9 d. Flemish per pound sterling, 5 per cent is gained; how goes the exchange, when by remittance I clear 10 per cent.?

31 s. 9 d. =1.5875 l.

As 105 : 1.5875 :: 110

110

105) 174.6250 (1.663095 = 1 l. Flem. 13 s. $3\frac{1}{7}$ grots, answer.

What is said above may be sufficient for reducing the coins of any country into sterling, and to render the following examples, and business of exchange in general, obvious to every common capacity.

HAMBURGH.

2 deniers gros	make one	sol lubs.
12 deniers gros or 6 sols lubs	make one	sol gros.
16 sols lubs	make one	Mark M^k
2 Marks M^k.	make one	dollar.
3 Marks	make one	rix-dollar = 48 sols lubs.
$7\frac{1}{2}$ M^k. = $2\frac{1}{2}$ R. D.	make one	livre gros or pound Flemish, or 120 sols lubs = 20 sols gros.

12. Reduce 1541 Mk 14⅓ sols lubs bank money of Hamburgh, into sterling money of England, exchange at 32⅓ sols gros per pound sterling.

				l.		
32⅓	:	1	::	1541	14⅓	
6				16		
194 sols lubs.				24656		
2				1541		
388 deniers.				24670⅓ sols lubs.		
3				2		
1164 thirds of den.				49340⅔ deniers.		
				3		

l. s. d.
1164) 148022 (127 3 4, the answer.
3162
8342
194
20
3880
388
12
4656

13. In 127 l. 3 s. 4 d. sterling, how many Hamburgh marks, &c.?

l.
127
32⅓
254
381
42 4, being ⅓ of 127.
5 5, being ⅙ of 32⅓ nearly.
20) 411.1 9, deniers gros.

Livres

```
Livres 205  11   9
          7½        marks in one livre.
       ------
       1435
        102   8,   for ½ livres.
          3  12,   for ½ of 7½ marks.
          -   6,   for 1/10 of 60 fols.
          -   3,   for ½ of 6 deniers.
          -   1½,  for ½ of 3 deniers.
  Mk. 1541  14½, fols lubs, the anfwer and proof of the
                                   foregoing example.
```

14. In 750 l. 14 s. 7 d. fterling, exchange at 32 s. 8 d. how many rix-dollars banco of Hamburgh.?

```
     l.   s.   d.
    750  14   7
    375   7   3½
     75   1   5½
     25   -   5¾
   ---------------
   1226   3   9¾ Flemifh.
      2½          And 3   fols gros  =  18 } fols.
   ----               9¾ deniers     =   5 }
   2452                                 ---
    613                                  23
 R.D. 3065  23 fols. lubs, the anfwer.
```

15. In 3065 rix-doflars, 23 fols lubs, how many pounds fterling, exchange at 32 s. 8 d.?

```
      s.   d.    l. ft.     R. D. S. L.
   As 32   8   :   1   ::   3065   24
      12                       3
    -----                   ------
     392 deniers.           9195
                              16
                          --------
                          147143 fols lubs.
                               2
                    392) 294286 (750 l. 14 s. 7 d. anfwer.
                          1988
                          ----
                           286
                            20
                          ----
                          5720
                          1800
                          ----
                           232
                            12
                          ----
                          2784
```


16. In

16. In 584 rix-dollars, 9 sols gross slight money, agio $4\frac{9}{16}$ per cent. exchange 35s. $8\frac{1}{2}$d. how many pounds sterling? 1 rix-dollar being 48 sols lubs. ∴ $\frac{9}{16}$ = 27 sols lubs.

```
                         R. D. S. G.
 104 27 : 100 :: 584 : 9
     48               8
    ---             ----
    832             4681 sols gros.
   416                 6
   ----            -----
   5019 sols lubs.  28086
                      100
              -----------
         5019) 2808600 (559 R. D. 28 S. L. banco.
                29910
                48150
               ------
                 2979
                   48
               ------
                23832
               11916
               ------
               142992
                42612
               ------
                 2460

 s.  d.      l.      R. D. S. L.
 35  8½  :   1   ::   559   28
 12                    48
 ---                 -----
 428                  4472
   2                 2236
 ---                 ------
 857                 26860 sols lubs.
                         4
                   -------
          857) 107440 ( 125 l. 7 s. 4 d. ster. answ.
                2174
                4600
                ----
                 315
                × 20
                ----
                6300
                 301
                × 12
                ----
                3612
                 184
```

17. In 1075 Mk. 14 sols lubs current, agio $8\frac{3}{8}$ per cent. and 384 dollars, 2 sols gross slight, agio $4\frac{7}{8}$ per cent. exchange 35 s. 7 d. how many rix dollars banco, and pounds sterling?

As

Mk. S. L. Mk. $108\frac{3}{8}$ = 108 ft. 6 sols lubs.
As 108 6 : 100 :: 1075 14 sols lubs.
16 16

1734 17214
× 100

Mk. S. L. D.
1734) 1721400 (992 11 1 banco.
16080
4740
1272
16
30352
3012
1278

Dol. Dol. S. L.
$104\frac{7}{8}$ = 104 28
2 sols gros. = 12

Dol. S. L. Dol. S. L.
104 28 : 100 :: 384 12
32
768
1152
12300
100

3356) 1230000 (366 16 banco.
22320
21840
1740
32
3408
5112
54528
20968
832

2
733 Mk ban.
992 $11\frac{1}{2}$
3) 1725 $11\frac{1}{2}$
575 $11\frac{1}{2}$ banco.

R. D. S. L. R. D. S. L.
As $7\frac{1}{2}$: 1 :: 575 $11\frac{1}{2}$
16 48

120 4600
2 2300

240 27611
2

l. F. s. d.
24.0) 5522.3 (230 1 6 ban.

s. d. l. l. F. s. d.
35 7 : 1 :: 230 1 6
12 20

427 4601
12

427) 55218 (129 l. 6 s. $-\frac{1}{2}$ d. sterling, answer.

Amsterdam,

Amsterdam, January 10, 1764.

Invoice, or factory of 10 pieces of Holland, 10 pieces of cambric, 9 pieces of Ghentish cloth, laden by me Jonas Diligent, on board the Josias, Thomas Cook, master, for the proper account and risque of Henry Porter, merchant, in London, under the mark per margin: contents, cost and charges, viz.

10 pieces of Holland.

								gil.	st.	pen.
N° 1	qt. $31\frac{1}{2}$	N° 6	$33\frac{1}{4}$	5 pieces, qt.	$158\frac{1}{4}$					
	$33\frac{1}{2}$		32	ditto -	163					
to	32	to	34	in all -	$321\frac{1}{4}$	ells, at 1 gil. 11 st. per ell -		497	18	1
	31		$31\frac{1}{2}$							
5	$31\frac{1}{2}$	10	$32\frac{1}{4}$							

9 pieces of cambric, qt. $124\frac{1}{2}$ ells Flemish, at 1 gil. 3 stiv. per ell - - - - - - - - 143 10 6

9 pieces of Ghenting, q. $105\frac{1}{2}$ ells Flemish, at 19 stivers per ell - - - - - - - - 100 4 6

C H A R G E S.

HP

To custom and brokerage of the Hollands, 3 gil. per piece - - -	30	0			
To charges in buying - - - - - - - - - - - - - - -	2	5			
To custom of cambric and Ghenting - - - - - - - - -	19	11			
To fledage and boatage - - - - - - - - - - - - -	3	16			
To warehouse room - - - - - - - - - - - - - - -	4	3			
To average and portage - - - - - - - - - - - - -	1	11			
			61	6	-
To my commission at $2\frac{1}{2}$ per cent. - - - -			20	1	5
Flor.			823	-	6

Errors excepted.

From your humble servant, JONAS DILIGENT.

18. What

18. What ſterling doth the invoice on the other ſide amount to; viz. 823 gilders. 6 pennings, at 34 s. 6 d. Flemiſh for 1 l. ſterling?

s.	d.		l.		Flem.
34	6	:	1	::	823
12					20
414					16460
					2

414) 32921 (79 l. 10 s. $4\frac{1}{2}$ d. the anſ.
3941
———
215
20
———
4300
———
160
× 12
———
1920
———
264

FRANCE.

England exchanges with France on the crown of three livres Tournois, or 60 ſols French, and gives pence, ſterling, more or leſs, for this exchange crown.

12 deniers } make one { ſol.
20 ſols } make one { livre of France.
3 livres } make one { ecu or crown.

The exchange between France and other countries are more variable than any, owing to the frequent alteration of their coin; which is ſo great, that Mr. Poſtlethwaite affirms he has known, in the ſpace of a few years, the crown or ecu of three livres from 5 d. to near 60 d. Engliſh; but that the firſt indeed was payable in their bank-notes then (viz. anno 1720) in great diſcredit: ſo that there can be no other way of aſcertaining the part of exchange with that kingdom, but by an actual aſſay, and weighing their ſpecie at the times.

19. In

19. In 27 l. 16 s. 8 d. ſterling, exchange at $31\frac{1}{2}$ d. per ecu; how many livres Tournois?

```
     d.    liv.   l.  s.  d.
As 31½ : 3 :: 27  16  8
               20
             ----
              556
               12
             ----
             6680
                2
            -----
            13360
                3
            -----
    63) 40080 (636 liv. 3 ſol. 9⅔ den. anſ.
         228
          390
         ----
           12
           20
         ----
          240
         ----
           51
           12
         ----
          612
         ----
           45
```

20. In 5731 crowns, 45 ſols, how much ſterling, exchange at $31\frac{1}{8}$ per crown?

```
crown.     crowns. ſols.
  1 : 31⅛ :: 5731  45
       8          60        1 crown = 60 deniers.
     ----    ------
      249    343905
                249
            -------
            3095145
           1375620
           687810
         ----------
      60| 85632345
       8| 1427205
      12|  178400⅝ pence ſterling.
      20|   14866 8½
         ------------
        £ 743 6 8½ ſterling, anſwer.
```

21. Sup-

21. Suppose Paris owes London 4186 livres, 7 sols, 5 deniers, and remits the same sum to London at $31\frac{5}{8}$ per crown.

```
                  liv. sols. den.
   3 : 31 5/8 :: 4186    7    5
        8        × 20
      ----      -----
       253      83727
                 × 12
               -------
               1004729
                   253
               -------
               3014187
              5023645
             2009458
            ----------
         80|2 541964|37
           |----------
          9|  3177455
           |----------
          8|   353050
           |----------
         12|    44131 1/4, pence sterling.
           |----------
         20|     3677  7 1/4
           |----------------
            £ 183  17  7 1/4 sterling, answer.
```

22. What comes 175·96 quintals to, at 2 liv. 17 sol. 7 den. per quintal, of 100 lb. per invoice on the next page?

```
            quint.
            175·96
                 2
           -------
       1/4 |351·92
  1/6, 1/2 | 87·98
       1/9 | 43·99
       1/4 | 17·598
       1/6 |  4·3995
           |   ·73325
           |---------      liv. sols. den.
           506.62075 = 506   12    5
```

Bourdeaux,

Bourdeaux, November 22, 1764.

Invoice of half a tun of wine, and 20 parcels of prunes, shipped on board the Canary-Merchant, John King, master, for accompt of Valentine Austin, merchant, in London, marked as in the margin.

		liv. so. de.
To 2 hogsheads of Graves claret at 50 crowns per tun	- - - - - -	75 — —
To 20 puncheons of prunes, bought of Mr. Tart and company, viz.		

		lb.		l.	
	Nº 1	1000	Nº 11	955	Tare $97\frac{1}{2}$ lb. per cask.
	2	1000	12	960	20
	3	1000	13	960	
	4	1005	14	955	1950
	5	990	15	900	
VA	6	995	16	925	9990 ten puncheons.
	7	905	17	950	9556 ditto.
	8	1045	18	981	
	9	1000	19	930	Gr. 19546.20 puncheons.
	10	1000	20	1040	Tare 1950

	liv. sol. den.
17596 lb. net, at 2 17 7 per quintal	506 12 5

CHARGES.

	liv.	sol.	den.			
To custom and brokerage of wine, 20 livres per ton - - - - - - -	10	—	—			
To charges in buying, 15 sols per ton - - - - - - - - - - - - -	—	7	6			
To fledage and boatage of the said wine - - - - - - - - - - - -	—	15	—			
To custom of prunes, 4 livres, 15 sols per ps. or puncheon - - - -	95	—	—			
To fledage and boatage, at 9 sols per ps. - - - - - - - - - - -	9	—	—			
To the ship-broker for the prunes, 10 sols per ton - - - - - - -	4	17	9			
To average poor's box, 27 sols per ton - - - - - - - - - - - -	13	3	10			
				133	4	1
				714	16	6
To my commission, at $2\frac{1}{2}$ per cent. - - Livres				17	17	5
Livres				732	13	11

Errors excepted.

SAMUEL KENT.

23. Three

23. Three hoghsheads of Graves claret, at 50 crowns per tun, per invoice.

```
                      crowns.
                        50
                      × 3 livres in one crown.
                      ----
                    2) 150
                      ----
                        75
20 puncheons of prunes 506  12   5
Charges  -  -  -       133   4   1
                      -----------
     What comes livres 714  16   6 to, at 2½ per cent.
                                2½
                      -----------
                      1429  13   -
                       357   8   3
                      -----------
                      17.87  1   3
                         20
                    ---------
                      17.51
                         12
                      -----            liv. sols. den.
                       6.15 Answer,    17   17    6
                      -----
```

24. What ought the 175.96 quintals of prunes to weigh in London, one quintal at Bourdeaux being 110 lb.?

```
              quint.
              175.96
                 110
              ------  cwt.  qr.  lb.
         112) 19355.6 (172   3    7, weight in London.
                815
                 315
                 ---
             28) 91
                 --
                  7
                 --
```

25. What

25. What comes 732 livres, 13 sols, 11 deniers to in London, at $57\frac{1}{2}$d. per crown at Boudeaux?

liv. cr. den.

60 : $57\frac{1}{2}$: : 732 13 11
12 20

720 14653
12

175847
$57\frac{1}{2}$

1230929
879235
87923.5

12)
720) 1011120.25 ($14043\frac{1}{4}$
291
312 20) 1170 $3\frac{1}{4}$
240
24 £ 58 10 $3\frac{1}{4}$
4

96

24

SPAIN.

4 Maravedis vellon, or $2\frac{1}{8}$ Maravedis plate	make one	Quartas.
$8\frac{1}{2}$ Quartas, or 34 Maravedis vellon	make one	Rial vellon.
16 Quartas, or 34 Maravedis plate	make one	Rial of plate.
8 Rials of plate	make one	Piso, piaster, piece of $\frac{8}{8}$, or dollar.

N. B. A rial vellon is $\frac{17}{32}$ of a rial of plate, and $\frac{17}{256}$ of a piaster.

26. Reduce 1387 piasters, 3 rials, 3 maravedis of Spain, into pounds, &c. sterling, of England, exchange at $45\frac{5}{8}$ sterling piaster?

piast. ri. mar.
1387 3 3
$45\frac{5}{8}$

6935
5548
$693\frac{4}{8} = \frac{1}{2}$ } 1387 piasters.
$173\frac{3}{8} = \frac{1}{8}$
$11\frac{3}{8} = 2$ rials or $\frac{1}{4}$ } of the exchange.
$5\frac{5}{8} = 1$ rial or $\frac{1}{8}$
$\frac{4}{8} = 3$ maravedis

12 | $64299\frac{1}{4}$
20 | 5374 $11\frac{1}{4}$
£ 263 14 $11\frac{1}{4}$, the answer.

27. In 572 l. 18 s. 9 d. how many pieces of $\frac{8}{8}$, exchange at $42\frac{5}{8}$ per piaster?

$42\frac{5}{8}$: 1 :: 572 18 9 (l. s. d.)
8 ... 20
341 ... 11458
12
137505
8

341)1100040(3225 7 13, the answer. (piast. ri. mar.)
770
884
2020
315
8
2500
133
34
532
399
4522
1112
89

28. In

28. In 274 dollars, 4 rials, 7 quartas, how many pounds sterling, exchange at 48 d. per dollar?

dol.	ri.	qr.
274	4	7
48		

2192
1096
24 = 4 rials or $\frac{1}{2}$ exchange.
$2\frac{1}{2}$ = 7 quartas.

12 | $13178\frac{1}{2}$
20 | 1098 $2\frac{1}{2}$
£ 54 18 $2\frac{1}{2}$

29. In 58794 quartas, how many pounds sterling, exchange at $40\frac{1}{8}$ d. per piaster?

1 ps. $\frac{8}{8}$	d.	quartas.
8 :	$40\frac{1}{8}$: :	58794
16	8	321
128	321	58794

117588
176382

128)18872874(147444
607
952
568
567
554
42

8 | 147444
12 | $18430\frac{1}{2}$
20 | 1535 $10\frac{1}{2}$
Answer, £ 76 15 $10\frac{1}{2}$

30. What is the brokerage of 15066 rials of plate in the invoice following, at $\frac{1}{4}$ per cent?

rials.
4) 150.66
$37\frac{1}{2}$, brokerage.

 31. What

31. What is the commiſſion of 15147·5 rials of plate, at $2\frac{1}{2}$ per cent.?

$$\begin{array}{r} \text{rials.} \\ 151.475 \\ 2.5 \\ \hline 757375 \\ 302950 \\ \hline 378.6875 \end{array}$$

378.6875, or $378\frac{1}{2}$ rials commiſſion.

32. What ſterling money does the whole 15526 rials of plate in the invoice following amount to, exchange at 52d. ſterling per piece of $\frac{8}{8}$?

15526 rials of plate.

$$\begin{array}{r|l} & 15526 \\ & 52 \\ \hline & 31052 \\ & 77630 \\ \hline 8 & 807352 \\ \hline 12 & 100919 \\ \hline & \text{d.} \\ 20 & 8409\ \ 11 \\ \hline & \text{s.}\ \ \text{d.} \\ & 420\ \ 9\ \ 11 \end{array}$$

420 9 11 ſterling, the anſwer.

Laus Deo, in Cadiz, November 23, 1764.

Invoice of one barrel, cont. one ſeron of caſcarilla, ſhipped on board the Seville-merchant, Captain John Tatam, commander, for account and riſque, as per advice of Henry-Euſtace Johnſon, Eſq; merchant in London; the mark as per margin.

HE

		R. Pl.
One ſeron, quantity net $209\frac{1}{4}$ lb. of caſcarilla, at 9 pieces of $\frac{8}{8}$ per lb.		15066
To diſpatch, 4 pieces of $\frac{8}{8}$, is	32	
To porterage to the houſe and boat	4	
To boat-hire aboard	8	
To brokerage, at $\frac{1}{4}$ per cent.	$37\frac{1}{2}$	
		$81\frac{1}{2}$
To my commiſſion, at $2\frac{1}{2}$ per cent.		$378\frac{1}{2}$
R. Plate		15526

Errors excepted.

John Hickson.

PORTUGAL.

Portugal exchanges with London on the milrea, and London gives from 60 to 66 pence ſterling for the ſame.

400 reas - -	make one	cruſadoe.
1000 reas, or $2\frac{1}{2}$ cruſadoes	make one	milrea.

33. In 2729 cruſadoes, 372 reas, how much ſterling money, exchange 62 d. per milrea?

cruſ. reas.
2729 372
400

1091.972
5 s. 2 d.

5455
181 10 d. being $\frac{1}{6}$ of 1091 milreas.
2 7 = 31 d. being $\frac{1}{2}$ exchange for 500 reas.
2 $-\frac{3}{4}$ = $\frac{1}{5}$ or double exchange for 400 reas.
$2\frac{1}{2}$ = $\frac{1}{10}$ of 400 = 40 reas.
$1\frac{1}{4}$ = $\frac{1}{2}$ of 40 = 20 reas.
$\frac{1}{2}$ = $\frac{1}{2}$ of 20 = 10 reas.

20) 5641 10

£ 282 1 10 d. ſterling, the anſwer.

34. In 754 l. 18 s. 6 d. ſterling, how many cruſadoes, exchange $64\frac{1}{2}$ d. per milrea?

d. mil. l. s. d.
$64\frac{1}{2}$: 1 :: 754 18 6
2 20

129 15098
12

181182
2

129) 362364 (2809 milreas.
1043
1164
3

2809
$2\frac{1}{2}$

5618
$1404\frac{1}{2}$

$7022\frac{1}{2}$ cruſadoes, the anſwer.

35. What

35. What sterling money does the invoice following, viz. 187 milreas, 686 reas amount to, at 40 reas for 3 d.?

$$\begin{array}{r|l}
 & \text{mil. rea.} \\
40 & 187.686 \quad \text{Rem. } 6 = \tfrac{3}{20} \text{ of } 3\text{d. or } \tfrac{9}{20} \text{ of } 1\text{ d.} \\
\hline
\tfrac{1}{80} & 4692\tfrac{1}{4} \\
\hline
 & 58 \;\; 13 \;\; -\tfrac{1}{4} \text{ sterling, the answer.}
\end{array}$$

Also 2) 175.150, at $\frac{1}{2}$ per cent.
is .875, brokerage.

And 176.025, at 3 per cent.
× 3

is 5.280, commission.

Oporto, November 23, 1764.

Invoice of wine laden by Charles Colby, on board the Savanna, Richard Delamore, maſter, for William Blaydwin and company, and conſigned to Spelman Swain, Eſq; in Dantzick.

		M.	R.
	To coſt for 10 pipes of wine, bought of Anthony de Minas, at 16 M. per pipe	160	—
	To cuſtom, at 1055 reas per pipe	10	550
	To trimming, &c. at 400 reas	4	—
Nº 1	To primage, at 60 reas per pipe	—	600
to 10	To brokerage, at ½ per cent	—	876
	To commiſſion at 3 per cent.	5	280
	To port charges of the ſaid ſhip	6	380
	M. R.	187	686

Errors excepted

CHARLES COLBY.

GENOA.

GENOA.

In St. George's bank at Genoa, accounts are kept in piafters, or pezzoes, which are divided into folidi and denari, as the pound fterling.

But fome merchants keep their accounts in lires, or liras folidi, and denari, divided as before, which money is only $\frac{1}{5}$ in value of the bank money.

The exchange runs from 45 to 54d. per piafter.

36. In 784 pez. 19s. 6d. lire money, how much money of exchange?

pez.	s.	d.	
5)784	19	6,	lire money.
156	19	$10\frac{4}{5}$,	exchange money, anfwer.

37. Reduce 156 pez. 19s. $10\frac{4}{5}$, exchange money, to livres.

pez.	s.	d.	
156	19	$10\frac{4}{5}$	
		5	
784	19	6,	lire money, anfwer.

38. London is indebted to Genoa in 1710l. 16s. 4d.; for how many pezzoes may Genoa value on London, the exchange at $47\frac{1}{2}$d.?

d. $\frac{1}{2}$P.pez. l. s. d. $\frac{1}{2}$P.

$47\frac{1}{2} = 95 : 1 :: 1710\ 16\ 4 = 821192$

s. d.

pezzoes 8644 2 6, anfwer.

39. Genoa is indebted to London in 8644 pez. 2s. 6d. for how much fterling may London value on Genoa, the exchange at $47\frac{1}{2}$ per pezzoe?

l.	s.	d.	
6)8644	2	6	
8)1440	13	9,	for 40d.
2) 180	1	$8\frac{5}{8}$,	for 5.
90	–	$10\frac{3}{8}$,	for $2\frac{1}{2}$.
1710	16	4,	anfwer.

40. Lon-

40. London draws on Genoa for 1710 l. 16 s. 4 d. sterling; how much lire money will pay the draught, exchange at 48d. per piaster?

	l.	s.	d.	
	1710	16	4	
			5	times 4 s. in a pound sterling.
Pezzoes	8554	1	8	of exchange.
			5	livres in a pezzoe.
	42770	8	4	lire money, answer.

LEGHORN.

N. B. At Leghorn a dollar is valued at 6 livres, at Genoa but five.

12 denarii	make one	soldi.
20 soldi	make one	lira, or piaster of Leghorn.
$5\frac{1}{6}$ soldi	make one	grossi.
24 grossi	make one	ducat.

In Leghorn accounts are kept in piasters, soldi, and denari, divided as at Genoa. Some likewise keep their accounts in liras, or lires, divided as the piaster; but this money is only $\frac{1}{6}$ of the money of exchange.

41. In 278 l. 17 s. 9 d. sterling, how many pezzoes of Leghorn, exchange at $47\frac{3}{8}$d. per pezzoe?

d. pez. l. s. d.
$47\frac{3}{8}$: 1 : : 278 17 9
8 20
379 5577
12
66933
8
379) 535464 (1412 pez. 16 sol. 9 den. answer.
1564
486
1074
316
× 20
6320
2590
314
12
3768
357

42 Lon-

42 London is indebted to Leghorn in 7456 piast. 9s. 6d. lire money; what sterling stands as an equivalent in the London merchant's books, the exchange being at $49\frac{7}{8}$ d. per piaster?

	piast.	s.	d.	
6)	7456	9	9	
6)	1089	4	11,	money of exchange.
5	181	10	$9\frac{5}{6}$,	at 40 d.
8	36	6	$1\frac{3}{4}$	
2	4	10	$9\frac{1}{4}$	
2	2	4	$4\frac{1}{2}$	
	1	2	$8\frac{1}{4}$	
	–	11	4	
£.	226	7	$1\frac{1}{2}$,	answer.

Factory

Factory of the coſt and charges of one hundred barrels of anchovies, ſhipped on board the Tortoiſe, captain William Raſtal, for account of Mr. John Hourd, of London, merchant, and conſigned to himſelf, as under mark per margin.

J H	To prime coſt of the ſaid 100 barrels of anchovies, at piece of $\frac{8}{8}$ per barrel $2\frac{3}{4}$				1650	-	-
	To portage and warehouſe	Liv. 6	-	-			
	To jeſſing, with coopers pains	15	-	-			
	To warehouſe room and Leviathan	25	-	-			
	To portage and boatage abroad	20	-	-			
	To brokerage, $\frac{1}{2}$ per cent.	8	5	-			
	To primage	5	-	-			
					79	5	-
	To my proviſion, 3 per cent.				51	17	6
	Livres				1781	2	6

Leghorn, November 23, 1773.

Errors excepted.

Per WILLIAM TODKIL.

43. At

43. At 115 sols per piece of $\frac{8}{8}$, viz. the foregoing invoice, how much sterling may Mr. Hourd credit his factor, exchange at 4s. 6d. sterling per piece of $\frac{8}{8}$?

```
        liv. sols. den.
        1781   2   6
          20
        ---------
115)35622.5(309.76087 pieces of 8/8.
        1122
          875
          700
           1000
             800
             ---
   309.76087
   ---------
    61.95217
     7.74402
   ---------
    69.6962 = 69 l. 13 s. 11 d. sterling, the answer.
   ---------
```

Provision of 1729 liv. 5 sols. at 3 per cent.

```
      liv.
      17.2925
            3
      -------
      51.8775 = 51 liv. 17 sols. 6 den. provision.
      -------
```

VENICE.

Money of exchange is always understood to be that of ducats in bank, which is imaginary, 100 whereof make 120 ducats current money; so that the difference betwixt bank and current money is an agio of 20 per cent. Though the brokers have invented another agio to be added, which is more or less, according to bargain.

The course of exchange of a ducat of the bank of Venice is from 45 to 50d. sterling.

44. Ve-

44. Venice draws on London for 2850 ducats, 10 ſols, $10\frac{1}{7}\frac{4}{3}$ deniers banco, exchange at $45\frac{5}{8}$d. per ducat; how much ſterling will pay the draught?

	duc.	ſol.	den.	
6)	2850	10	$10\frac{1}{7}\frac{4}{3}$	
8)	475	1	$9\frac{2}{3}$,	at 40d.
8)	59	7	9,	at 5d.
	7	8	$5\frac{1}{3}$,	at $\frac{5}{8}$.
£	541	18	–,	at $45\frac{5}{8}$, anſwer.

45. Reduce 1459 ducats, 18 ſols, 1 denier d'or bank money of Venice, into ſterling money, exchange at $47\frac{3}{4}$ d. ſterling per ducat?

duc.	ſol.	den.
1459	18	1
$47\frac{3}{4}$		
10213		
5836		
$729\frac{4}{8} = \frac{1}{2}$	of 1459.	
$364\frac{6}{8} = \frac{1}{4}$		
$23\frac{7}{8}$,	for 10 ſolidi.	
$11\frac{7}{8}$,	for 5 ditto.	
$4\frac{7}{8}$,	for 2	
$2\frac{3}{8}$,	for 1.	
$\frac{1}{8}$,	for 1 denier.	
$69710\frac{1}{2}$	pence ſterling.	

12	$69710\frac{1}{2}$	
20	5809	$2\frac{1}{2}$
£ 290	9	$2\frac{1}{2}$, anſwer.

46. Venice is indebted to London in 4789 ducats, 19 s. 3 d. current money; how much ſterling may London draw for, agio at 20 per cent. when the exchange is at 4s. 1d. per ducat banco?

	duc.	s.	d.	
6 : 5 : :	4789	19	3,	current money.
			5	
6)	23949	6	3	
10)	3991	12	$8\frac{1}{2}$	
2)	399	3	$3\frac{1}{2}$,	at 2
	199	11	$7\frac{5}{8}$,	at 1
12)	199	11	$7\frac{5}{8}$,	at 1
	16	12	$7\frac{5}{8}$,	at – 1
£	814	19	$2\frac{1}{8}$,	at 4 1

When

When London exchanges on the piece of foreign money, as the French crown, Venetian ducat, &c. London ought to remit when the exchange is low, and draw when it is high, to negotiate with advantage. The reason will be obvious for 100 l. will go farther in purchasing ducats, crowns, milreas, &c. when the course of exchange is at 40 d. than when it is at 50 d. and 100 crowns will go farther in paying a debt due by France to London, when the exchange is at 32 d. than when it is only at par.

POLAND and PRUSSIA.

3 shillings, or 18 phenningen	make one	grosch.
3 grosch		ditkin.
2 ditkins		sixer.
3 sixers		tymph.
$7\frac{1}{2}$ grosch		ach de halbers.
4 ach de halbers		florin or gilder.
3 florins or gilders		current dollar.
4 gilders		specie dollar.

Danzick and Koningsberg exchange with London by way of Amsterdam and Hamburgh, 270 Polish grosch being = 1 l. gross banco in Holland, 110 Polish grosch being = 1 rix-dollar banco of Hamburgh.

47. Let 5850 florins be changed into sterling money, 270 groschi Poli per pound Flemish and 33 s. 4 d. Flemish per pound sterling.

G. P. flor.
270 : 1 :: 5850
30

30 | 175500

9 | 5850

£ 650 Flemish.

33 s. 4 d. : 1 :: 650
12 20

400 13000
× 12

400) 156000

£ 390 sterling, the answer.

RUS-

RUSSIA.

3 copecs - -	make one	altine.
10 copecs - -		grievener.
25 copecs - -		polpolitin.
2 polpolitons -		poltin.
2 poltins - -		rubble.
2 rubbles - -		ducat.

The Ruſſian rubbles are converted into florins current money of Amſterdam, and the current into bank money, according to the agio of three or five per cent. and bank money into ſterling, according to the courſe of exchange between England and Amſterdam.

48. In 4675 rubbles, 46 copecs, exchange 122 copecs per rix-dollar current, agio three per cent. and 34 s. 7 d. Flemiſh per pound ſterling, how much ſterling money?

rub. cop.
4675 40
100

rix dollars.
122) 467546 (3832.34426
× 2.5

1916172130
766468852
9580.86065 florins current.

103 : 100 :: 9580.86065

103) 958086.065 (9301.80645 florins banco.
310 40
186
830 372072.258
665
470
580

34 s. 7 d. = 415) 372072.258 (896 l. 11 s. $2\frac{1}{4}$ d. the anſw.
4007
2722

232.2
20

4644
&c.

IRE-

IRELAND.

In Ireland accounts are kept in pounds, ſhillings, and pence Iriſh, divided as in England; but having no coins of their own, they are ſupplied by the different countries with which they traffic.

The par of exchange between England and Ireland is 100 l. ſterling for 108 l. 6 s. 8 d. Iriſh, or 1 s. Engliſh = 13 d. Iriſh.

The courſe of exchange is from five to 12 per cent. according to the balance of trade.

49. London remits to Ireland 787 l. 15 s. ſterling; how much Iriſh muſt London be credited, exchange at $11\frac{5}{8}$ per cent.?

	l.	s.	d.			
$\frac{1}{10}$	787	15				
$\frac{1}{10}$	78	15	6	for	10	per cent.
$\frac{1}{2}$	7	17	$6\frac{1}{2}$		1	
$\frac{1}{4}$	3	18	$9\frac{1}{4}$		$\frac{1}{2}$	
	–	19	$8\frac{1}{4}$		$\frac{1}{8}$	
£	879	6	6, the anſwer.			

50. Dublin draws upon London for 879 l. 6s. $6\frac{1}{4}$d. Iriſh, exchange at $11\frac{5}{8}$ per cent. how much ſterling muſt London pay Dublin, to diſcharge this bill?

111.625 : 100 : : 879.3260416 l.

111.625) 87932 60416(787.75 = 787 l. 15s.
979510
865104
837291
559166

AMERICA AND THE WEST-INDIES.

In exchange with our colonies in America and the Weſt-Indies, accounts are kept in pounds, ſhillings, and pence, divided as in England, and their money is called currency.

The ſcarcity of caſh obliges them to ſubſtitute a paper currency for carrying on their trade; which being ſubject to caſualties, ſuffers a very great diſcount for ſterling in the purchaſe of bills of exchange.

51. Philadelphia is indebted to London 1575 l. 14 s. 9 d. currency; what ſterling may London reckon to be remitted, when the exchange is 75 per cent.?

As 175 : 100 :: 1575 l. 14 s. 9 d.

By dividing the two firſt terms by 25.

	cur.	ſt.	l.	s.	d.
As	7 :	4 ::	1575	14	9
					4
			7)6302	19	–
Anſwer,			900	8	$5\frac{1}{7}$, ſterling.

52. London receives a bill of exchange from Philadelphia for 900 l. 8 s. $5\frac{1}{7}$ d. ſterling; for how much currency was London indebted, exchange being at 75 per cent.?

	l	s.	d.	
$\frac{1}{2}$	900	8	$5\frac{1}{7}$	
$\frac{1}{2}$	450	4	$2\frac{4}{7}$,	at 50 per cent.
	225	2	$1\frac{2}{7}$,	at 25 per cent.
Anſwer, £	1575	14	9,	currency.

53. London conſigns to Virginia goods, per invoice, amounting to 578 l. 19 s. 6 d. which are ſold for 847 l. 15 s. 6 d. currency; what ſterling ought the factor to remit, deducting five per cent. for commiſſion and charges; and what does London gain per cent. upon the adventure, ſuppoſing the exchange at 30 per cent.?

$130 + 5 = 135 = 5 \times 9 \times 3$

135 : 100 ::	847	15	6
			10
	8477	15	–
		10	
5	84777	10	
3	16955	10	
9	5651	16	8
£	627	19	$7\frac{1}{9}$, to be remitted.
–	578	19	6, conſigned,
£	49	–	$1\frac{1}{2}$ ſterling, gained.

578 975)4900.625(8.45403 = 8 l. 9 s. $3\frac{1}{4}$ d. per cent.

54. Vir-

54. Virginia is indebted to London 575 l. 19 s. 6 d. ſterling; with how much currency will London be credited at Virginia, when the exchange is $33\frac{1}{3}$ per cent.?

	l.	s.	d.
$\frac{1}{3}$	575	19	6
	191	19	10
Anſwer, £	767	19	4, currency.

SECT. II.

SIMPLE ARBITRATION of EXCHANGE.

WHEN a factor has orders from his employers to remit a certain ſum of money to any place, and then draws upon the laſt place to ſome other; as the par of exchange is continually fluctuating, there may happen to be a loſs in the executing one part of the commiſſion, and a gain in the other part thereof; which the ſkilful factor ſhould endeavour (if poſſible) ſo to improve to the benefit of his employer, to make the gain ſuperior to the loſs; or in caſe the negotiation would be to his conſtituent s loſs, he may write to him for new orders, or wait till the courſe of exchange be more in his favour.

Arbitration of exchange may be performed by one or more operations in the rule of three.

1. V, of Amſterdam, draws upon X, of Hamburgh, at 67 d. Flemiſh per dollar of 32 ſols Lubeck; and on Y, of Nuremberg, at 70d. Flemiſh per florin of 65 crutzers current. If V has orders to draw on X, in order to remit to Y at the ſaid prices, how would run the exchange between Hamburgh and Nuremberg?

67d. : 32 ſter. : : 70d. : $30\frac{29}{67}$ ſols Lubeck per florin.

2. M, of Amſterdam, orders N, of London, to remit to O, of Paris, at 54d. ſterling, and to draw on P, of Antwerp, for the value, at $33\frac{1}{2}$ ſhillings Flemiſh per pound ſterling; but as ſoon as N received the commiſſion, the exchange was on Paris at $54\frac{1}{2}$ d. per crown: pray at what rate of ex-

change ought N to draw on P, to execute his orders, and be no loser?

d. s. d.

Reciprocally, 54 : 33.5. : : 54.5

$$\begin{array}{r}
33.5 \\
54 \\
\hline
1340 \\
1675 \\
\hline
54.5)1809.0(33\text{s. } 2\tfrac{14}{109}\text{d. Flem. the answer.} \\
1740 \\
\hline
105 \\
12 \\
\hline
1260 \\
\hline
170 \\
\hline
\end{array}$$

3. London changes with Amsterdam on par at $33\frac{1}{3}$ s. Flemish, for one pound sterling; Amsterdam changes on Middleburgh, at 2 per cent. advance: how stands the exchange between London and Middleburg?

Flem. l.

As 100 : 102 : : $33\frac{1}{3}$ = 1.6̸

$$\begin{array}{r}
1.6 \\
\hline
9)612 \\
\hline
680 \\
102 \\
\hline
100)\ 170.0\ (1.7\text{ l.} = 1\text{ l. } 14\text{ s. Fl. per lb. ster.} \\
\hline
\end{array}$$

4. Amsterdam changes on London at 34 s. 4 d. per pound sterling, and on Lisbon at 52 d. Flemish for 400 reas; how then ought the exchange to go between London and Lisbon?

s. d. d. d.

As 34 4 = 412 : 240 : : 52

$$\begin{array}{r}
52 \\
\hline
480 \\
1200 \\
\hline
412)12480(30\tfrac{30}{103}\text{ pence for 400 reas.} \\
120 \\
\hline
\end{array}$$

$\therefore 30\frac{30}{103} \times 2\frac{1}{2} = 75\frac{75}{103}$d. sterling for 1000 reas.

5. Q.

5. Q, of Amsterdam, remits to R, of Paris, 2000 crowns, 91 pence Flemish per crown, at double usance, or two months, and pays $\frac{3}{20}$ per cent. brokerage; with orders to remit him again the value at 93d. per crown, allowing at the same time $\frac{1}{3}$ per cent. for provision: what is gained per cent. per annum, by a remittance thus managed?

$100\frac{3}{20} : 100 :: 91 : 90\frac{1730}{20003} = 90\frac{520730}{602903}$.
$100\frac{1}{3} : 100 :: 93 : 92\frac{208}{301} = 92\frac{416624}{602903}$.
Thus $92\frac{416624}{602903} - 90\frac{581170}{602903} = 1\frac{438357}{602903}$.
$\because 2 : 1\frac{438357}{602903} :: 12 : 10\frac{218530}{602903}$, the answer.

6. A, of Paris, draws on B, of London, 1200 crowns, at 55d. sterling per crown; for the value whereof B. draws again on A, at 56d. sterling, besides reckoning half per cent. did A get or lose by this transaction, and what?

```
        cr.    s. d.      l.  l.   l.
 1/5 | 1200 at 4  7   As 100 : 5 : : 275
     |-----                           .5
 1/5 |  240                        -----
 1/8 |   30                100) 137.5 = 1l. 7s. 6d.
     |    5                     -----
     |-----
 £ 275 - -                   d.    cr.      d.
     1 7 6 commission.   As 56  :  1  : :  66330
   -------                  56) 166330 (1184 13/28.
 £ 276 7 6 = 66330 pence.
```

$\because 1200 - 1184\frac{13}{28} = 15\frac{15}{28}$ A's gain by this transaction.

7. A, of Amsterdam, owes B, of Paris, 2000 florins of current specie, which he is to remit him, by order, the exchange $90\frac{1}{2}$d. per crown of 60 sols Turnois, the agio of the bank being four per cent. better than specie; but when this was to be negotiated the exchange was down at $89\frac{1}{2}$d. per crown, and the agio raised to five per cent. what did B. get by this turn of affairs?

Florins 2000 × 40 = 80000 Flemish pence.

$\left.\begin{array}{l} 90.5)\ 80000\ (883.9778 \\ 89.5)\ 80000\ (893.8558 \end{array}\right\}$ crowns Turnois.

$\left.\begin{array}{l} \text{As } 105 : 100 :: 893.8558 : 851.2912 \\ \phantom{\text{As }} 104 : 100 :: 883.9787 : 849.9787 \end{array}\right\}$ agio accounted. (crowns.)

cr. fl. den.
Difference 1.3125 = 1 18 9 in favour of B. Q. E. F.

 8. But

8. But arbitration of exchange may commonly be more readily performed by a numerical equation; viz. Let us ſuppoſe that the exchange between London and Amſterdam is at 34s. 6d. for 1l. ſterling; and between London and France $31\frac{3}{4}$d. ſterling, for 1 ecu or crown.

To find the proportional arbitrated price between Amſterdam and Paris,

Make the following numerical equation;
viz. 1 crown Paris $= 31\frac{3}{4}$d. ſterling,
And 240 ſterling $=$ 34s. 6d. $=$ 414d. Flemiſh.

The right-hand numbers conſtitute (by being multiplied continually into one another) a general dividend; the left-hand a general diviſor, the quotient of which will give a true ſolution to the queſtion.

But theſe may be reduced in lower terms, or leſs proportional numbers, by obſerving the axiom in reduction of vulgar fractions.

$$\begin{array}{r|c|l}
 & 1 & \cancel{1}\text{ cr.} = \cancel{31\frac{3}{4}} \\
 & 2 & 240\text{ d.} = 34 \times 6\text{d.} = 414 \\
\hline
1 \times 4 & 3 & 4 = 127 \\
2 \div 6 & 4 & 40 = 69 \\
 & 5 & \dfrac{127 \times 69}{4 \times 40} = \dfrac{8763}{160} = 54\frac{123}{160}\text{, the anſwer. Flem pence.}
\end{array}$$

This operation is thus performed:

$31\frac{3}{4} \times 4 = 127$, which place under the line on the ſame ſide, and place 4 on the other ſide to balance it.

Divide 240 and 414 each by ſix, and the quotes will be 40 and 69; which place on the ſame ſide with their dividends, cancelling all numbers as they are done with.

The reſt are ſo plain and eaſy, it needs no explanation.

9. Again, ſuppoſe the exchange between Paris and Amſterdam is at $54\frac{123}{160}$, and on London $31\frac{1}{4}$; the proportional arbitrated price between London and Amſterdam is required?

$$\begin{array}{ccc}
\text{l. ſter.} & & \text{d.} \\
1 & = & \cancel{240} \\
\cancel{31\frac{3}{4}} & = & \cancel{54}\frac{123}{160} \\
\hline
\cancel{160} & & 8763 \\
127 & & \cancel{4} \\
\cancel{4} & & 6
\end{array}$$

$$\therefore \frac{8763 \times 6}{127} = \frac{52578}{127} = 414 \text{ Flem. pence} = 34\text{ s. } 6\text{d.}$$

The

The foregoing operation is performed thus:

$54\frac{123}{160} \times 160 = 8763$ placed underneath, and 160 set on the other side to balance.

Then $31\frac{3}{4} \times 4 = 127$, to balance which place 4 on the other side.

Then I perceive, that 160 and 240 are each divisible by 40, the quotes whereof are 4 and 6.

Lastly, finding 4 on each side, they cancel each other.

You are desired, as before directed, to cancel every figure as it is done with.

10. Lastly, exchange Amsterdam on Paris at $54\frac{123}{160}$, and Amsterdam on London at 34s. 6d. what is the arbitrated price between London and Paris?

1 crown Paris	=	$\cancel{54}\frac{123}{160}$
$\cancel{34}$s. $\cancel{6}$d. or $\cancel{414}$d.	=	$\cancel{240}$d.
$\cancel{160}$		$\cancel{8763}$
2		$\cancel{3}$
$\cancel{138}$		2921
46		1

$\therefore \frac{2921}{92} = 31\frac{69}{92} = 31\frac{3}{4}$d.

The common measure of the fraction being 23.

11. London exchanges on Amsterdam at 34s. 9d. for 1l. sterling, and on Lisbon at 5s. $5\frac{5}{8}$d. per milrea; what is the arbitrated price between Amsterdam and Lisbon?

1 crusado Lisbon	=	$\cancel{400}$ reas.
$\cancel{1000}$ reas	=	$\cancel{65}\frac{5}{8}$d. sterling.
$\cancel{240}$d. sterling	=	$\cancel{34}\ \cancel{9} = 417$ Flemish pence.
$\cancel{8}$		$\cancel{525}$
$\cancel{5}$		$\cancel{7}$
$\cancel{48}$		$\cancel{105}$
1		$\cancel{21}$
4		$\cancel{21}$
16		1
		7

$\therefore \frac{417 \times 7}{16 \times 4} = \frac{2919}{64} = 45\frac{39}{64}$d. Flemish, for one crusado Lisbon.

 12. Am-

12. Amsterdam exchanges on Lisbon at $45\frac{39}{64}$ Flemish pence, and on London at 34 s. 9 d. what is the arbitrated price of exchange between London and Lisbon?

$$\begin{array}{rcl} & 1 \text{ milrea} = & \cancel{1000} \text{ reas.} \\ 1 \text{ crusado} = \cancel{400} \text{ reas} & = & \cancel{45}\tfrac{39}{64}\text{d. Amsterdam,} \\ \cancel{417}\text{ d. Fl.} & = & \cancel{240}\text{ d. sterling.} \end{array}$$

$$\begin{array}{cc} \cancel{64} & 2919 \\ \cancel{7} & 5 \\ 139 & \cancel{80} \\ 8 & \cancel{10} \\ & 5 \end{array}$$

$\therefore \dfrac{2919 \times 5 \times 5}{139 \times 8} = \dfrac{72975}{1112} = 65\frac{5}{8}$d. sterling per milrea.

N.B. The common measure of the fraction being 139.

13. Lisbon exchanges on Amsterdam at $45\frac{39}{64}$ per crusado, on London at 5 s. $5\frac{5}{8}$ d. per milrea; what is the arbitrated price between London and Amsterdam?

$$\begin{array}{lcl} 1 \text{ l. sterling} & = & \cancel{240}\text{d. sterling.} \\ \cancel{65}\tfrac{5}{8} \text{ sterling} & = & \cancel{1000} \text{ reas Portugal.} \\ \cancel{400} \text{ reas} & = & 45\tfrac{39}{64} \text{ Flemish pence.} \end{array}$$

$$\begin{array}{cc} \cancel{525} & \cancel{8} \\ \cancel{7} & \cancel{5} \\ \cancel{64} & \cancel{2919} \\ \cancel{8} & \cancel{30} \\ \cancel{105} & \cancel{5} \\ \cancel{21} & \cancel{6} \\ \cancel{3} & 417 \end{array}$$

Here the answer comes out exactly 417 Flemish pence, or 34s. 9d. of Amsterdam; and this will frequently happen, and the operation performed on the thumb-nail by the expert accomptant.

These examples prove the truth of this method in regard to each other.

14. Amsterdam hath orders to remit a certain sum to Cadiz; at the time of this order Amsterdam can remit to Cadiz at $94\frac{1}{4}$d. per ducat of 375 maravedis, and London to Cadiz at 38 d. per piaster of 272 maravedis. Quere, which will be most advantageous to Amsterdam, to remit directly

to

to Cadiz, or by London, the exchange between Amsterdam and London being 35 st. 10 guil. per pound sterling?

$\cancel{272}$ maravedis = 38 d. sterling.
$\cancel{240}$ d. sterling = $\cancel{35}$ s. 10 guil. = $\cancel{430}$ d. Amsterdam.

$$\begin{array}{lll} \hline & & \cancel{375} \\ 136 & & \hline \\ \cancel{24} & & 19 \\ 8 & & 43 \\ \hline & & 125 \\ & & \hline \end{array}$$

$$\therefore \frac{19 \times 43 \times 125}{136 \times 8} = \frac{102125}{1088} = 93\tfrac{941}{1088} \text{ pence Amsterdam}$$

for 375 maravedis; which is 18 s. 8¼ d. for every 100 l. sterling in favour of Amsterdam; viz.

$$\frac{941}{1088} = .86489, \text{ nearly.}$$

As 94·75 : 93·86489 :: 100 : 99·06584 = 99 l. 1 s. 3¾ d.
Then 100 l. — 99 l. 1 s. 3¾ d. = 18 s. 8¼ d. as above.

SECT. III.

COMPOUND ARBITRATION *of* EXCHANGE.

WHEN the price of exchange is given betwixt one country and another, betwixt that second and a third, and betwixt that third and a fourth, &c. to find the arbitrated price between the first and the last, observe the following

RULE.

Place the antecedents in one column, and the consequents in another, to the right of the antecedents; so as to form a numerical equation in the algebraic way of analysis, in which the first antecedent and the last consequent, to which an antecedent is required, must always be of the same denomination or species; the first consequent must be of the same denomination with the second antecedent; the second

ſecond conſequent with the third antecedent, &c. throughout. If a fraction is annexed to any of the numbers, both the antecedent and conſequent muſt be multiplied into the denominator of that fraction, and the proportion will ſtill be the ſame. The terms being thus diſpoſed, cancel the quantities that are the ſame on both ſides of the equation, and abridge ſuch quantities as are commenſurable; then multiply all the antecedents into one another for a general diviſor, and all the conſequents for a general dividend, and the quotient will be the anſwer, or value of the antecedent required.

1. Suppoſe London to remit 500 l. to Spain, by the way of Holland, at 35 s. per pound; thence, by the way of France, at 58 grotes per crown; thence to Venice, at 100 crowns per 60 ducats banco; and from Venice to Spain, at 360 maravedis per ducat banco; how many piaſters of 272 maravedis will it amount to in Spain, excluſive of charges?

anteced.		conſeq.
1 pound	=	$\cancel{4}\cancel{2}\cancel{0}$ d. Flemiſh.
$\cancel{5}\cancel{8}$ grotes	=	1 crown.
$\cancel{1}\cancel{0}\cancel{0}$ crowns	=	$\cancel{6}\cancel{0}$ ducats.
1 ducat	=	$\cancel{3}\cancel{6}\cancel{0}$ maravedis.
$\cancel{2}\cancel{7}\cancel{2}$ maraved.	=	1 piaſter.

How many piaſters for 500 l.?

These reduced, will be,

1	- -	21
29	- -	1
1	- -	3
1	- -	45
17	- -	1
		500

$$\frac{21 \times 3 \times 45 \times 500}{29 \times 17} = 2875\tfrac{125}{493} = 2875\tfrac{1}{4}, \text{ nearly, the anſwer.}$$

2. A banker in Paris remits to his factor in Amſterdam 455 crowns Tournois; firſt to London, at 30 d. per crown; from London to Rome, at 65 d. per ſtampt crown; from Rome to Venice, at 100 ſtampt crowns for 140 ducats banco; from Venice to Leghorn, at 100 ducats banco for 100 piaſters of Leghorn; and from Leghorn to Amſterdam, at 86 Fle-

86 Flemish pence per piaster, how many guilders banco will be received at Amsterdam?

anteced.		conseq.
1 crown Paris	=	30 d. sterling.
65 d. sterling	=	1 crown Rome.
100 crowns Rome	=	140 ducats Venice.
100 ducats Venice	=	100 piasters Leghorn.
1 piaster Leghorn	=	86 pence Flemish.
		455 crowns Tournois.
13		6
5		7
		91

$$\therefore \frac{86\times6\times7\times91}{13} = \frac{228692}{13} = 25284 \text{ Flemish pence.}$$

And 40)25284

Answer, 632 guilders, 2 stivers.

3. A merchant of London hath credit for 1360 piasters of Leghorn, from which there is advice that a remittance can be made at 50 d. per piaster. The London merchant, finding he could make no more by drawing for them, orders them to be remitted in the following manner; viz. first to Venice, at 94 piasters for 100 ducats banco; thence to Cadiz, at 320 maravedis per ducat; thence to Lisbon, at 630 reas per piaster of 272 maravedis; thence to Amsterdam, at 50 grotes per crusado of 400 reas; from thence to Paris, at 56 grotes per crown; and lastly, he brings them home at $31\frac{1}{3}$d. per crown: what will be the arbitrated price per piaster between London and Leghorn, and how much will be received at London, without reckoning charges?

anteced.		conseq.
94 piasters	=	100 ducats banco.
1 ducat	=	320 maravedis.
272 maravedis	=	630 reas.
400 reas.	=	50 grotes.
56 grotes	=	1 crown.
3 crowns	=	94 pence sterling $= 31\frac{1}{3}\times 3$.
What	=	1 piaster?

1	- - - - -	1
1	- - - - -	5
34	- - - - -	15
1	- - - - -	25
1	- - - - -	1
1	- - - - -	1

$$\therefore \frac{5 \times 15 \times 25}{34} = \frac{1875}{34} = 55\tfrac{5}{34} \text{ d. per piaster.}$$

	l.	s.	d.
1360 piasters, at $55\frac{5}{34}$ d. per piaster =	312	10	–
Ditto at 50 d. per piaster - - - - =	283	6	8
Gained by the negotiation £	29	3	4

4. Amsterdam being to remit to London 750 l. Flemish, he first sends it to France, at 54 d. per crown; from thence to Venice at 100 crowns for 56 ducats banco; from thence to Hamburgh, at 100 grotes per ducat; from thence to Portugal, at 45 grotes per crusado of 400 reas; and from Portugal to London, at 5s. 3d. for 1000 reas; and suppose the commission, &c. at each place be half per cent. quere, how much sterling money must be received in London; and whether more or less, than if it was remitted directly from Amsterdam to London, at 35s. 6d. Flemish per pound sterling?

$\cancel{54}$ pence	=	1 crown.
$\cancel{100}$ crowns	=	$\cancel{56}$ ducats banco of Venice.
1 ducat V.	=	$\cancel{100}$ grotes Hamburgh.
$\cancel{45}$ gr. Ham.	=	$\cancel{400}$ reas Portugal.
$\cancel{100}$ reas	=	$\cancel{63}$
		£ $\cancel{750}$ Flemish, at $35\frac{1}{2}$ s. per pound sterl.

$\cancel{8}$	2
$\cancel{8}$	$\cancel{7}$
$\cancel{77}$	28
9	$\cancel{36}$
	10

∴ 2

$\therefore \frac{2 \times 7 \times 28 \times 10}{9} = \frac{3920}{9} = 435.5$ l. sterling.

$\times .005 = \frac{1}{2}$ per cent. at Port.

$35\frac{1}{2}$ s. $= 1.775$ Flemish — 2.177

433.377

1.775) 750.000 (422.5352, had it been remitted directly to London - - - $\times .005 = \frac{1}{2}$ per cent. Hamb.

2.1668

431.2108

$\times .005 = \frac{1}{2}$ per cent. Venice.

2.156

429.0548

$\times .005 = \frac{1}{2}$ per cent. Paris.

2.1453

426.9095 received in London.
422.5352 direct remittance.

Answer, $4.3743 = 4$l. 7 s. $5\frac{3}{4}$ d. London gains by the remittance above.

SECT. IV.

Comparison of WEIGHTS *and* MEASURES.

IT is a very necessary way (of great importance to the merchant) to be acquainted with the weights and measures of the different countries where he deals; to facilitate which knowledge, I have in the following pages exhibited authentic tables of the conformity which weights and measures in the most noted trading places in Europe have with one another.

1. Suppose 100 lb. of Amsterdam be equal to 100 lb. of Paris; and 100 lb. of Paris to be 150 lb. in Genoa; and 100 lb. of Genoa to be 70 lb. in Leipsick; and 100 lb. of Leipsick

Leipsick to be 160 lb. in Milan; how many Milan pounds will equiponderate 548 lb. Amsterdam?

lb.		lb.
$\cancel{100}$ Amsterdam	=	100 Paris.
$\cancel{100}$ Paris - - -	=	$\cancel{150}$ Genoa.
$\cancel{100}$ Genoa - -	=	$\cancel{70}$ Leipsick.
$\cancel{100}$ Leipsick -	=	$\cancel{160}$ Milan.
Quere, Milan -	=	548 Amsterdam?

$\cancel{2}$ - - - - - -	3
25 - - - - - -	$\cancel{4}$
	7
	2

$548 \times 3 \times 7 \times 2 = 23016$

25) 23016 ($920\frac{16}{25}$ Milan, the answer.

2. If 7 aunes of Paris make 9 yards of London, 36 yards of London 49 aunes of Holland, 7 aunes of Holland 9 braces of Milan, 3 braces of Milan 2 vares of Aragon, 5 vares of Aragon 2 canes of Montpelier, 9 canes of Montpelier 10 canes of Thoulouse, and 4 canes of Thoulouse 9 canes of Troyes, in Champaigne; how many aunes of Troyes will measure 100 aunes of Paris?

$\cancel{7}$ aunes of Paris -	=	$\cancel{9}$ yards of London.
$\cancel{36}$ yards of London	=	$\cancel{49}$ aunes of Holland.
$\cancel{7}$ aunes of Holland	=	$\cancel{9}$ braces of Milan.
$\cancel{3}$ braces of Milan	=	$\cancel{2}$ vares of Aragon.
$\cancel{5}$ vares of Aragon	=	$\cancel{2}$ canes of Montpelier.
$\cancel{9}$ canes of Montpelier	=	$\cancel{10}$ canes of Thoulouse.
4 canes of Thoulouse	=	$\cancel{9}$ aunes of Troyes.
How many aunes of Troyes	=	100 aunes of Paris?

1 - - - - - - - - -	3
$\cancel{4}$ - - - - - - - - -	$\cancel{2}$
2 - - - - - - - - -	1

$10 \times 3 = 300$ dividend. 2) 300 (150 aunes of Troyes.

A TABLE

A TABLE *representing the conformity which the weights of the principal trading cities of Europe have with each other, taken from Postlethwayt's Dictionary.*

As the weight of Amsterdam, Paris, Bourdeaux, Besançon, and several other places, have but a very trifling difference, they are comprehended under those of Amsterdam, as those of Nuremburg are under Frankfort, and others in the same manner.

		A England, Scotland, and Ireland.	B Amsterdam, Paris, &c.	C Antwerp, or Brabant.	D Rouen the viscounty weight.	E Lyons, the city weight	F Rochelle.	G Thoulouse, and Upper Languedoc	H Marseilles, and Provence.	I Geneva.	K Hamburgh.
A	100 lb. of England, Scotland, Ireland, and London	100	91 8	96 8	88	106	90 9	107 11	113	81 7	93 5
B	100 lb. of Amsterdam, Paris, &c.	109 8	100	105 8	96 4	116	99	118	123 8	89	102
C	100 lb. of Antwerp, or Brabant	103 2	94 12	100	91 4	110	93 13	111 12	117	84 5	96 10
D	100 lb. of Rouen, the viscounty	113 14	104	109 12	100	120 8	102 15	122 11	128 8	92 9	106
E	100 lb. of Lyons, the city	94 3	86	90 12	82 12	100	85 2	101 8	105 4	76 8	87 12
F	100 lb. of Rochelle	110 9	101	106 8	97 3	117	100	119 3	124 12	89 14	103
G	100 lb. of Thoulouse, and Upper Languedoc	92 6	84 12	89 6	81 8	98 5	83 15	100	105 4	75 7	86 7
H	100 lb. of Marseilles, and Provence	88 11	81	85 8	78	94	80 3	95 9	100	72	82 10
I	100 lb. of Geneva	123	112 6	118 8	108	130 5	111 6	132 9	128 4	100	114 10
K	100 lb. of Hamburgh	107 5	98	103 6	94 4	113 10	97	115 10	121	87 4	100
L	100 lb. of Frankfort	111 11	102	107 8	98 3	118 5	101	120 6	126	90 12	104
M	100 lb. of Leipsic	104 5	95 4	100	91 12	110 8	94 4	112 6	117 12	84 12	92 2
N	100 lb. of Genoa	73	66	70 5	64	77 5	66	78 10	82 5	59 5	68
O	100 lb. of Leghorn	75 8	69	72 12	66 6	80	68 5	81 6	85 4	61 6	70 6
P	100 lb. of Milan	65 3	59 8	62 12	57 4	69	58 14	70 3	73 8	53	60 1
Q	100 lb. of Venice	65 11	60	63 6	57 12	69 10	59 6	70 13	74 2	53 6	61 3
R	100 lb. of Naples	64 10	59	62 4	57	68 7	58 6	69 10	72 14	52 8	60 2
S	100 lb. of Seville, Cadiz, &c.	103 7	94 8	99 12	91	109 10	93 9	111 8	116 11	84 2	96 6
T	100 lb. of Portugal	95 4	87 8	92	84 4	101 8	86 10	103 4	08	77 14	89 4
U	100 lb. of Liege	104	95	100 3	91 7	110 3	94	112	117 5	84 8	96 14

make

Continued.

		L	M	N	O	P	Q	R	S	T	U
	England, &c.	Frankfort, &c.	Leipsick, &c.	Genoa.	Leghorn.	Milan.	Venice.	Naples.	Seville, Cadiz, &c.	Portugal.	Liege.
A 100 lb of England, &c. make	100	89 7	96 1	137 4	132 11	153 11	152	154 10	97	104 13	96 5
B 100 lb of Amsterdam, &c.	109 8	98	105	150	145	168	166	169	106	114 8	105 4
C 100 lb. of Antwerp, &c.	103 12	92 13	99 8	142 2	137 6	159 3	157 1/3	160 2	108	108 8	99 11
D 100 lb of Rouen, &c.	113 14	102	109 4	156	150 13	174 11	172 2/3	175 12	110 4	119	109 7
E 100 lb. of Lyons, &c.	94 3	84 4	90 5	129	124 11	144 8	142 3/4	145 6	91 3	98 8	90 8
F 100 lb. of Rochelle	110 9	99	106	151 8	146 7	169 11	167 10	170 11	107	115 10	106 5
G 100 lb. of Thoulouse, &c.	92 6	83	89 8	127 2	122 14	142 6	140	143 4	89 13	97	89 3
H 100 lb. of Marseilles, &c.	88 11	79 6	85 8	121 2	117 7	136 1	134 8	136 14	85 13	92 12	85 4
I 100 lb. of Geneva	123	110 2	118	168 9	163	188 13	186 8	189 14	119 2	128 8	118 4
K 100 lb. of Hamburgh,	107 5	89 11	102 15	147	142 2	164 10	162 11	165 10	103 13	112 4	103 2
L 100 lb. of Frankfort	111 11	100	107 1	153	147 14	171 6	169 5	172 6	108 2	116 13	107 6
M 100 lb. of Leipsick	104 5	93 5	100	142 13	138 1	160	158 2	61	101	109	100 4
N 100 lb. of Genoa	73	65 5	70	100	96 11	112	110 11	112 11	70 11	76 5	70 6
O 100 lb. of Leghorn	75 8	67 19	72 8	103 8	100	116	114 8	116 9	73	79	72 10
P 100 lb. of Milan	65 3	58 5	62 8	89 4	86 4	100	98 12	100 8	63	68 2	62 10
Q 100 lb. of Venice	65 11	58 13	63	90	87	100 13	100	101 6	63 9	68 11	63 2
R 100 lb. of Naples	64 10	57 13	62	88 8	85 8	99 2	98	100	62 8	67 9	62
S 100 lb. of Seville, &c.	103 7	92 10	99 4	141 12	137	158 12	156 14	159 12	100	108 3	99 14
T 100 lb. of Portugal	95 4	85 12	91 13	131 14	126 13	147	145 4	148	92 12	100	92
U 100 lb. of Liege	101	93	99 12	142 8	137 12	159 9	157 11	160 10	100 9	108 12	100

N. B. By means of this TABLE may be easily discerned, at one view, the conformity which the weights of one place therein exhibited have with those of another: for example, if you would know how many pounds 100 lb. weight English make at Amsterdam, look for England in the first column, and from thence pass your eye along the line till you come to the column under the title of Amsterdam at the top, and you will find that 91 lb. 8 oz. 16 oz. to the pound, equal 100 lb. English; and in like manner you may find the agreement between any other weight of these places specified in the TABLE.

A TABLE

A TABLE *representing the conformity which the long with each other, taken from Postlethwait's Commercial*

	The ells of Amsterdam, Haerlem, Leyden, the Hague, Rotterdam, and other cities of Holland, as well as the ell of Nuremberg, are equal among themselves. They are also comprehended under the ell of Amsterdam, as that of Osnaburgh is under that of France and England; and the ell of Bern and Basil under that of Hamburgh, Frankfort and Leipsick.		A Yards of England, Scotland, and Ireland.	B Ells of France and England.	C Ells of Holland and Amsterdam.	D Ells of Antwerp and Brussels.
A	100 yards of Engl. Scotl. and Ireland	make	100	78	$133\frac{1}{3}$	$131\frac{2}{3}$
B	100 ells of France and England - -	make	$128\frac{1}{4}$	100	$173\frac{1}{2}$	$166\frac{2}{3}$
C	100 ells of Holland or Amsterdam -	make	75	$57\frac{2}{3}$	100	$98\frac{1}{4}$
D	100 ells of Antwerp and Brussels - -	make	76	60	$101\frac{1}{4}$	100
E	100 ells of Hamburgh, Frankfort, &c.	make	$62\frac{1}{2}$	$48\frac{7}{8}$	$83\frac{1}{3}$	$82\frac{1}{3}$
F	100 ells of Breslau, in Silesia - - -	make	60	$46\frac{3}{5}$	80	79
G	100 ells of Dantzick - - - - - -	make	$66\frac{3}{4}$	52	89	$87\frac{3}{4}$
H	100 ells of Bergue and Drontheim -	make	$67\frac{1}{2}$	$52\frac{1}{4}$	90	89
I	100 ells of Sweden or Stockholm - -	make	$65\frac{3}{4}$	$51\frac{1}{4}$	$87\frac{1}{2}$	$86\frac{1}{2}$
K	100 ells of St. Gall, for linen - - -	make	87	$67\frac{1}{4}$	116	$114\frac{1}{2}$
L	100 ells of St. Gall, for cloth - - -	make	67	$52\frac{1}{2}$	$89\frac{1}{3}$	$88\frac{3}{4}$
M	100 ells of Geneva - - - - - - -	make	$124\frac{3}{4}$	$97\frac{1}{2}$	$166\frac{2}{3}$	$164\frac{3}{5}$
N	100 canes of Marseilles and Montpelier	make	$214\frac{1}{2}$	$167\frac{1}{3}$	286	$282\frac{4}{5}$
O	100 canes of Thoul. and Upper Lang.	make	$199\frac{3}{4}$	156	$266\frac{2}{3}$	$263\frac{1}{3}$
P	100 canes of Genoa, of 9 palmos - -	make	$245\frac{1}{4}$	$191\frac{1}{3}$	327	323
Q	100 canes of Rome - - - - - - -	make	$227\frac{1}{4}$	$177\frac{1}{4}$	303	$299\frac{1}{5}$
R	100 vares of Castille and Biscay - -	make	$93\frac{3}{4}$	$73\frac{1}{5}$	125	$123\frac{2}{5}$
S	100 vares of Cadiz and Andalusia - -	make	$91\frac{3}{4}$	$71\frac{1}{7}$	$122\frac{1}{4}$	119
T	100 vares of Portugal or Lisbon - -	make	123	96	164	162
V	100 covedos of Portugal or Lisbon -	make	74	$58\frac{1}{2}$	100	$98\frac{3}{4}$
W	100 brasses of Venice - - - - - -	make	$73\frac{1}{2}$	$57\frac{1}{3}$	98	$96\frac{3}{4}$
X	100 brasses of Bergamo, &c. - - -	make	$72\frac{1}{4}$	$55\frac{3}{5}$	95	$93\frac{4}{5}$
Y	100 brasses of Florence, Leghorn, &c.	make	$65\frac{1}{5}$	50	$85\frac{1}{2}$	$84\frac{4}{5}$
Z	100 brasses of Milan - - - - - - -	make	58 1/[illegible]	$45\frac{3}{5}$	78	77

measures of the principal trading cities of Europe have Dictionary.

	E	F	G	H	I	K	L	M
	Ells of Hamburgh, Frankfort, Leipsick and Cologn.	Ells of Breslau, in Silesia.	Ells of Dantzick.	Ells of Bergue & Drontheim.	Ells of Sweden or Stockholm.	Ells of St. Gall, for linen.	Ells of St. Gall, for cloth.	Ells of Geneva.
A	160	166$\frac{3}{5}$	150	146$\frac{2}{3}$	154	114$\frac{2}{3}$	149$\frac{1}{3}$	80
B	205$\frac{1}{4}$	213$\frac{3}{4}$	192$\frac{4}{5}$	188	195$\frac{3}{4}$	147	191$\frac{1}{2}$	102$\frac{3}{5}$
C	120	125	112$\frac{1}{2}$	110	114$\frac{1}{2}$	86	112	60
D	121$\frac{1}{2}$	126$\frac{3}{4}$	114	111$\frac{4}{5}$	116	87	113$\frac{4}{5}$	60$\frac{3}{5}$
E	100	104$\frac{1}{5}$	92$\frac{3}{5}$	91$\frac{3}{4}$	95$\frac{1}{2}$	71$\frac{2}{5}$	91$\frac{1}{3}$	50
F	96	100	89$\frac{4}{5}$	88	91$\frac{1}{2}$	68$\frac{3}{4}$	89$\frac{3}{5}$	48
G	96$\frac{3}{4}$	111$\frac{1}{4}$	100	98	102	76$\frac{3}{4}$	99$\frac{3}{4}$	53$\frac{3}{5}$
H	108	112$\frac{1}{2}$	101$\frac{1}{2}$	100	103	77$\frac{4}{5}$	100$\frac{4}{5}$	54
I	105	109$\frac{1}{4}$	97$\frac{3}{4}$	96$\frac{1}{4}$	100	75$\frac{1}{4}$	98	52$\frac{1}{2}$
K	139$\frac{1}{5}$	145	130$\frac{1}{2}$	127$\frac{3}{5}$	133	100	130	69$\frac{3}{5}$
L	107$\frac{1}{5}$	111$\frac{3}{4}$	100$\frac{1}{2}$	98$\frac{1}{4}$	102$\frac{1}{4}$	76$\frac{4}{5}$	100	53$\frac{3}{5}$
M	200	208$\frac{1}{3}$	187$\frac{1}{2}$	183$\frac{1}{4}$	191	143$\frac{1}{3}$	130$\frac{2}{3}$	100
N	343$\frac{1}{5}$	357$\frac{1}{2}$	321$\frac{3}{4}$	314$\frac{3}{5}$	327$\frac{1}{2}$	246	320$\frac{1}{2}$	171$\frac{3}{4}$
O	320	333$\frac{1}{3}$	300	193$\frac{1}{3}$	304	229$\frac{1}{3}$	298$\frac{2}{3}$	160
P	392$\frac{4}{5}$	408$\frac{3}{4}$	367$\frac{4}{5}$	359$\frac{3}{4}$	374$\frac{1}{2}$	281$\frac{1}{4}$	366$\frac{1}{4}$	196$\frac{1}{4}$
Q	363$\frac{3}{5}$	378$\frac{3}{4}$	340$\frac{4}{5}$	333$\frac{1}{3}$	347	260$\frac{3}{5}$	339$\frac{4}{5}$	181$\frac{4}{5}$
R	150	156$\frac{1}{4}$	140$\frac{2}{3}$	137$\frac{1}{2}$	143$\frac{1}{4}$	107$\frac{1}{2}$	140	75
S	146$\frac{3}{5}$	152$\frac{3}{5}$	138	134$\frac{1}{2}$	139$\frac{4}{5}$	105	137	73$\frac{1}{3}$
T	196$\frac{4}{5}$	205	184$\frac{1}{2}$	180$\frac{4}{5}$	187$\frac{3}{4}$	141	183$\frac{3}{5}$	94$\frac{4}{5}$
V	120	125	112$\frac{1}{2}$	110	114$\frac{1}{2}$	86	112	60
W	117$\frac{3}{5}$	122$\frac{1}{2}$	104$\frac{1}{4}$	107$\frac{3}{4}$	112$\frac{1}{4}$	84$\frac{1}{4}$	109$\frac{3}{4}$	58$\frac{3}{4}$
X	114	118$\frac{3}{4}$	106$\frac{3}{4}$	104$\frac{1}{2}$	108$\frac{3}{4}$	81$\frac{3}{4}$	106$\frac{4}{5}$	57
Y	102$\frac{3}{5}$	106$\frac{4}{5}$	96$\frac{2}{5}$	94	98	73$\frac{1}{2}$	95$\frac{3}{4}$	51$\frac{3}{5}$
Z	93$\frac{3}{5}$	97$\frac{1}{2}$	87$\frac{3}{4}$	85$\frac{4}{5}$	89$\frac{1}{4}$	67	87$\frac{3}{5}$	46$\frac{4}{5}$

A TABLE *representing the conformity which the long with each other, taken from Postlethwayt's Commercial*

Continued.	N Canes of Marseilles and Montpelier.	O Canes of Thoulouse, Albi, and Castres.	P Canes of Genoa, of 9 palmos.	Q Canes of Rome.
A 100 yards of England, Scotland, &c.	$46\frac{2}{3}$	50	$40\frac{2}{3}$	44
B 100 ells of France and England - -	$59\frac{4}{5}$	$64\frac{1}{5}$	$52\frac{1}{5}$	$56\frac{4}{5}$
C 100 ells of Holland and Amsterdam -	35	$37\frac{1}{2}$	$30\frac{1}{2}$	33
D 100 ells of Antwerp and Brussels - -	$35\frac{4}{5}$	38	$30\frac{4}{5}$	$33\frac{3}{5}$
E 100 ells of Hamburgh, Frankfort, &c.	$29\frac{1}{5}$	$31\frac{1}{4}$	$25\frac{4}{5}$	$27\frac{1}{2}$
F 100 ells of Breslau, in Silesia - - -	28	39	$24\frac{4}{5}$	$26\frac{4}{5}$
G 100 ells of Dantzick - - - - - -	$31\frac{1}{5}$	$33\frac{4}{5}$	$27\frac{1}{5}$	$29\frac{4}{5}$
H 100 ells of Bergue and Drontheim -	$31\frac{1}{2}$	$33\frac{3}{4}$	$27\frac{4}{5}$	$29\frac{3}{4}$
I 100 ells of Sweden or Stockholm - -	$30\frac{1}{2}$	$32\frac{3}{4}$	$26\frac{3}{4}$	$28\frac{3}{4}$
K 100 ells of St. Gall, for linen - - -	$40\frac{2}{5}$	$43\frac{1}{2}$	$35\frac{4}{5}$	$38\frac{1}{3}$
L 100 ells of St. Gall, for cloth - - -	$31\frac{1}{4}$	$33\frac{1}{2}$	$27\frac{1}{5}$	$29\frac{1}{2}$
M 100 ells of Geneva - - - - - - -	$58\frac{1}{3}$	$62\frac{1}{2}$	$50\frac{4}{5}$	55
N 100 canes of Marseilles and Montpelier	100	$107\frac{1}{4}$	$87\frac{1}{4}$	$94\frac{4}{5}$
O 100 canes of Thoulouse, &c. - - -	$93\frac{1}{3}$	100	$81\frac{1}{3}$	88
P 100 canes of Genoa, of 9 palmos - -	$114\frac{1}{2}$	$122\frac{3}{5}$	100	108
Q 100 canes of Rome - - - - - - -	116	$113\frac{2}{3}$	$92\frac{4}{5}$	100
R 100 vares of Castille and Biscay - -	$43\frac{3}{4}$	$46\frac{4}{5}$	$38\frac{1}{5}$	$41\frac{1}{4}$
S 100 vares of Cadiz, &c. - - - - -	$42\frac{2}{3}$	45	$37\frac{1}{4}$	$40\frac{2}{3}$
T 100 vares of Portugal or Lisbon - -	$57\frac{4}{5}$	$61\frac{1}{2}$	50	$54\frac{1}{5}$
V 100 covedos of Portugal or Lisbon -	35	$37\frac{1}{2}$	$30\frac{1}{2}$	33
W 100 brasses of Venice - - - - - -	$34\frac{1}{3}$	$36\frac{3}{4}$	$29\frac{3}{4}$	$32\frac{1}{3}$
X 100 brasses of Bergamo, &c. - - -	$33\frac{1}{4}$	$35\frac{3}{5}$	29	$31\frac{3}{5}$
Y 100 brasses of Florence, &c. - - -	30	32	$26\frac{1}{5}$	$28\frac{1}{5}$
Z 100 brasses of Milan - - - - - -	$27\frac{3}{5}$	$29\frac{1}{4}$	$23\frac{4}{5}$	$25\frac{3}{4}$

measures of the principal trading cities in Europe have Dictionary.

R	S	T	V	W	X	Y	Z
Vares of Castille and Biscay.	Vares of Cadiz and Andalusia.	Vares of Portugal or Lisbon.	Covedos of Portugal or Lisbon.	Brasses of Venice.	Brasses of Berg. Boulogn, Modena, and Mantua.	Brasses of Florence, Leghorn, and Lucca.	Brasses of Milan.
107	$109\frac{1}{5}$	$81\frac{1}{3}$	$133\frac{1}{3}$	136	$104\frac{1}{3}$	$154\frac{3}{4}$	$171\frac{1}{3}$
$136\frac{7}{8}$	140	$104\frac{1}{3}$	171	$174\frac{4}{5}$	179	$199\frac{1}{5}$	$219\frac{1}{4}$
80	$81\frac{4}{5}$	61	100	102	$105\frac{1}{4}$	$116\frac{1}{2}$	$128\frac{1}{2}$
81	84	$61\frac{3}{5}$	$101\frac{1}{4}$	$103\frac{3}{5}$	$106\frac{3}{5}$	118	130
$65\frac{4}{5}$	$68\frac{1}{3}$	$50\frac{4}{5}$	$83\frac{1}{3}$	85	$88\frac{1}{2}$	97	107
64	$65\frac{1}{5}$	$48\frac{4}{5}$	80	$81\frac{3}{5}$	$84\frac{4}{5}$	$93\frac{1}{5}$	$102\frac{4}{5}$
$71\frac{1}{5}$	$72\frac{1}{2}$	$54\frac{1}{2}$	89	$90\frac{3}{4}$	$93\frac{3}{5}$	$103\frac{3}{4}$	$114\frac{1}{3}$
72	$74\frac{3}{4}$	55	90	$91\frac{3}{4}$	$94\frac{3}{4}$	105	$115\frac{3}{4}$
70	$71\frac{1}{2}$	$53\frac{1}{2}$	$87\frac{1}{2}$	$89\frac{1}{4}$	92	102	$112\frac{1}{2}$
$92\frac{4}{5}$	$95\frac{1}{5}$	$70\frac{3}{4}$	116	$118\frac{3}{4}$	122	$135\frac{1}{5}$	149
$71\frac{1}{2}$	73	$54\frac{1}{2}$	$89\frac{1}{3}$	$91\frac{1}{5}$	94	104	$114\frac{3}{4}$
$133\frac{1}{3}$	$136\frac{2}{5}$	$101\frac{2}{5}$	$166\frac{2}{3}$	170	$172\frac{4}{5}$	$193\frac{1}{5}$	$214\frac{1}{5}$
$228\frac{4}{5}$	234	$174\frac{3}{4}$	286	$291\frac{1}{4}$	301	$333\frac{1}{5}$	$367\frac{1}{2}$
$213\frac{1}{3}$	218	$162\frac{2}{3}$	$266\frac{2}{3}$	272	$280\frac{3}{5}$	$309\frac{1}{3}$	$342\frac{2}{3}$
$261\frac{3}{5}$	$268\frac{1}{5}$	$199\frac{1}{2}$	327	$333\frac{1}{2}$	$344\frac{1}{5}$	381	$420\frac{1}{5}$
$242\frac{4}{5}$	$245\frac{3}{5}$	$184\frac{4}{5}$	303	309	319	353	$389\frac{4}{5}$
100	$102\frac{1}{3}$	$76\frac{1}{4}$	125	$127\frac{1}{2}$	$131\frac{1}{2}$	$145\frac{3}{5}$	$159\frac{2}{3}$
$97\frac{2}{3}$	100	$74\frac{2}{3}$	$122\frac{1}{4}$	$125\frac{1}{4}$	179	142	157
$131\frac{1}{3}$	134	100	164	$167\frac{1}{4}$	$172\frac{3}{5}$	191	$210\frac{3}{4}$
80	$81\frac{4}{5}$	61	100	102	$105\frac{1}{4}$	$116\frac{1}{2}$	$128\frac{1}{2}$
$78\frac{4}{5}$	$80\frac{1}{5}$	$59\frac{3}{4}$	98	100	$103\frac{1}{3}$	$114\frac{1}{5}$	126
76	$78\frac{1}{5}$	58	95	97	100	$100\frac{3}{5}$	122
$68\frac{4}{5}$	$70\frac{1}{3}$	$52\frac{1}{5}$	$85\frac{1}{2}$	$87\frac{1}{5}$	95	100	$109\frac{4}{5}$
$62\frac{4}{5}$	$63\frac{2}{3}$	$47\frac{1}{5}$	78	$79\frac{1}{5}$	$82\frac{1}{5}$	91	100

3. Sup-

3. Suppose you owe 100 anees of wheat at Lyons, and would know what quantity you would purchase at Macon to replace them, and have no other means of knowledge but the following; viz.

$\cancel{3}$ anees of Lyons - - -	=	$\cancel{4}$ setiers of Paris.
1 setier of Paris - - - -	=	2 bushels of Bourdeaux.
$\cancel{30}$ bushels of Bourdeaux	=	$\cancel{77}$ muds of Amsterdam.
$\cancel{77}$ muds of Amsterdam	=	$\cancel{40}$ fanegas of Cadiz.
$\cancel{130}$ fanegas of Cadiz - -	=	$\cancel{87}$ anees of Macon.
How many of Macon - -	=	$\cancel{100}$ of Lyons?

9 - - - - - - - - - -	$\cancel{2}$
1 - - - - - - - - - -	19
$\cancel{8}$ - - - - - - - - - -	$\cancel{1}$
1 - - - - - - - - - -	20

$2 \times 19 \times 20 = 760$

9) 760 ($84\frac{4}{9}$ anees of Macon, the answer.

4. Suppose a merchant of Hamburgh, not knowing the proportion between the ell of that place and yard of London, and having orders to procure 81 yards of cloth, of which 7 ells of Hamburgh must be had for 3l. sterling; how shall he discover how many pounds sterling the 81 yards will amount to, only by knowing that 7 ells of France make 9 yards of London, and 7 ells of Holland make 4 ells of France, and that 1 ell of Holland makes $1\frac{1}{5}$ of Hamburgh?

Note, since $1 = 1\frac{1}{5}$; consequently, $5 = 6$, which dispatches the fraction.

$\cancel{9}$ yards of London	=	$\cancel{7}$ ells of France.
$\cancel{4}$ ells of France -	=	7 ells of Holland.
5 ditto of Holland	=	$\cancel{6}$ ditto of Hamburgh.
$\cancel{7}$ ditto of Hamburgh	=	$\cancel{3}$ pound sterling.
How much sterling for 81 yards?		

$\cancel{3}$ - - - - - - - - -	1
2 - - - - - - - - -	$\cancel{3}$

$81 \times 7 = 567$, divisor.

$2 \times 5 = 10$, dividend.

∴ 567 = 56 l. 14 s. the answer required.

THE END of the SECOND BOOK.

Arithmetical Collections AND IMPROVEMENTS.

BOOK III.

Containing the more abstruse and curious part of ARITHMETICK.

CHAPTER I.

ALLIGATION.

WHEN corn, wine, spices, metal, &c. are required to be mixed together, the method of proportioning such mixtures is called the rule of alligation.

SECT. I.

ALLIGATION MEDIAL.

BY alligation medial the mean rate or price of any mixture is found, when the particular quantities and their prices are given.

RULE.

First find the sum of all the quantities proposed to be mixed, and also the sum of their particular rates; then as the sum of all the quantities : is to the sum of all the rates : : so is any part of the mixture : to the mean rate or price of that part.

1. A vintner mixeth $31\frac{1}{2}$ gallons of Malaga sack, worth 7 s. 6 d. a gallon, with 18 gallons of Canary, at 6 s. 9 d. a gallon; $13\frac{1}{2}$ gallons of cherry, at 5 s. a gallon; and 27 gallons of white wine, at 4 s. 3 d. a gallon; what is a gallon of this mixture worth?

gal.			s.	d.		l.	s.	d.
$31\frac{1}{2}$	- - sack	- - at	7	6	- - =	11	16	3
18	- - Canary	- at	6	9	- - =	6	1	6
$13\frac{1}{2}$	- - cherry -	- at	5	–	- - =	3	7	6
27	- - white wine	at	4	3	- - =	5	14	9

27 l.

l. $10 \times 9 = 90$.

$$\begin{array}{r|l} 10 & 27 \\ 9 & \text{— s. d.} \\ & 2\ 14\ - \\ \hline & -\ 6\ - \text{ per gallon, the answer.} \end{array}$$

2. With 13 gallons of Canary, at 6s. 8d. a gallon, I mixed 20 gallons of white wine, at 5s. a gallon, and to these added 10 gallons of cyder, at 3s. a gallon; at what rate must I sell a quart of this mixture, so as to clear 10 per cent.?

gal.			s. d.		l. s. d.
13	- -	Canary - at	6 8	- - -	4 6 8
20	- -	white wine at	5 -	- - -	5 - -
10	- -	cyder - - at	3 -	- - -	1 10 -
43 = 172 quarts		- - - - - -			£ 10 16 8 = 10.83

10) 10.833
1.083

172) 11.916 (.06928 = 1s. 4.627 d. the answer.

SECT. II.

ALLIGATION ALTERNATE.

IS when the particular rate of every ingredient, and the mean rate, are given, to discover the particular quantity of each ingredient concerned in a mixture.

RULE.

Place the mean rate so, that it may be easily compared with the particular rates; setting down the differences between the mean rate and the particular rates, alternately, and they will be the quantities required.

1. A

1. A grocer would mix a quantity of sugar, at 10 d. per pound, with other sugars of $7\frac{1}{2}$d, 5 d. and $4\frac{1}{2}$ d. per pound, intending to make up a commodity worth 6 d. per pound; in what proportions is he to take of those sugars?

(1)
$$6\left\{\begin{array}{l}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{array}\right.\left|\begin{array}{l}1\frac{1}{2}\\1\\1\frac{1}{2}\\4\end{array}\right|$$

(2)
$$6\left\{\begin{array}{l}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{array}\right.\left|\begin{array}{l}1\\1\frac{1}{2}\\4\\1\frac{1}{2}\end{array}\right|$$

When one branch is linked to two or more other branches, the differences ought to be as often transcribed as it is so diversly linked.

(3)
$$6\left\{\begin{array}{l}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{array}\right.\left|\begin{array}{l}1\frac{1}{2}\\1\frac{1}{2}+1\\1\frac{1}{2}\\4+1\frac{1}{2}\end{array}\right|\begin{array}{l}1\frac{1}{2}\\2\frac{1}{2}\\1\frac{1}{2}\\5\frac{1}{2}\end{array}\Bigg|$$

(4)
$$6\left\{\begin{array}{l}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{array}\right.\left|\begin{array}{l}1\frac{1}{2}+1\\1\\4+1\frac{1}{2}\\4\end{array}\right|\begin{array}{l}2\frac{1}{2}\\1\\5\frac{1}{2}\\4\end{array}\Bigg|$$

(5)
$$6\left\{\begin{array}{l}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{array}\right.\left|\begin{array}{l}1\frac{1}{2}+1\\1\frac{1}{2}\\4\\4+1\frac{1}{2}\end{array}\right|\begin{array}{l}2\frac{1}{2}\\1\frac{1}{2}\\4\\5\frac{1}{2}\end{array}\Bigg|$$

(6)
$$6\left\{\begin{array}{l}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{array}\right.\left|\begin{array}{l}1\\1+1\frac{1}{2}\\4+1\frac{1}{2}\\1\frac{1}{2}\end{array}\right|\begin{array}{l}1\\2\frac{1}{2}\\5\frac{1}{2}\\5\frac{1}{2}\end{array}\Bigg|$$

(7)
$$6\left\{\begin{array}{l}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{array}\right.\left|\begin{array}{l}1+1\frac{1}{2}\\1+1\frac{1}{2}\\4+1\frac{1}{2}\\4+1\frac{1}{2}\end{array}\right|\begin{array}{l}2\frac{1}{2}\\2\frac{1}{2}\\5\frac{1}{2}\\5\frac{1}{2}\end{array}\Bigg|$$

2. A proveditor for the army intending to mix wheat at 4 s. a bushel, with rye at 3 s. a bushel, with barley at 2 s. a bushel, with pease at 1 s. 4 d. a bushel, and with oats at 12 d. a bushel, is desirous to know in what proportion to mix them, so that the mass may be worth 1 s. 8 d. per bushel?

There are divers ways of alligating or linking these numbers together, viz.

$$20\left\{\begin{array}{l}48\\36\\24\\16\\12\end{array}\right.\left|\begin{array}{l}8\\4\\4\\16+4\\28\end{array}\right|\begin{array}{l}8\\4\\4\\20\\28\end{array}\left|\begin{array}{l}2\\1\\1\\5\\7\end{array}\right|$$

, or,

$$20\left\{\begin{array}{l}48\\36\\24\\16\\12\end{array}\right.\left|\begin{array}{l}4\\8\\4\\28+4\\16\end{array}\right|\begin{array}{l}4\\8\\4\\32\\16\end{array}\left|\begin{array}{l}1\\2\\1\\8\\4\end{array}\right|\begin{array}{l}W\\R\\B\\P\\O\end{array}$$

or,

or,

	48	8	8	2
	36	4	4	1
20	24	8	8	2
	16	16	16	4
	12	28 + 4	32	8

	48	8	8	2	W
	36	8	8	2	R
20	24	4	4	1	B
	16	4	4	1	P
	12	28 + 16	44	11	O

	48	4	4	4	1
	36	8	8	8	2
20	24	8	8	8	2
	16	28	28	28	7
	12	16 + 4	20	20	5

	48	8	8	2	W
	36	4 + 8	12	3	R
20	24	4	4	1	B
	16	16 + 4	20	5	P
	12	28 + 16	44	11	O

	48	4	4	1
	36	4	4	1
20	24	8	8	2
	16	28 + 16	44	11
	12	4	4	1

	48	4 + 8	12	3	W
	36	4	4	1	R
20	24	8	8	2	B
	16	28 + 16	44	11	P
	12	28 + 4	32	8	O

	48	8	8	2
	36	4	4	1
20	24	8 + 4	12	3
	16	16 + 4	20	5
	12	28 + 4	32	8

	48	4 + 8	12	3	W
	36	4	4	1	R
20	24	4	4	1	B
	16	28 + 16 + 4	48	12	P
	12	28	28	7	O

	48	4	4	1
	36	4 + 8	12	3
20	24	4	4	1
	16	28 + 16 + 4	48	12
	12	16	16	4

	48	4	4	1	W
	36	4	4	1	R
20	24	8 + 4	12	3	B
	16	28 + 16 + 4	48	12	P
	12	4	4	1	O

	48	8	8	2
	36	8	8	2
20	24	8 + 4	12	3
	16	4	4	1
	12	28 + 16 + 4	48	12

	48	8	8	2	W
	36	4 + 8	12	3	R
20	24	8	8	2	B
	16	16	16	4	P
	12	28 + 16 + 4	48	12	O

	48	4 + 8	12	3
	36	8	8	2
20	24	8	8	2
	16	28	28	7
	12	28 + 16 + 4	48	12

	48	8 + 4	12	3	W
	36	8	8	2	R
20	24	8 + 4	12	3	B
	16	28 + 4	32	8	P
	12	28 + 16 + 4	48	12	O

	48	4 + 8	12	3
	36	4 + 8	12	3
20	24	9	8	2
	16	28 + 16	44	11
	12	28 + 16 + 4	48	12

	48	8	8	2	W
	36	8 + 4	12	3	R
20	24	8 + 4	12	3	B
	16	16 + 4	20	5	P
	12	28 + 16 + 4	48	12	O

or,

or,

	48	8 + 4	12	3
	36	8 + 4	12	3
20	24	4	4	1
	16	28+16+4	48	12
	12	28 + 16	44	11

	48	4 + 8	12	3	W
	36	4	4	1	R
20	24	4 + 8	12	3	B
	16	21 + 16+4	48	12	P
	12	28 + 4	32	8	O

	48	4	4	1
	36	8 + 4	12	3
20	24	8 + 4	12	3
	16	28+16+4	8	12
	12	15 + 4	20	5

	48	4 + 8	12	1	W
	36	4 + 8	12	1	R
20	24	4 + 8	12	1	B
	16	28+16+4	48	4	P
	12	28+16+4	48	4	O

Here you have 24 different anſwers by the various ways of alligating or linking the prices together, which may be increaſed infinitely by doubling, tripling, &c. the quantities; or they may be leſſened by making the pecks, pints, or any leſs quantity.

The reaſon of theſe combinations, and the alternate placing of their differences, will appear from this plain conſideration, viz. that whatſoever is loſt by ſelling any quantity whoſe price exceeds the mean, is gained again on the quantity alligated thereto, whoſe given price is leſs than the mean.

When two kinds of things only are given to be mixed, the rule of alligation will give but one anſwer.

3. Suppoſe it is required to mix brandy, at 8 s. per gallon, with cyder, at 1 s. per gallon; ſo that the mixture may be worth 5 s. per gallon.

5	8	4 gallons of brandy.
	1	3 gallons of cyder.

If three kinds of things are given to be mixed, the rule of alligation will give but one anſwer; but then (as might have been obſerved in mixture of two things) all numbers that are in the ſame proportion between themſelves, and the number which compoſes that anſwer, will alſo ſatisfy the queſtion.

But by an artifice explained by the ingenious Mr. James Dodſon, in the 18th edition of Wingate's Arithmetic, innumerable other anſwers may be obtained, compoſed of numbers in a different proportion.

4. Let it be required to mix brandy, at 8 s. per gallon, with wine at 7 s. per gallon, and cyder at 1 s. per gallon,

gallons, ſo that the mixture may be worth 5 s. per gallon?

8	4	4 brandy.
7	4	4 wine.
1	3 + 2	5 cyder.

Now ſuppoſe, that if it be determined to uſe five gallons of cyder in the mixture; but to uſe any quantity of brandy and wine that will anſwer the queſtion.

Then may the quantity of brandy be increaſed or diminiſhed by 2; the difference between the prices of the wine and mixture, if at the ſame time the quantity of wine be diminiſhed or increaſed by 3, the difference of the prices of the brandy and mixture.

Thus, 4 + 2 = 6 brandy, and 4 — 3 = 1 wine; ſo that ſix gallons of brandy, one gallon of wine, and five gallons of cyder, will alſo anſwer the queſtion, as may be eaſily provided by alligation medial.

Again, 4 — 2 = 2 brandy, and 4 + 3 = 7 wine.

So that two gallons of brandy, ſeven gallons of wine, and five gallons of cyder, will alſo anſwer the queſtion, as may be proved.

But inſtead of the numbers of the firſt anſwer, 4, 4 and 5, larger numbers in the ſame proportion, viz. 12, 12 and 15 were taken, the following eight anſwers would be found by increaſing and diminiſhing the quantities of brandy and wine, as above directed, the quantity of cyder remaining conſtantly 15.

Brandy	18	16	14	12	10	8	6	4	2
Wine	3	6	9	12	15	18	21	24	27
Cyder	15	15	15	15	15	15	15	15	15

And if inſtead of theſe ſtill larger numbers in that proportion, or in proportion to any of the laſt found anſwers, be aſſumed, a greater number of other anſwers may be found.

But if inſtead of ſuppoſing the quantity of cyder invariable, the quantity of brandy be taken for ſuch; then an infinite number of anſwers may be found, by continually increaſing the quantity of wine by 4, the difference between the prices of the cyder and mixture; and the quantity of cyder by 2, the difference between the prices of the wine and mixture.

Thus, aſſuming the ſecond anſwer, 6, 1 and 5, and making the ſix gallons of brandy invariable:

Brandy

Brandy	5	6	6	6	6	6	6	6	6, &c.
Wine	1	5	9	13	17	21	25	29	33, &c.
Cyder	5	7	9	11	13	15	17	19	21, &c.

Or by taking the third anſwer, 2, 7 and 5, as the baſis, and making the ſeven gallons of wine invariable; increaſing the quantity of brandy by 4, the difference between the price of the cyder and mixture; and the quantity of cyder by 3, the difference of the prices of the brandy and mixture:

Brandy	2	6	10	14	14	22	26	30, &c.
Wine	7	7	7	7	7	7	7	7, &c.
Cyder	5	8	11	14	17	20	23	26, &c.

When there are four kinds of things to be mixed, and two of them of greater value, and the other two of leſſer value than the mixture, the rule of alligation will give ſeven anſwers, as may be obſerved by queſtion 1, in this rule; with any of which, or with any numbers in the ſame proportion, innumerable other anſwers may be found, conſiſting of numbers in different proportion among themſelves, by making any two invariable, and changing the reſt in the manner as above, obſerving alſo the following

RULE.

The numbers by which the quantity of any ſimple is to be varied, is always the difference between the price of the mixture and the price of the other ſimple, which in any operation is conſidered as variable.

Secondly, That if the ſimples, which in any operation are conſidered as variable, be both of greater, or both of leſs value than the mixture, then, while the one is increaſed, the other muſt be diminiſhed; but if one be of greater value than the mixture, and the other of leſs, then they muſt both be increaſed, or both diminiſhed.

5. Let it be required to mix brandy, at 8 s. wine, at 7 s. cyder, at 1 s. and water at nothing per gallon, together; ſo that the mixture may be worth 5 s. per gallon?

I ſhall only alligate the ſeveral values of the ſimples together by the following method:

5	8	5 + 4	9	brandy,
	7	5 + 4	9	wine.
	1	3 + 2	5	cyder.
	0	3 + 2	5	water.

Now

Now making the wine and cyder invariable:

Brandy	9	14	19	24	29	34	39,	&c.
Wine	9	9	9	9	9	9	9,	&c.
Cyder	5	5	5	5	5	5	5,	&c.
Water	5	8	11	14	17	20	23,	&c.

Making the brandy and cyder invariable:

Brandy	9	9	9	9	9	9	9,	&c.
Wine	9	14	19	24	29	34	39,	&c.
Cyder	5	5	5	5	5	5	5,	&c.
Water	5	7	9	11	13	15	17,	&c.

Making the wine and water invariable:

Brandy	9	13	17	21	25	29	33,	&c.
Wine	9	9	9	9	9	9	9,	&c.
Cyder	5	8	11	14	17	20	23,	&c.
Water	5	5	5	5	5	5	5,	&c.

Making the brandy and wine invariable:

Brandy	9	9	9
Wine	9	9	9
Cyder	10	5	–
Water	1	5	9

Or taking four other numbers in the same proportion:

As 9 . 9 . 5 and 5, viz. 36 . 36 . 20 and 20.

Brandy	36	36	36	36	36	36	36	36
Wine	36	36	36	36	36	36	36	36
Cyder	40	35	30	25	20	15	10	5
Water	4	8	12	16	20	24	28	32

Lastly, making the cyder and water invariable:

Brandy, &c.	44	42	40	38	36	34	32	30	28,	&c.
Wine, &c.	24	27	30	33	36	39	42	45	48,	&c.
Cyder, &c.	20	20	20	20	20	20	20	20	20,	&c.
Water, &c.	20	20	20	20	20	20	20	20	20,	&c.

Not only the sets of numbers thus found, but their sums and differences, will also be answers.

Thus

	bran.	wine.	cyd.	wat.
Thus from or to	42 .	27 .	20 .	20
Take or add -	9 .	9 .	10 .	1
The remainder -	33 .	18 .	10 .	19
And ſum - - -	51 .	36 .	30 .	21

will be anſwers to the queſtion.

These anſwers may all be proved by alligation medial; I ſhall only prove the laſt, viz. the difference, and leave the reſt to exerciſe the young arithmetician;

	gal.		s.		l.	s.	d.
viz.	33	brandy,	at 8	- - - - -	13	4	–
	18	wine,	at 7	- - - - -	6	6	–
	10	cyder,	at 1	- - - - -	–	10	–
	19	water,	at –	- - - - -	–	–	–
	80			£	20	–	–

80 gal. : 20l. : : 1 gal. : 5s.

* *

SECT. III.

ALLIGATION PARTIAL.

ALLIGATION PARTIAL is when, having the ſeveral rates of divers ingredients and the quantity of one of them given, we diſcover the ſeveral quantities of the reſt in ſuch ſort, that the quantities ſo found, being mixed with the quantity given, that mixture may bear a certain rate propoſed.

Having ſet down the mean rate, the particular rates and their differences, as before, ſay,

RULE,

As the difference oppoſite to the known quantity is to : the known quantity, ſo is : : any other difference : to the quantity of its oppoſite name.

1. Let it be required to mix brandy, at 8s. per gallon, and wine, at 7s. per gallon, with 10 gallons of cyder, at 1s.

1 s. per gallon, so that the mixture may be worth 5 s. per gallon.

$$\left\{\begin{array}{l}8\\7\\1\end{array}\right. \quad \begin{array}{|l}4\\4\\3+2\end{array} \quad \begin{array}{|l}4\\4\\5\end{array}$$

$$\overline{13}$$

5 : 10 : : 4 : 8 gallons each of brandy and wine.
5 : 10 : : 13 : 26 gallons, the whole mixture.

Now, having found one answer by the above proportion, others may be found by the method before delivered.

Brandy	12	10	8	6	4	2
Wine	2	5	8	11	14	17
Cyder	10	10	10	10	10	10

By which means five other answers are obtained.

2. A tobacconist has by him 120 lb. of fine Oroonoko tobacco, worth 2 s. 6 d. a pound; to this he would put as much York-river ditto, at 20 d. with other inferior tobacco, at 18 d. and 15 d. a pound, as will make up a mixture answerable to 2 s. a pound: what will this parcel weigh?

$$24\left\{\begin{array}{l}30\\20\\18\\15\end{array}\right. \quad \begin{array}{|l}4+6+9\\6\\6\\6\end{array} \quad \begin{array}{|l|}19\\6\\6\\6\end{array}$$

Then $19+6+6+6=37$.

$\because$ 19 : 37 : : 120 : $233\frac{13}{19}$, the answer required.

But as some answers in whole numbers may also be obtained by the foregoing method, putting 38, 12, 12 and 12 instead of those found by alligation, the two last being invariable.

38	42	46	50	54	58	62
12	18	24	30	36	42	48
12	12	12	12	12	12	12
12	12	12	12	12	12	12

Now, taking the sum of the two last sets of numbers,

	at 2 s. 6 d.	at 1 s. 8 d.	at 1 s. 6 d.	at 1 s. 3 d.
viz.	58	42	12	12
	62	48	12	12
	120 +	90 +	24 +	24 = 258,

being a second answer.

By

By making the ſecond and laſt invariable:

32	38	44	50
12	12	12	12
6	12	18	24
12	12	12	12
at 2s. 6d.	at 1s. 8d.	at 1s. 6d.	at 1s. 3d.
32	12	6	12
38	12	12	12
50	12	24	12
120 +	36 +	42 +	36 = 234,

the third anſwer.

Laſtly, making the ſecond and third invariable:

11	20	29	38	47	56	65	74	83	92	101
12	12	12	12	12	12	12	12	12	12	12
12	12	12	12	12	12	12	12	12	12	12
—6	0	6	12	18	24	30	36	42	48	54

	2s. 6d.	1s. 8d.	1s. 6d.	1s. 3d.
	101	12	12	54
	101	12	12	54
	47	12	12	18
	11	12	12	— 6
	260	48	48	120
ſubtr.	20	12	12	-
2)	240	36	36	120
	120 +	18 +	18 +	60 = 216,

the fourth anſwer.

Theſe laſt three anſwers may each be made the baſis of divers others in different proportion, by making the firſt term with any one of the others invariable; and the other two variable to their utmoſt limits, which I ſhall leave for the practice of young ſtudents in arithmetic; having (I think) been copious enough upon this ſubject.

Hh SECT.

SECT IV.

ALLIGATION TOTAL.

ALLIGATION TOTAL is ſo called, when the particular rates, the mean rate, and the whole quantity of the ingredients to be mixed, are given, and the particular quantity of each ingredient is required. To find which, obſerve the following

RULE.

Having found the ſeveral differences as before directed, ſay, as the ſum of all the differences: is to the whole quantity of the mixture :: ſo is each particular difference : to its particular quantity.

1. Let it be required to mix brandy, at 8s. wine, at 7s. and cyder, at 1s. per gallon together; ſo that the mixture may contain 26 gallons, and be worth 5s. per gallon.

$$5\begin{cases}8\\7\\1\end{cases}\quad\left|\begin{array}{l}4\\4\\3+2\end{array}\right|\begin{array}{l}4\\4\\5\end{array}\Bigg|\qquad \frac{}{13}$$

$$13:26::\begin{cases}4\\4\\5\end{cases}\begin{array}{c}:\\:\\:\end{array}\begin{array}{l}8\text{ brandy.}\\8\text{ wine.}\\10\text{ cyder.}\\\hline 26\end{array}$$

One anſwer being thus obtained, the reſt may be found by the following

RULE.

I. Let the quantity of that ingredient, whoſe value alone is greater or leſs than the value of the mixture, be increaſed or diminiſhed by the difference or differences between the prices of the other two ingredients, and the price of the mixture.

II. Of the remaining two ingredients, let the quantity of that ingredient, whoſe value is fartheſt from the value of the mixture, be increaſed or decreaſed (according as the former is) by the ſum of the differences between the prices of the other two ingredients, and that of the mixture.

III. Let

III. Let the quantity of the remaining ingredient be decreased or increased, also, by the sum of the differences between the prices of the other two ingredients; and that of the mixture; but observe, that the quantity of this ingredient is to be decreased, when those of the two former are increased; and the contrary.

I. The value of the cyder alone is less than the value of the mixture.

Also, $8-5=3$, $7-5=2$, and $3-2=1$, the difference of those differences.

$\because$ $10+1=11$, and $10-1=9$, are the quantities of cyder.

II. Of the other two the value of the brandy is furthest from that of the mixture.

Also, $7-5=2$, $5-1=4$, and $2+4=6$, sum of their differences.

$\because$ $8+6=14$, and $8-6=2$, are the quantities of brandy.

Lastly, $8-5=3$, $5-1=4$, and $4+3=7$.

$\because$ $8-7=1$, and $8+7=15$, are the quantities of the wine.

Thus we have obtained two answers more, which make in all three different answers to this question.

$$\left.\begin{array}{lccc} \text{Brandy} & 14 & 8 & 2 \\ \text{Wine} & 1 & 8 & 15 \\ \text{Cyder} & 11 & 10 & 9 \end{array}\right\} \text{all which may be proved by alligation medial.}$$

But if there be four or more ingredients out of which the mixture is to be compounded, then one or more of them must be considered as invariable; so that there may be only three variable, and those so, that one of them will be of a contrary value, with respect to the price of the mixture, from the other two.

2. It is required to mix such a quantity of brandy, at 8s. wine at 7s. cyder at 1s. and water at nothing per gallon, as will make a hogshead, or 63 gallons of the mixture, worth 5s. per gallon.

Then by the process in alligation alternate, question 5th, the two following proportions may be found; viz.

	Bran.	wine.	cyd.	wat.
Among the first found answers	9	9	5	5
Among the third - - - -	9	14	5	7

$$18+23+10+12=63.$$

Then

Then making the water invariable, we have, by the foregoing rule,

Brandy	36	30	24	18	12	6
Wine	2	9	16	23	30	37
Cyder	13	12	11	10	9	8
Water	12	12	12	12	12	12

II. Making the cyder invariable, produceth

Brandy	32	25	18	11	4
Wine	7	15	23	31	39
Cyder	10	10	10	10	10
Water	14	13	12	11	10

III. Making the wine invariable, gives

Brandy	19	18	17
Wine	23	23	23
Cyder	2	10	18
Water	19	12	15

Lastly, making the brandy invariable, we have

Brandy	18	18	18
Wine	24	23	22
Cyder	3	10	17
Water	18	12	6

If you are desirous to find more answers, you may, for

Water makes any number invariable from	5 to 19
Cyder from - - - - - - - - - - - -	2 to 18
Wine from - - - - - - - - - - - -	2 to 39
Brandy from - - - - - - - - - - - -	4 to 36

But if instead of gallons you mix by pints; viz.

instead of 24 . 16 . 11 . 12 gallons,

you take 192 . 128 . 88 . 96 pints for the basis of the operation, a still greater number of answers may be produced; viz.

B.	192	191	190	189	188	187	186	185	184	183	182	181	180	179
W.	128	128	128	128	128	128	128	128	128	128	128	128	128	128
C.	88	96	104	112	120	128	136	144	152	160	168	176	184	192
W.	96	89	82	75	68	61	54	47	40	33	26	19	12	5

And,

B.	192	193	194	195	196	197	198	199	200	201	202
W.	128	128	128	128	128	128	128	128	128	128	128
B.	88	80	72	64	56	48	40	32	24	16	8
W.	96	103	110	117	124	131	138	145	152	159	166

CHAP.

CHAPTER II.

Specific Gravity *of* Metals, *&c.*

THE specific gravity of a body, is the relation the weight of a body of one kind hath to the weight of an equal magnitude of a body of another kind.

Gold is the heavieſt of all known bodies, the moſt malleable and ductile of all metals; is incapable of ruſt, and not ſonorous when ſtruck upon; requires a ſtrong fire to melt it, is the moſt diviſible of all bodies; and its ductility is ſuch, that wire-drawers can extend a leaf of gold to the 12000000th part of an inch in thinneſs, over a flatted ſilver wire, which will be perfectly covered, though viewed with a microſcope; by which means an ounce of gold may be made to reach more than 155½ miles.

Silver is the fineſt, pureſt, moſt ductile, and moſt precious metal, according to its natural properties, except gold.

Lead is the heavieſt of all metals next to gold; it is the ſofteſt of all, is leaſt ſonorous, except gold, very ductile, and the moſt ready fuſible of all, except tin.

Tin is a white ſhining metal, of ſo pliable a nature, that it may be bent into any form; its harſhneſs is between ſilver and lead, and is the lighteſt of all metals.

Copper is an hard ſonorous metal, difficult in fuſion, and is mixed with gold and ſilver, in order to harden them, and render them more uſeful either in coin or utenſils, which would otherwiſe be too ſoft and flexible.

Iron is the leaſt heavy of all metals, except tin, but conſiderably the hardeſt of them all; fire renders it more ductile, being moſt of all maleable when hotteſt; when wrought into ſteel, is leſs maleable; it is more capable of ruſt than any other metal; it is very ſonorous, and requires the ſtrongeſt fire of all the metals to melt it.

In the compariſon of the weights of bodies, it will be the moſt convenient to conſider one body the ſtandard or unit to which others are to be compared.

Rain-water is nearly alike in all places, a cubic foot of which hath, by repeated experiments, been found to weigh 62½ pounds averdupoiſe.

 A TA-

A TABLE *shewing the specific gravity to rain-water of inch of each in parts of a pound averdupoise, taken from of an ounce from Ward: the deficiencies in both authors*

Bodies.	sp. gra.	wt. lb. av.	wt. oz. tr.
Fine gold - - - -	19.640	0.7103587	10.359273
Standard gold - -	19.520	0.7060185	9.962625
Coast Gold - - -	18 888	0.6828703	9.911707
Quickfilver - - -	13.762	0.4976574	7.384411
Lead - - - - - -	11.313	0.4091696	5.984010
Fine filver - - -	11.091	0.4011501	5.850035
Standard filver - -	10.629	0.3844400	5.556769
Cast filver - - - -	10.528	0.3807870	5.503967
Copper - - - -	8.769	0.3171658	4.747121
Plate brafs - - -	8.350	0.2942593	4.404273
Cast brafs - - - -	8.104	0.2929832	4.272409
Steel - - - - - -	7.850	0.2839265	4.142127
Bar iron - - - -	7.764	0.2808159	4.031361
Block tin - - - -	7.238	0.2617901	3.861519
Cast iron - - - -	7.135	0.2580647	3.806568
Loadftone - - - -	5.106	0.1816788	2.724083
Blue flate - - - -	3.500	0.1264914	1.867272
Veined marble - -	2.702	0.0977286	1.429411
Common glafs - -	2.600	0.0940393	1.360841
Flint ftone - - -	2.582	0.0933883	1.351419
Portland ftone - -	2.570	0.0929543	1.345139
Freeftone - - - -	2.352	0.0915788	1.231038
Brick - - - - -	2.000	0.0723379	1.046801
Alabafter - - - -	1.888	0.0683061	0.988456
Ivory } Horn } - - - -	1.832	0.0662606	0.958489

metals and other bodies; and the weight of a cubic Robertson's Mensuration, and of ounces troy, and parts supplied.

Bodies.	ſp. gra.	wt. lb. av.	wt. oz. tr.
Brimſtone - - - -	1.800	0.0651042	0.949424
Clay - - - - - -	1.712	0.0619213	0 902498
Lignum vitæ - - -	1.327	0.0479862	0.699936
Coal - - - - - -	1.255	0.0453921	0 661959
Pitch - - - - - -	1.150	0.0415943	0.606576
Mahogony wood - -	1.063	0.0384475	0.560691
Dry box wood - -	1.030	0.0372530	0.543282
Milk / Sea water - - - -	1.033	0.0372530	0.542742
Rain water - - -	1.000	0 0361690	0.527458
Red wine - - - -	0.993	0.0359158	0.523766
Bees wax - - - -	0.995	0.0359881	0.524820
Linſeed oil - - -	0.932	0.0337095	0.491591
Proof ſpirits or brandy - -	0.927	0.0335503	0.489268
Dry oak - - - -	0.915	0.0330946	0 489008
Olive oil - - - -	0.913	0.0330222	0 481569
Beech - - - - - -	0.854	0.0308883	0.450449
Dry elm / Dry aſh - - - -	0.800	0.0289352	0.421966
Dry wainſcot - - -	0.747	0.0270182	0.394011
Dry yellow fir - -	0.657	0.0237630	0.346539
Cedar - - - - - -	0.613	0.0221715	0.323332
Dry white deal - -	0.569	0.0205801	0.300123
Cork - - - - - -	0.240	0.0186805	0.126590
Air - - - - - -	0.0012	0.0000434	0.000633

When a heavy body is weighed in any fluid, it loſes therein ſo much of its weight, as an equal bulk of that fluid is found to weigh: as for inſtance,

A cubic inch of lead $=$ 5.984010 } ounces troy, &c.
A cubic inch of water $=$ 0.542742 }

Their difference is $=$ 5.441268, the weight of a cubic inch of lead in the water, &c.

1. An irregular piece of lead ore, taken from the Yorkſhire pit, weighs in the ſcale juſt 12 ounces; but weighed in water loſes 5 ounces of that weight; ſo that a quantity of water of the bigneſs of the ore weighs juſt 5 ounces: from the Derbyſhire pit a rough fragment of ore weighs, out of water, $14\frac{1}{2}$ ounces; and in water 9 ounces: the comparative or the ſpecific weight of theſe two ores is required?

$14\frac{1}{2} - 9 = 5\frac{1}{2}$ lb. weight of water of an equal bulk.

$$\left.\begin{array}{l}\text{Then } 14\tfrac{1}{2} \times 5 = 72\tfrac{1}{2} \text{ Derbyſhire} \\ \text{And } 12 \times 5\tfrac{1}{2} = 66 \text{ to Yorkſhire}\end{array}\right\} \text{ore's gravity. Q.E.F.}$$

2. An irregular fragment of glaſs in the ſcale weighs 171 grains; another of magnet 102 grains: in water the firſt fetches up no more than 120 grains: and the other 79: then 51 and 23 are the ſeveral weights of their comparative bulks of water: what then will their ſpecific gravities turn out to be?

$$\left.\begin{array}{l}171 \times 23 = 3933 \text{ glaſs to} \\ 102 \times 51 = 5202 \text{ magnet}\end{array}\right\} \text{or as } 437 \text{ to } 578.$$

The ſolidity of any body, multiplied by the tabular weight correſponding, will give the weight in pounds averdupoiſe or ounces troy.

3. What is the weight of a piece of oak, of a rectangular form, whoſe length is 56 inches, breadth 18, and depth 12 inches?

First, $56 \times 18 \times 12 = 12096$ cubic inches.

Then $12096 \times .0330946 = 400.3122816$ lb. Q. E. F.

4. What is the diameter of an iron ſhot, weighing 42 pounds averdupoiſe?

First, .2580647) 42.0000000 (162.7499.

Then .5236) 162.7499 (310.84778457.

$\sqrt[3]{310.84778457} = 6.7743$, the diameter required.

5. What is the weight of an iron bombſhell of three inches thick, the greateſt diameter being 16 inches?

Firſt, $16 - 6 = 10$, the diameter of the concavity.

Alſo $16 \times 16 \times 16 = 4096$.

And $4096 \times .5236 = 2144.6656$.

Again, $10 \times 10 \times 10 = 1000$.

Alſo

Alſo $1000 \times .5236 = 523.6$

Then $2144.6656 - 523.6 = 1621.0656$, the ſolidity of the ſhell.

$\because$ $1621.0656 \times .2580647 = 418.3398$ lb. the weight required.

6. In the walls of Balbeck, in Turkey, there are three ſtones laid end to end, now in ſight, that meaſure in length 61 yards; one of which in particular is 63 feet long, 12 feet thick, and four yards over: now, if this block was marble, what power would balance it, ſo as to prepare it for moving?

Firſtly, $63 \times 12 \times 12 = 9072$ ſolid feet.
Alſo $9072 \times 1728 = 15676416$ cubic inches.
Then $15676416 \times .0977286 = 1532034.1887$ lb.
$\because$ 2240) 1532034 (683 tun, 18 cwt. 98 lb. Q. E. F.

7. Required the weight of one of the Portland key-ſtones to the middle arch of Weſtminſter-bridge; the diameter of the arch being 76 feet; the height of the key-ſtone five feet; the chord of its greateſt breadth to the front of the arch three feet four inches; and its depth in the arch four feet?

Firſt, $76 + 5 = 81$; alſo 3 f. 4 in. $= 3.\dot{3}$, greater breadth.
As $81 : 3.\dot{3} :: 76 : 3.127572$, its leaſt breadth.

Here the chords and their arches being nearly equal, viz. ſo ſmall a part of ſo large a circle differs very little from a right line, the figure of the key-ſtone may be reckoned a priſmoid, and meaſured accordingly;

viz. $3.\dot{3} \times 4 = 13.\dot{3}$; alſo $3.127572 \times 4 = 12.510288$.

Then $\dfrac{13.\dot{3} + 12.510288}{2} = 12.9218105$.

Alſo $12.92181 + 12.510288 + 13.\dot{3} = 38.76543$.
And $38.76543 \times \frac{5}{3} = 64.60905$ ſolid feet.
Then $64.60905 \times 1728 = 111644.4384$ cubic inches.
$\because$ $111644.4384 \times .0929543 = 10377.83$ lb.
Anſwer, 10377.83 lb. = 4 ton, 12 cwt. 2 qrs. 17.83 lb.

The weight of any body in pounds averdupoiſe, or ounces troy, being divided by the tabular weight correſponding, the quotient will be the ſolidity in cubic inches.

8. What

8. What will a block of marble, weighing 8 tons, 14 cwt. come to, at 6 s. a foot solid?

8 ton, 14 cwt. = 19488 lb.
.0977286) 19488.0000000 (199409.4 inches.
1728) 199409.4 (115.4 cubic feet.
1154. × .3 = 34.62 l. = 34 l. 12 s. 4¾ d.

The absolute weight of a body floating in a fluid, is precisely equal to the weight of such part of the fluid as shall be thrust away thereby, and displaced; or, in other words, to the immersed part of the body.

9. Suppose that a man of war, with all its ordnance, rigging, and appointments, draws so much water as to displace 1300 tons of sea-water, London beer measure; the weight of this vessel is required?

First 1300 × 4 = 5200 hogsheads.
Also 5200 × 15228 = 79185600 cubic inches
And 79185600 × 037253 = 2949901 lb. averdupoise.
Answer, 2949901 lb. = 26338 cwt. 1 qr. 17 lb.

10. How many inches will a cubic foot of dry elm sink in common water?

1728 × .0289352 = 50.0000256 lb. is the weight of a foot of elm, or of the water displaced.
0.36169) 50.0000256 (1382.4 cubic inches immersed.
∴ 144) 1382.4 (9 6 inches, the answer.

11. Suppose a seaman hath a gallon of brandy in a glass bottle, that weighs 3½ lb. troy on board; and to conceal it from the king's officers, throws it into the sea; if it will sink, how much force will just buoy it up?

First, 3½ lb. troy = 42 ounces.
Also 1.360841) 42.000000 (30.864 cubic inches.
Then 231 × 489268 = 113 020908 ounces brandy.
And 42 × 113 020908 = 155.020908 ounces in all.
Again, 231 + 30.864 = 261.864 inches of water.
Also 261.864 × .542742 = 142.12462 ounces of water.
∴ 155.020908 — 142 12462 = 12.896288 ounces heavier than the same bulk of salt water.

12. Another of the mariners has half an anchor of brandy; the cask suppose measures ⅛ of a cubic foot; what

what quantity of lead is juſt requiſite to keep the caſk and liquor under water?

Firſt, 8) 1728 (216 cubic inches, the caſk.
Alſo 231 × 5 = 1155 cubic inches of brandy.
Then 216 × .489008 = 105 625728, wt. of the caſk.
Alſo 1155 × .489268 = 565.104540 oz. weight of the brandy.
Again, 216 + 1155 = 1371 cubic inches.
Then 1371 × .542742 = 744 099282, weight of water of an equal bulk.
Alſo 105 625728 + 565.10454 = 670.730268.
And 744.099282 — 670.730268 = 73.268914.
Alſo one inch of lead 5.98401 — .542742 = 5.441268, weight of one inch of lead in water.
∴ Recip. 5.98401 : 73.268914 : : 5.441268 : 80½ ounces troy of lead to keep the caſk, with its contents, juſt under water. Q. E. F.

13. How thick muſt be the metal of a concave copper ball, ſix inches in its outſide diameter, ſo as to ſink to its center in common water?

Firſt, 6 × 6 × 6 = 216, cube of the diameter.
Alſo 216 × .5236 = 113.0976 cubic inches, the ſolidity of the ſphere.
2) 113.0976 (56.5488 cubic inches to be immerſed, or of water to be removed.
∴ 56.5488 × .036169 = 2.0453lb. weight of the copper ball.
And .3171658) 2.0453000 (6.448678 cubic inches of copper in the ball.
Again, 6 × 6 × 3.1416 = 113.0976 ſquare inches, ſuperficies of the ball. N. B. The ſolidity and ſuperficies of this ball are equal.
∴ 113.0976) 6 44870 (.057, or about $\frac{1}{18}$ of an inch in thickneſs. Q. E. F.

14. What will a chain of ſtandard gold weigh in water, that raiſes a fluid an inch in a veſſel three inches ſquare, when put into it? And ſuppoſing the workman had adulterated the ſaid chain with 14½ ounces of ſilver; how much higher would the water be, upon its immerſion being raiſed in the veſſel?

Firſt, 3 × 3 = 9 ſolid inches in the gold chain.

Then

Then $9 \times 9.962625 = 89.663625$, its weight in air.
And $9 \times 0.527458 = 4.747122$, wt. of its bulk of water.

Weight of the gold 84.916503 in water. Q. E. F.

A solid inch of silver is 5.556769 ounces troy.
As $5.556769 : 1 :: 14.5 : 2.6094$ inches of silver.
Then $89.663625 - 14.5 = 75.163625$ ounces of gold.
9.962625) 75.163625 (7.5546, space taken up by the gold.
2.6094, by the silver, as above.

Sum 10.164, by both
Then $10.164 - 9 = 1.164$.
∴ 9) 1 164 (.1293. Q. E. F.

15. Hiero, king of Sicily, ordered his jeweller to make him a crown, containing 63 ounces of gold; the workman thought substituting part silver therein a proper perquisite; which taking air, Archimedes was appointed to examine it, who, on putting it into a vessel of water, found it raised the fluid, or that itself contained 8.2245 cubic inches of metal; and having discovered that the cubic inch of gold more critically weighed 10 36 ounces, and that of silver but 5 85 ounces; he, by calculation, found what part of his majesty's gold had been changed: and you are desired to repeat the process?

10.36) 63.00 (6.08108 inches in solidity, had it been gold
5.85) 63.00 (10.76923 solid inches, if all silver.

$$\text{Then, by alligation, } 8.2245\left\{\begin{matrix} 6.08108 \\ 10.76923 \end{matrix}\right\} \quad \left\{\begin{matrix} 2.54473 \\ 2.14342 \end{matrix}\right.$$

4.68815

4.68815) 2.54473 (.5428, part gold.
4 68815) 2.14342 (.4572, part silver.
∴ $.5428 \times 63 = 34.1884 = 34$ oz. 3 dwt. $22\frac{1}{4}$ gr. of gold.
And $.4572 \times 63 = 28.8036 = 28$ 16 $1\frac{1}{4}$ of silver.
Q. E. F.

Since gold and silver are always weighed, bought and sold, by troy weight, which weights are seldom in the possession of gentlemen in the country, graziers, or farmers; therefore to prevent their being imposed on by Jews, and other itinerant traders, I shall give an example concerning the reduction of troy into averdupoise weight.

In

In the year 1696 an experiment was made by authority (by a balance which would turn with six grains put into either scale) when it was found, that 15 pounds averdupoise weight, were equal to 18lb. 2 ozs. 15 dwts. troy = 105000 grains.

∴ 15) 105000 (7000 grains troy = 16 oz. averdupoise.
Also 16) 7000 ($437\frac{1}{2}$ grains troy = 1 oz. averdupoise.
And 480)437.5(.9114583 oz. troy = 1 oz. averdupoise.

First, suppose a silver tankard weighs 2 lb. 11 oz. 8 dr. averdupoise, its weight in ounces troy is required?

First, 2 lb. 11 oz. 8 dr. = 43.5 ounces.

And 43 5 × .9114583 = 39.6484375 = 39 oz. 12 dwt. $23\frac{1}{4}$ gr. by the experiment above.

CHAPTER III.

POSITION; OR, THE RULE OF FALSE.

THE rule of position, or supposition, is so called, because we suppose some uncertain number, in order, that by reasoning from them we may gain the true number; and because those suppositions are taken at adventure, it is also called the rule of false.

SECT. I.

SINGLE POSITION.

BY single position are solved such questions as require only one supposition to discover the true result.

RULE.

When you have made choice of your position, work it according to the nature of the question as if it were the true number; and if by the ordering your position you find the result either too much or too little, you may then find out the number sought by this proportion;

viz.

viz. as the result of your position is to: the position, so is:: the given number: the number sought.

1. Three persons, viz. A, B, C, thus discourse together concerning their age; says B to A, I am as old and half as old again as you; then says C to B, but I am twice as old as you: A replied, I am sure the sum of all our ages is 165: now I demand each man's age?

$$\begin{aligned} &\text{Suppose } 24 = A. \\ &\text{Then will } 24 + 12 = 36 = B. \\ &\text{And } 36 \times 2 = 72 = C. \end{aligned}$$

$$\therefore \underline{132} : 24 :: 165 : 30, \text{ A's} \quad \text{Also } 45, \text{ B's} \quad \text{And } 90, \text{ C's} \left.\right\} \text{age. Q. E. F.}$$

$$\overline{165}$$

2. Three persons, Andrew, Benjamin, and Charles, are to go a journey of 235 miles; of this journey Andrew is to go a certain number of miles unknown; Benjamin is to go four times as many miles as Andrew, and three miles more; and Charles is to go twice as many miles as Benjamin, and five miles more: how many miles must each of these persons travel severally?

$$\begin{aligned} &\qquad\qquad\qquad\quad \text{miles.} \\ &\text{Suppose - - - - - } 10, \text{ Andrew.} \\ &\text{Then } \overline{10 \times 4} + 3 = 43, \text{ Benjamin.} \\ &\text{Again, } \overline{43 \times 2} + 5 = 91, \text{ Charles.} \end{aligned}$$

$$\overline{144}$$

$$\text{Also } 3 + \overline{3 \times 2} + 5 = \underline{14}$$

$$130 \text{ and } 235 - 14 = 221.$$

$$\left.\begin{aligned} \therefore 130 : 10 :: 221 : 17 \\ \text{Also } \overline{17 \times 4} + 3 = 71 \\ \text{And } \overline{71 \times 2} + 5 = 147 \end{aligned}\right\} \text{miles} \left\{\begin{aligned} &\text{Andrew.} \\ &\text{Benjamin.} \\ &\text{Charles.} \end{aligned}\right.$$

$$\overline{235}$$

3. There were in company together four persons, Adam, Edward, Charles, and William; Adam told Charles that he was older than him by two years; Edward told them, that he was as old as both of them together, and four years older; William, hearing them, said, I am just 96 years old, and that

that is equal to all your ages: how old was each of them severally?

Suppose Charles 20
Then Adam be 22
And Edward - 46

Then $2 + 2 + 4 = 8$.

$88 - 8 = 80$.

And $96 - 8 = 88$

$\because 80 : 20 :: 88 : 22 =$ Charles's
Also $22 + 2 = 24 =$ Adam's
And $22 + 24 + 4 = 50 =$ Edward's
Their sum $= 96 =$ William's
} age.

4. The captain, lieutenant, and cornet of a troop have taken among them from some enemy 478 crowns, which they agree to share in this manner; the captain is to have 24 times as much as the cornet, wanting only seven crowns; and the lieutenant is to have five times as much as the cornet, wanting three crowns; what is each officer's share?

Suppose the cornet to have 8
Then $8 \times 5 - 3 = 37$
And $8 \times 24 - 7 = 185$

230

Again, $7 + 3 = 10$
Also $478 + 10 = 488$
And $230 + 10 = 240$

$\because 240 : 8 :: 488 : 16\frac{4}{15} =$ cornet's
Also $16\frac{4}{15} \times 5 - 3 = 78\frac{5}{15} =$ lieutenant's
Lastly, $16\frac{4}{15} \times 24 - 7 = 383\frac{6}{15} =$ captain's
} share. Q. E. F.

5. Let 273 l. be divided amongst four persons, viz. Andrew, Bennet, Christopher, and Daniel; Andrew is to have a share unknown; Bennet is to have twice as much as Andrew, and 30 l. more; Christopher is to have three times as much as Andrew, wanting 52 l. and Daniel is to have five times as much as Andrew, and 20 l. more; how must this 273 l. be divided amongst them, so that every one may have his true share?

l.
Suppose - - - 20 A
Then must $20 \times 2 + 30 = 70$ B
Likewise $20 \times 3 - 52 = 8$ C
And $20 \times 5 + 20 = 120$ D

218

Again, $30 + 20 - 52 = 2$
Also $273 + 2 = 275$
Likew. $218 + 2 = 220$

	$220 : 20 :: 275 : 25$,	Andrew's	} share.
Then will	$25 \times 2 + 30 = 80$,	Bennet's	
Likewise	$25 \times 3 - 52 = 23$,	Christopher's	
And	$25 \times 5 + 20 = 145$,	Daniel's	

£ 273

6. Admit three merchants build a ship, which cost 1360 l. A pays a certain part unknown; B paid $2\frac{1}{2}$ as much, wanting 15.5 l. and C paid as much as both A and B, and 75.25 l. over; how much did each man pay?

1.

Suppose A paid 100

B $100 \times 2.5 - 15.5 = 234.5$	$75.25 - 31 = 44.25$
C $334.5 + 75.25 = 409.75$	$1360 - 44.25 = 1315.75$

$744.25 - 44.25 = 700$

	l.	s.	d.	
∴ $700 : 100 :: 1315.75 : 187.964286 =$	187	19	$3\frac{1}{2}$,	A } paid.
Then				
$187.964286 \times 2.5 - 15.5 = 454.410713 =$	454	8	$2\frac{1}{2}$,	B
And $642.375 + 75.25 = 717.625 =$	717	12	6,	C

1360.

7. As I walk'd forth to take the air,
The heavens and nature smiling were;
A grave old shepherd there I 'spy'd,
Close by a crystal fountain's side.
Unto this shepherd I did say,
How many sheep have you I pray?
But he reply'd, add to one half of these,
One fourth, $\frac{1}{5}$, $\frac{1}{8}$, and, if you please,
One tenth, $\frac{1}{20}$, and $\frac{1}{40}$ too;
These being made one sum by you,
Exactly to my age will be,
In this proportion, as 15 to 3.
What is your age, good Sir? said I.
To which the shepherd made reply;
One-half, one-fourth, one-fifth do take,
One-tenth, one twentieth, they will make;
If added, five score and ten more,
And now my age, Sir, pray explore?

And

And now methinks his age I'd know,
Which I must beg of you to show;
Likewise the number of the sheep,
Which this crabb'd shepherd there did keep?

Suppose 20 = shepherd's age.
Then $\frac{1}{2} = 10$, $\frac{1}{4} = 5$, $\frac{1}{5} = 4$, $\frac{1}{10} = 2$, and $\frac{1}{20} = 1$.
Also $10 + 5 + 4 + 2 + 1 = 22$.
As $22 : 20 :: 110 : 100$ = shepherd's age.
Again, suppose 40 = number of sheep.
Then will $\frac{1}{2} = 20$, $\frac{1}{4} = 10$, $\frac{1}{5} = 8$, $\frac{1}{8} = 5$, $\frac{1}{10} = 4$, $\frac{1}{20} = 2$, and $\frac{1}{40} = 1$.
Also $20 + 10 + 8 + 5 + 4 + 2 + 1 = 50$.
And $3 : 15 :: 100 : 500$, per question.
$\because$ $50 : 40 :: 500 : 400$, the number of sheep. Q. E. F.

SECT. II.

DOUBLE POSITION.

IN double position two suppositions are used; and if we miss in both, observe the nature of the errors whether they be greater or less, and accordingly mark them with the signs + or —; then,

RULE.

As the difference of the errors, if alike, or their sum, if unlike : is to the difference of the suppositions :: so is either of the errors : to a fourth number; which added to, or subtracted from, its proper supposition, gives the number sought.

1. A young gentleman walking in a garden, and meeting with a bevy of young ladies, began thus to address them: Bless you all 10 fair ladies! Sir, replies one, you are mistaken, we are not 10; but if we were twice as many more as we are, we should be as many above 10, as we are now below: what was their number?

Suppose 4, then $\overline{4 \times 2} + 4 = 12$.
Now as 4 is 6 less than 10, and 12 but 2 above 10;
$\because$ $2 - 6 = -4$, the first error.

 Again,

Again, ſuppoſe 7, then $\overline{7 \times 2} + 7 = 21$.
As 7 is 3 leſs than 10, and 21 greater by 11;
$\because 11 - 3 = + 8$, the ſecond error.
Then $4 + 8 = 12$, ſum of the errors.
And $7 - 4 = 3$, difference of the ſuppoſitions.
Alſo $12 : 3 :: 4 : 1$. $\because 4 + 1 = 5$. Q. E. F.
Or $12 : 3 :: 8 : 2$; and $7 - 2 = 5$, the anſwer, as above.

2. A gentleman hath two horſes of good value, and a ſaddle worth 50 l. which ſet on the back of the firſt horſe, made his value double that of the ſecond; but if ſet on the back of the ſecond horſe, makes his worth triple that of the firſt horſe: I demand the value of each horſe?

Suppoſe the firſt horſe to be worth 24 l.

Then $24 + 50 = 74$; alſo $\frac{74}{2} = 37$ l. value of the ſecond.

And $37 + 50 = 87$; but $24 \times 3 = 72$, leſs than 87 by 15.
So that $- 15 =$ firſt error.
Again, ſuppoſe the firſt horſe to be worth 34 l.

Then $34 + 50 = 84$; alſo $\frac{84}{2} = 42$, value of the ſecond.

And $42 + 50 = 92$; alſo $34 \times 3 = 102$, more than 92 by 10.
Hence $+ 10 =$ ſecond error.
Then $10 + 15 = 25 =$ ſum of the errors.
And $34 - 24 = 10 =$ difference of the ſuppoſitions.
$25 : 10 :: 15 : 6$. $\because 24 + 6 = 30$. Q. E. F.
Or $25 : 10 :: 10 : 4$. $\because 34 - 4 = 30$, as above.

3. A lady bought tabby at four ſhillings a yard, and Perſian at two ſhillings; the whole number of yards ſhe bought were eight, and the whole price 20 ſhillings; how many yards had ſhe of each ſort?

	s.
Suppoſe four yards of tabby, at	16
Then muſt ſhe have four of Perſian at	8
Sum of their values	24

So that the firſt error is $+ 4$.

	s.
Again, ſuppoſe ſhe had three yards of tabby, value	12
Then muſt ſhe have five of Perſian, value - -	10

The ſum of their values $= 22$ s.

So

So that the second error is $+ 2$.
Then $4 - 3 = 1$, difference of the suppositions.
Also $4 - 2 = 2$, difference of the errors.
As $2 : 1 :: 4 : 2$. $\therefore 4 - 2 = 2$ yards of tabby.
And $8 - 2 = 6$ yards of Persian.

	s.	s.
For two yards of tabby at	$4 =$	8
And six yards of Persian, at	$2 =$	12
	Sum	20, as was required.

4. A and B having a certain number of crowns, says B to A, give me one of your crowns, and I shall have as many as you; but says A to B, give me one of your crowns, and I shall have twice as many as you; how many had each?

Suppose A to have 5,
And B - - - - - 3.
Then $3 - 1 = 2$; and $5 + 1 = 6$.
The first error being $- 2$.
Again, suppose A to have 9.
And B - - - - - - - - 7.
Then $7 - 1 = 6$; and $9 + 1 = 10$.
The last error being $+ 2$.
Then $2 + 2 = 4$, sum of the errors.
And $9 - 5 = 4$, difference of the suppositions.
As $4 : 4 :: 5 : 5$. $\therefore 5 + 2 = 7$.
Also $4 : 4 :: 9 : 9$. $\therefore 9 - 2 = 7$.
$\frac{7+1}{2} = 4$; and $4 + 1 = 5$, B's crowns.

For $5 + 1 = 7 - 1$; and $7 + 1 = \overline{5 - 1} \times 2$, as was required.

5. There is a certain fish whose head is nine inches long, the tail is as long as the head, and half the body, and the body is as long as both the head and tail; I demand the whole length of the said fish?

Suppose the body be 20 inches.
Then $\frac{20}{2} + 9 = 19$, tail.
Also $19 + 9 = 28 - 20 = 8$.
So that the first error is $- 8$.

Again,

Again, ſuppoſe the body 24 inches.

Then $\frac{24}{2} + 9 = 21$, tail.

Alſo $21 + 9 = 30$; and $30 - 24 = 6$.

So that the ſecond error is — 6.

Then $8 - 6 = 2$, difference of the errors.

And $24 - 20 = 4$, difference of the ſuppoſitions,

$$\left.\begin{array}{l}\text{Alſo } 2 : 4 :: 8 : 16. \because 16 + 20 = 36 \\ \text{Or } 2 : 4 :: 5 : 12. \because 12 + 24 = 36\end{array}\right\}\text{body.}$$

Likewiſe $\frac{36}{2} + 9 = 27$, tail.

And $36 + 27 + 9 = 72$ inches. Q. E. F.

6. When firſt the marriage knot was ty'd,
Betwixt my wife and me,
My age did her's as far exceed,
As three times three doth three:
But after ten and half ten years
We man and wife had been,
Her age came up as near to mine,
As eight is to ſixteen:
Now tell me (you who can) I pray,
What were our ages on the wedding-day?

Suppoſe the wife's age 12 years.

Then muſt the huſband's be 36.

$$\text{And 15 years after}\left\{\begin{array}{l}\text{wife} \;\; - \;\; 27. \\ \text{huſband } 51.\end{array}\right.$$

Twice her age greater than his by 3.

$\because$ The firſt error is + 3.

Again, ſuppoſe the wife's age 18 years.

Then muſt the huſband's be 54.

$$\text{Alſo 15 years after}\left\{\begin{array}{l}\text{wife's} \;\; - \;\; 33. \\ \text{huſband's } 69.\end{array}\right.$$

And the ſecond error — 3.

Proceeding, $3 + 3 = 6$, ſum of the errors.

And $18 - 12 = 6$, difference of the ſuppoſitions.

$$\left.\begin{array}{l}\because 6 : 6 :: 12 : 12; \text{ alſo } 12 + 3 = 15 \\ \text{Or } 6 : 6 :: 18 : 18; \text{ and } 18 - 3 = 15\end{array}\right\}\text{the wife's age}$$

7. A man that was idle, and minded to ſpend
Both money and time, went to drink with his friend;
He ſaid to his hoſt, if you'll now to me lend
As much coin as I have, then my ſixpence I'll ſpend.

His

His hoſt lent the money, his ſixpence he ſpent,
And, having ſo done, to another houſe went,
Where the ſame he requeſted, and the ſame ſum he ſpent.
He went to a third houſe, where, Landlord, cries he,
Lend me as much money as I've left here you ſee;
Which having receiv'd, his ſixpence he ſpent,
So, all being gone, home the fuddle-cap went,
To caſt up his reck'nings; but his head aching ſore,
He begs you to do't, and he'll do ſo no more;
What had he at firſt, and how much on ſcore?

Suppoſe he had 8 d.
Then $8 + 8 = 16$
Alſo $16 - 6 = 10$
$10 + 10 = 20$
$20 - 6 = 14$
$14 + 14 = 28$
And $28 - 6 = 22$, which ſhould be nothing.
$\therefore$ The firſt error is 22.
Again, ſuppoſe he had 7 d.
Then $7 + 7 = 14$
$14 - 6 = 8$
$8 + 8 = 16$
$16 - 6 = 10$
$10 + 10 = 20$
And $20 - 6 = 14$
$\therefore$ The ſecond error is 14.
Then $22 - 14 = 8$, difference of the errors.
And $8 - 7 = 1$, difference of the ſuppoſitions.
As $8 : 1 :: 22 : 2\frac{3}{4}$. $\therefore$ $8 - 2\frac{3}{4} = 5\frac{1}{4}$ d.
Or $8 : 1 :: 14 : 1\frac{3}{4}$. $\therefore$ $7 - 1\frac{3}{4} = 5\frac{1}{4}$ d. Q. E. F.

RULE II.

Proceed as directed in the firſt rule, till you have found the errors and their ſigns; then

Multiply alternately the firſt ſuppoſition by the ſecond error, and the ſecond ſuppoſition by the firſt error; and divide the ſum of the products by the ſum of the errors, when the errors are of different kinds; or the difference of the products by the difference of the errors, when the errors are of the ſame kind, and the quotient is the number ſought.

8. A perſon finding ſeveral beggars at his door, gave each of them three pence a-piece, and had five pence remaining: he

he would have given them four-pence a-piece, but he wanted ſeven pence to do it; how many beggars were there?

Suppoſe 15 beggars.

15	15
3	4
45	60
+ 5	− 7
50	53

Then $53 - 50 = +3$, the firſt error.

Again, ſuppoſe 13 beggars.

13	13
3	4
39	52
+ 5	− 7
44	45

Alſo $45 - 44 = +1$, the ſecond error.

Then $13 \times 3 = 39$

Alſo $15 \times 1 = 15$

And $3 - 1 = 2)24(12$ beggars. Q. E. F.

9. A labourer agreed to thraſh 60 buſhels of corn, part of it wheat, and part oats, at the rate of 2 d. per buſhel for the wheat, and $1\frac{1}{2}$ d. for the oats; at laſt he received 8 s. for his labour; how much of each did he thraſh?

	d.
Firſt, I ſuppoſe 30 buſhels of wheat, price - -	60
Then muſt there alſo be 30 buſhels of oats - -	45
	105
Which ſhould be but - - - - - - - - - -	96
Therefore the firſt error is - - - - - - - - -	+ 9
	d.
Again, I ſuppoſe 18 buſhels of wheat, price - -	36
Then alſo muſt there be 42 buſhels of oats - -	63
	99
	− 96
Then will the ſecond error be - - - - - - -	+ 3

Alſo

Alſo $18 \times 9 = 162$
And $30 \times 3 = 90$

$\because 9 - 3 = 6) 72 (12$ buſhels of wheat at 2 d. 2 s.
Then will there be 48 buſhels of oats, at $1\frac{1}{2}$ d. is 6 s.

Sum 60 buſhels - - - - - - - 8 s.

10. Two merchants, A and B, lay out an equal ſum of money in trade; A gains 126 l. and B loſes 87 l. and A's money is now double to B's; what did each lay out?

Suppoſe each lays out	220	220
	126	87
	346	133
	266	× 2
Firſt error	+ 80	266

Again, ſuppoſe	350	350
	126	87
	476	263

$263 \times 2 = 526 - 476 = -50$, the ſecond error.

$$\left.\begin{array}{l} 350 \times 80 = 28000 \\ 220 \times 50 = 11000 \end{array}\right\} 39000.$$

$\because 80 + 50 = 130) 39000 (300$l. Q. E. F.

The following rules and examples I had from the ingenious Mr Emerſon's Arithmetic, page 146, &c.

RULE III.

"Proceed as directed in the firſt rule, till you have found the errors and their ſigns; then,

I. Multiply the difference of the ſuppoſed numbers by the leaſt error, and divide the product by the difference of the errors, if they are alike; or by the ſum, if unlike: the quotient is the correction of the number belonging to the leaſt error.

II. Obſerve whether this be the leſſer or greater number, as alſo whether the errors have like or unlike ſigns.

III. If it is the leſs number, and like ſigns, ſubtract the correction; if unlike ſigns, add it.

IV. If the greater number, and like ſigns, add the correction; if unlike ſigns, ſubtract it: ſo you'll have the true number required.

11. A certain man being aſked what was the age of his four ſons, anſwered, that his eldeſt was four years older than the ſecond, and the ſecond five years older than the third, and the third ſix years older than the fourth, which was half the age of the eldeſt! how old was each?

$$\text{Suppoſe}\left\{\begin{array}{l}16 \text{ eldeſt.}\\ 12 \text{ ſecond.}\\ 7 \text{ third.}\\ 1 \text{ youngeſt.}\end{array}\right. \qquad \text{Again,}\left\{\begin{array}{l}24 \text{ eldeſt.}\\ 20 \text{ ſecond.}\\ 15 \text{ third.}\\ 9 \text{ youngeſt.}\end{array}\right.$$

$\frac{1}{2}$ eldeſt $= 8 - 1 = -7$, firſt error.
$\frac{1}{2}$ eldeſt $= 12 - 9 = -3$, ſecond error.
Then $24 - 16 = 8$, difference of ſuppoſitions.
Alſo $8 \times 3 = 24$.
And $7 - 3 = 4$, difference of the errors.
$4)24(6$, the correction.
$\because 24 + 6 = 30$, the age of the eldeſt.
Alſo 26, ſecond.
21, third.
And 15, youngeſt. Q. E. F.

12. There is a crown weighing 60 lb. which is made of gold, braſs, tin, and iron; the weight of the gold and braſs together is 40 lb. of the gold and tin 45 lb. of the gold and iron 36 lb. Quere, how much gold was in it?

Suppoſe	32 lb. of gold	28
	8 braſs	12
	13 tin	17
	4 iron	8
	57	65

Then $60 - 57 = -3$, the firſt error.
And $65 - 60 = +5$, the ſecond error.
Alſo $5 + 3 = 8$, ſum of the errors.
Likewiſe $32 - 28 = 4$, difference of the ſuppoſitions.
Again. $4 \times 3 = 12$; and $\frac{12}{8} = 1\frac{1}{2}$, the correction.
$\because 32 - 1\frac{1}{2} = 30\frac{1}{2}$, the quantity of gold. Q. E. F.

13. Three

13. Three companies of soldiers passing by a shepherd, the first takes half his flock, and half a sheep; the second takes half the remainder, and half a sheep; the third takes half the remainder, and half a sheep; after which the shepherd had 20 sheep remaining; how many had he at first?

Suppose	60	- - - - - - -	80
First took	30.5	- - - - - - -	40.5
	29.5		39.5
The second took	15.25		20.25
	14.25	- - - - - -	19.25
The third took	7.625	- - - - - -	10.125
	6.625	- - - - - -	9.125

Then $20 - 6.625 = 13.375$, first } error.
Also $20 - 9.125 = 10.875 =$ second. }

Again, $80 - 60 = 20$, difference of the suppositions.

Also $13.375 - 10.875 = 2.5$, difference of the errors.

$20 \times 10.875 = 217.5$.

$2.5)217.5(87$, correction.

$\therefore$ $80 + 87 = 167$ sheep. Q. E. F.

SCHOLIUM.

By supposing one of the numbers 0, and the other 1, the work is sometimes shortened.

14. A factor delivers six French crowns, and four dollars, for 2 l. 13 s. 6 d. and at another time four French crowns, and six dollars, for 2 l. 9 s. 10d. what was the value of each?

Suppose $0 =$ value of a French crown.

Then will 4 dollars $= 53.5$ s.

Also $4)53\frac{1}{2}(13\frac{3}{8}$.

And 6 crowns $+$ 6 dollars $= 80\frac{1}{4}$.

Then $80\frac{1}{4} - 49\frac{11}{12} = + 30\frac{5}{12}$, first error.

Again, suppose 1 s. $=$ value of a French crown.

Then 6 crowns and 4 dollars $= 53\frac{1}{2}$.

Also $53\frac{1}{2} - 6 = 47\frac{1}{2} = 4$ dollars.

$4)47\frac{1}{2}(11\frac{7}{8}$, value of a dollar.

And 4 crowns $+$ 6 dollars $= 75\frac{1}{4}$.

Then $75\frac{1}{4} - 49\frac{11}{12} = + 25\frac{5}{12}$, second error.

Also $30\frac{5}{12} - 25\frac{5}{12} = 5$, difference of the errors.

$\therefore$ $5)30\frac{5}{12}(6\frac{1}{12} = 6$s. 1d. $=$ value of a crown. Q. E. F.

And

And 6 crowns = 36s. 6d.
53 s. 6 d. — 36 s. 6 d. = 17 s. value of 4 dollars.
∴ 4)17 4s. 3d. = value of a dollar. Q. E. F.

In this rule it is generally presumed, that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number. When this does not happen, the rule of false does not give the exact answer, except the two supposed numbers be taken very near the true one.

The errors are the difference between the true result, and each of the false results; so that if the errors are unlike, the true number lies between the supposed numbers; but if alike, the true number lies without both of the supposed ones.

A great many questions may be resolved by this rule, which cannot be resolved by any other rule in arithmetic; but there are many questions where it cannot be certainly known, whether they can be resolved by it or not, till they be tried."

CHAPTER IV.

CONCERNING DIVISORS.

IT being often necessary, in arithmetical calculations, to find such multipliers, or numbers, which may be divided by any number of given divisors, without any remainder, or to leave any assigned remainder, or remainders; by which means many pleasant questions, not reducible to any of the foregoing rules, may be solved.

First find the least number that can be divided by any number of given divisors without a remainder.

RULE.

Multiply all the prime numbers, and the roots of such as are square or cube numbers, continually; the product will be the least number required.

1. Shew me how to find what's the least number
 That you can divide without a remainder,

By

By giving divifors, as the digits nine,
For a true canon I'd give a pint o' wine ?

Ladies' Diary, 1719.

Divifors 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9.

But as $\sqrt{4} = 2$, that 6 may be cancelled, being compofed of 2×3; $\sqrt[3]{8} = 2$; and $\sqrt{9} = 3$.

$\because 1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 7 \times 2 \times 3 = 2520.$

Q. E. F.

2. What particular leaft whole number is that, which being divided by 2, 3, 4, 5, 6, 7, 8, 9, fhall leave a remainder of 1, 2, 3, 4, 5, 6, 7, 8, refpectively ?

It is plain by the queftion above, that 2520 is the leaft number that can be divided by nine digits, without a remainder. $\because 2520 - 1 = 2519$, the number required.

3. A country girl to town did go,
Some walnuts for to fell,
A gentleman fhe chanc'd to meet,
And thus it her befel :
My pretty maid, fays he to her,
What number have you here ?
I can't tell, fir, fays fhe to him,
But this I'll make appear ;
I told them o'er ere I came out,
By fix's, fives, four's, three's, two's,
And, ev'ry time I number'd them,
One remain'd overplus ;
I told them o'er by feven's at laft,
And there were no remains ;
If you can find the number out,
Pray take it for your pains.

Firft, the leaft number that can be divided by 1, 2, 3, 4, 5, 6, without a remainder, viz. $1 \times 2 \times 3 \times 2 \times 5 = 60$.

Then $60 + 1 = 61$, will leave 1, when divided by each number; but $7)61(8$, and 5 remains.

Alfo $\left.\begin{array}{l} \overline{60 \times 2} + 1 = 121 \\ \overline{60 \times 3} + 1 = 181 \\ \overline{60 \times 4} + 1 = 241 \end{array}\right\}$ none of which are divifible by 7 without a remainder.

But $60 \times 5 + 1 = 301$, is the leaft number which admits of the conditions of the queftion.

Then

Then to find the next least number, which admits of the same conditions, viz.

$$\left.\begin{array}{l} 60 \times 6 + 1 = 361 \\ 60 \times 7 + 1 = 421 \\ 60 \times 8 + 1 = 481 \\ 60 \times 9 + 1 = 541 \\ 60 \times 10 + 1 = 601 \\ 60 \times 11 + 1 = 661 \end{array}\right\} \text{none of which are divisible by 7, without a remainder.}$$

But $60 \times 12 + 1 = 721$; is the next number admitting the conditions aforesaid.

Also $721 - 301 = 420$, the common difference of all numbers answering the same conditions.

∴ 301, 721, 1141, 1561, 1981, 2401, 2821, &c. *ad infinitum*, will answer the conditions of this question.

4. To find the least number of guineas, which being divided by 6, 5, 4, 3, and 2 respectively, shall leave 5, 4, 3, 2 and 1 respectively remaining? *L. Diary*, 1748.

As by the foregoing question, $1 \times 2 \times 3 \times 2 \times 5 = 60$, the least number, which divided by 1, 2, 3, 4, 5 and 6, leaves no remainder.

∴ $60 - 1 = 59$. Q. E. F. as may be easily proved.

5. Required the least number, that being divided by 9, shall leave for a remainder 6; if divided by 8, the remainder will be 5; if divided by 7, the remainder will be 4; and so on, each time leaving for a remainder three less than the divisor, till, divided by 3, the remainder will be nothing?

As 2520 is the least number which can be divided by the nine digits, or by the seven highest of them without a remainder.

∴ $2520 - 3 = 2517$. Q. E. F. as may be easily proved.

6. Required the three least numbers, which divided by 20 shall leave 19 for a remainder; but, if divided by 19, shall leave 18, if divided by 18, shall leave 17; and so on (always leaving one less than the divisor) to unity?

Gentlemen's Diary. 1747.

First, 1, 2. 3, 5, 7, 11, 13, 17, and 19, are prime numbers.

Also $\sqrt{4} = 2$, $\sqrt[3]{8} = 2$, $\sqrt{9} = 3$, and $\sqrt[4]{16} = 2$;

And all the rest are composite numbers.

∴ $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 \times 11 \times 13 \times 2 \times 17 \times 19 = 232792560$, the least number that can be divided by the

the given divisors without a remainder; also $232792560 \times 2 = 465585120$; and $232792560 \times 3 = 698377680$, being divided by the given divisors, will leave no remainder.]

$$\left.\begin{array}{r} \because 232792560 - 1 = 232792559 \\ 465585120 - 1 = 465585119 \\ \text{and } 698377680 - 1 = 698377679 \end{array}\right\} \text{the three least numbers. Q. E. F.}$$

Agreeing with the algebraic process, by Mr. Robinson, in the Gentleman's Diary, 1748,

7. A jolly fine girl did ride on the way,
With plums in a basket, it being market-day;
She rode on but softly, the weather being hot,
So I ask'd her what number of plums she had got:
She said, the just number she did not well know,
But I'll tell you which way will the true number show.
If you count them by two's, there will then remain one;
If you count them by three's, there rests two when you've done;
If you count them by four's, the remainder is three;
If by five's, then just four the remainder will be;
If by six at a time you do count them again,
You'll find, when you've done, that just five will remain;
But if seven at a time, you do count them o'er all,
The remainder will be then just nothing at all:
Now what is the number, and to what do they come,
At fourteen a penny, I'd fain know the sum?

By the third question it appears, 60 is the least number that can be divided by the first six digits, without a remainder, $60 - 1 = 59$, the least number that can be divided by the said six digits, leaving each division one less than the divisor; but 59, divided by 7, leaves a remainder.

Then $\overline{60 \times 2} - 1 = 119$, the least number that answers the conditions of this question.

Also 420 is found, by question 3, to be the common difference of numbers, answering the same conditions. ∵ 119, 539, 959, 1379, &c. will admit of the same conditions.

Q. E. F.

$$\left.\begin{array}{l} \because 14)119(8\tfrac{1}{2}\text{ d.} \\ \text{Or } 14)539(38\tfrac{1}{2} = 3\text{ s. } 2\tfrac{1}{2}\text{ d.} \end{array}\right\} \text{their value.}$$

8. Once old mother Gripe to a market went,
Some butter to sell it was her intent;
At a certain rate per pound she it sold,
What she got for it all, as I have been told,

Were

Were two ſhillings and two pence farthing juſt :
Now how much butter had the old toaſt,
And how ſhe might ſell her butter per pound,
Is what is required to be found?
Of various anſwers this queſtion will admit,
Find them all out, and they will whet thy wit.

Firſt, 2s. $2\frac{1}{4}$d. = 105 farthings, which is compoſed of theſe odd numbers, viz. $1 \times 3 \times 5 \times 7 = 105$.

		lb.		d.
	So that	105 lb.	at	$\frac{1}{4}$
3	105	35	- - -	$\frac{3}{4}$
5	105	21	- - -	$1\frac{1}{4}$
7	105	15	- - -	$1\frac{3}{4}$
15	105	7	- - -	$3\frac{3}{4}$
21	105	5	- - -	$5\frac{1}{4}$
35	105	3	- - -	$8\frac{3}{4}$
	Or	1lb.	at 2 s.	$2\frac{1}{4}$

per pound, all anſwer the conditions of this queſtion. Q. E. F.

This queſtion, and the foregoing, was taken from Tapper's Delight for the Ingenious, for July, Auguſt and September, 1711; the ſolutions my own.

CHAPTER V.

PROGRESSION, VARIATION, COMBINATION, &c.

SECT. I.

ARITHMETICAL PROGRESSION.

ARithmetical Progreſſion is a rank or ſeries of numbers increaſing or decreaſing by a common difference, or by a continual addition or ſubtraction of ſome equal numbers.

As $\left\{\begin{matrix} 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 \\ 9 . 8 . 7 . 6 . 5 . 4 . 3 . 2 . 1 \end{matrix}\right\}$ common diff. 1.

Or 1 . 3 . 5 . 7 . 9 . 11 . 13, common difference 2.
Alſo 42 . 35 . 28 . 21 . 14 . 7. common difference 7.

In arithmetical progreſſion are five things; any three of which being given, the other two may be found, which admit of twenty different propoſitions.

1. The

1. The first term, commonly the least } extreme.
2. The last term, commonly the greatest }
3. The number of terms.
4. The common excess, or difference.
5. The aggregate, or sum of all the terms.

We shall only concern ourselves with some few of them, but let us premise, that,

1. If any three numbers are in arithmetical progression, the sum of the two extremes, viz. the first and last, will be equal to the double of the mean or middle number.

As in these, 3 . 8 . 13; viz. $3+13=8+8$
Or 1 . 7 . 13 - - - $1+13=7+7$
And 7 . 14 . 21 - - - $7+21=14+14$

2. If four numbers are in arithmetical progression, the sum of the two extremes will be equal to the sum of the two means.

As 1 . 3 . 5 . 7; viz. $1+7=3+5$
And 5 . 8 . 11 . 14 - - - $5+14=8+11$

3. Also if many numbers be in arithmetic progression, the sum of the two extremes will be equal to the sum of any two means that are equally distant from the extremes.

7 . 9 . 11 . 13 . 15 . 17;
viz. $7+17=9+15=11+13$.

Or if the numbers be odd:

1 . 3 . 5 . 7 . 9 . 11 . 13;
viz; $1+13=3+11=5+9=7+7$.

4. Every series of numbers in arithmetic progression is composed of the excess or common difference, so often repeated as there are terms in the progression, except the first.

As in these, 2 . 5 . 8 . 11 . 14 . 17, &c.

Here the common difference being 3.

Then will $2+3=5$. $5+3=8$. $8+3=11$. $11+3=14$. $14+3=17$, &c.

Hence may be observed, that the difference between the two extremes (2 and 17) is composed of the common difference, multiplied into the number of all the terms, except the first.

In

In the aforesaid progression, 2 . 5 . 8 . 11 . 14 . 17.
The number of terms without the first is 5 } multipl.
The common difference - 3 }

The difference of the two extremes 15

PROPOSITION I.

The two extremes, and the number of terms, being given, to find the sum of all the series,

RULE.

Multiply the sum of the two extremes into the number of terms, and divide the product by 2, the quotient will be the sum of all the series.

1. How many strokes do the clocks of Venice (which go on to 24 o'clock) strike in the compass of a natural day?

$1 + 24 = 25$, sum of the extremes.
24, number of terms.

100
50

2)600(300 strokes. Q. E. F.

2. The length of my garden is 94 feet; now if eggs be laid along the pavement a foot asunder, to be fetched up singly to a basket, removed one foot from the last, how much ground must he travel that does it?

$2 + 188 = 190$, sum of the exremes.
94, number of terms.

2)17860(8930 feet.
Feet in a mile 5280|8930(1 mile, 5 fur. 21 poles, $3\frac{1}{2}$ feet.
Q. E. F.

660|3650

16.5| 350

$3\frac{1}{2}$

P R O-

PROPOSITION II.

The firſt term, the common exceſs, and the number of terms being given, to find the ſum of all the ſeries.

RULE.

From the product of the number of terms in the common exceſs; ſubtract the common exceſs, and to the remainder add the double of the firſt term; half the product of that ſum multiplied by the number of terms, gives the ſum of all the ſeries.

3. A gentleman bargains with a bricklayer to ſink him a well twenty fathoms deep, upon theſe terms, viz. to pay him three ſhillings for the firſt fathom, five for the ſecond, ſeven for the third, &c. raiſing two ſhillings every fathom; what will be due to the bricklayer for compleating the ſame?

First, $20 \times 2 = 40$; alſo $40 - 2 = 38$.
Again, $38 + 6 = 44$; and $44 \times 20 = 880$.
$\because 880 \div 2 = 440$ ſhillings $= 22$ l. Q. E. F.

PROPOSITION III.

The firſt term, number of terms, and ſum of all the ſeries given, to find the common exceſs.

RULE.

Divide the double ſum of all the ſeries by the number of terms, and from the quotient ſubtract double the firſt term; divide the remainder by the number of terms leſſened by unity, the quotient will be the common exceſs.

4. A gentleman travelled 100 leagues in eight days, and every day travelled equally farther than the preceding day; it is known that the firſt day he travelled two leagues, how many leagues did he travel each of the other days?

$200 \div 8 = 25$; alſo $25 - 4 = 21$; and $8 - 1 = 7$.
7) 21 (3, the common difference ſought.
Then 3 added to 2, and every other term reſpectively,

$$\text{gives}\ \left.\begin{array}{l} 5 \text{ for the ſecond} \\ 8 \text{ - - - third} \\ 11 \text{ - - - fourth} \\ 14 \text{ - - - fifth} \\ 17 \text{ - - - ſixth} \\ 20 \text{ - - - ſeventh} \\ 23 \text{ - - - eighth} \end{array}\right\}\text{day's journey. Q. E. F.}$$

100 leagues.

PROPOSITION IV.

The two extremes, and number of terms being given, to find the common difference.

RULE.

The difference of the two extremes, divided by the number of terms, less unity, the quotient will be the common excess.

5. One had 12 children that differed alike in their ages, the youngest was nine years old, the elder $36\frac{1}{2}$; what was the difference of their ages, and the age of each?

Here $36.5 - 9 = 27.5$, difference of the extremes.
Also $12 - 1 = 11)\,27.5\,(2.5$, common excess.

Which add to the age of the youngest, and so on continually to the rest.

viz. youngest	9	years old.
11th	$11\frac{1}{2}$	
10th	14	
9th	$16\frac{1}{2}$	
8th	19	
7th	$21\frac{1}{2}$	
6th	24	
5th	$26\frac{1}{2}$	
4th	29	
3d	$31\frac{1}{2}$	
2d	34	
eldest	$36\frac{1}{2}$	

6. A debt is to be discharged at 11 several payments in arithmetic progression; the first payment to be 12 l. 10 s. and the last 63 l. what is the whole debt, and what must each payment be?

First, $12.5 + 63 = 75.5$, sum of the extremes.
11, number of terms.

2)830.5 (415.25 = 415 l. 5s. whole debt.
Then $63 - 12.5 = 50.5$, difference of the extremes.
$11 - 1 = 10)\,50.5\,(5.05 = 5$ l. 1 s. common difference.

Therefore

	l.	s.		
Therefore	12	10	= first	payment.
	17	11	= second	
	22	12	= third	
	27	13	= fourth	
	32	14	= fifth	
	37	15	= sixth	
	42	16	= seventh	
	47	17	= eighth	
	52	18	= ninth	
	57	19	= tenth	
	63	—	= eleventh	
	415	5,	whole debt, as before.	

7. A man is to travel from London to a certain place in ten days, and to go but two miles the first day, increasing every day's journey by an equal excess, so that the last day's journey may be 29 miles; what will each day's journey be, and how many miles is the place he goes to distant from London?

First, 29 — 2 = 27, difference of the extremes.

10 — 1 = 9) 27 (3, the common difference.

Which added to each day's journey,

gives	2	miles for the	first.
	5		second.
	8		third.
	11		fourth.
	14		fifth.
	17		sixth.
	20		seventh.
	23		eighth.
	26		ninth.
	29		tenth.
	155	miles from London.	

PROPOSITION V.

The two extremes, and the common excess given, to find the number of terms.

RULE.

Divide the difference of the two extremes by the common excess, the quotient plus unity is the number of terms.

 8. A

8. A man going a journey, his first day's travel was five miles, his last day's travel 35 miles; he increased his journey every day three miles; how many days did he travel?

First, $35 - 5 = 30$, difference of the extremes.
Then 3) 30 (10; and $10 + 1 = 11$ days journey. Q. E. F.

PROPOSITION VI.

The common excess, number of terms, and sum of all the series given, to find the first term.

RULE.

Divide the sum of all the series by the number of terms, and from the quotient subtract half the product of the common excess into the number of terms less unity, the remainder will be the first term.

9. A man is to receive 300 l. at 12 several payments, each payment to exceed the former by four pounds; he is willing to bestow the first payment on any one that can tell him what it is; what must the arithmetician have for his pains?

First, 12) 300 (25; also $12 - 1 = 11$.
Then $11 \times 4 = 44$; and 2) 44 (22.
$\therefore 25 - 22 = 3$, the artist's reward. Q. E. F.

10. Suppose it 100 leagues between London and Carlisle, two couriers set out from each place on the same road; that from London towards Carlisle travelling every day two leagues more than the day before; the other from Carlisle to set off one day after, travelling every day three leagues more than the preceding one; and that they meet exactly half way, the first at the end of five days, and the other at the end of four; how many leagues did each travel each day?

First, 5) 50 (10; also $5 - 1 = 4$.
Then $4 \times 2 = 8$; and 2) 8 (4.

$$\left.\begin{array}{l} \therefore 10 - 4 = 6, \text{ his first} \\ 6 + 2 = 8 \text{ - - second} \\ \quad\quad 10 \text{ - - third} \\ \quad\quad 12 \text{ - - fourth} \\ \quad\quad 14 \text{ - - fifth} \end{array}\right\} \text{day's journey.}$$

Sum 50

Again,

Again, 4) 50 (12.5; alfo $4 - 1 = 3$.
Then $3 \times 3 = 9$; and 2) 9 (4.5.

$$\therefore\ 12.5 - 4.5 = 8, \text{ his firft} \quad 8 + 3 = 11 \text{ - - fecond} \quad 14 \text{ - - third} \quad 17 \text{ - - fourth} \left.\right\} \text{day's journey.}$$

50 leagues.

PROPOSITION VII.

The laft term, number of terms, and common excefs given, to find the firft term.

RULE.

Multiply the common excefs into the number of terms lefs unity, the product fubtracted from the laft term leaves the firft.

11. A man in fix days went from London to Manchefter, every day's journey was greater than the preceding one by four miles, his laft day's journey was 40 miles, what was the firft?

Number of terms $6 - 1 = 5$
Common excefs 4

20

Then $40 - 20 = 20$, the firft day's journey. Q. E. F.

I fhall now add one propofition more, exclufive of the 20 abovementioned.

PROPOSITION VIII.

When one perfon or thing moves with an equal, and another the fame way by a progreffive motion, to find in what time the firft will be overtaken.

RULE.

Add the common excefs of the purfuer's day's journey to double the fpace gone each day by the purfued; from that fum fubtract double the fpace that the purfuer travelled the firft day, and divide the remainder by the com-

mon excefs, the quotient will give the number of days in which the purfued will be overtaken by the purfuer.

12. A noted highwayman having committed a robbery, not fufpecting a purfuit, fled northward at the rate of eight leagues a day; Jonathan Wyld, upon the fcent, follows him, in a progreffive motion, only three leagues the firft day, five the next, feven the third, and fo on, increafing every day two leagues; in how many days will the highwayman be overtaken?

First, $2+16=18$; alfo $18-6=12$.

$\therefore$ 2) 12 (6 days. Q. E. F.

For $6\times8=48$ leagues, the fpace travelled by the robber.

Then, by Prop. II. $6\times2=12$; alfo $12-2=10$; and $10+6=16$.

Alfo $16\times6=96$. $\therefore$ 2) 96 (48 leagues, when the thieftaker comes up with the highwayman.

13. Y. Z. made the following bet for 1000 guineas, to be decided the Monday, Tuefday, and Wednefday, in Whitfun-week, on Barnham Downs, between the hours of eight in the morning, and eight at night. The propofer has ten choice cricketers in full exercife, who on this occafion are to be diftinguifhed by the firft 10 letters of the alphabet. Thefe are to run and gather up, and carry fingly, 1000 eggs, laid in a right line, juft two yards afunder, putting them gently into a bafket placed juft a fathom behind the firft. They are to work one at a time, in the following order: A is to fetch up the firft 10 eggs, B the fecond, C the third, and fo forward to K, whofe turn it will be to fetch up the 100th egg. After which, A fets out again for the next 10, B takes the next, and fo forward alternately, till K fhall have carried up the 1000th egg, at 100 eggs per man. The fellows are to have 300 l. for their three days work, if they do it, and it is to be diftributed in proportion to the ground each man fhall in his courfe have gone over. It is required, firft, How many miles each perfon will have run? Secondly, What part of the 300 l. will come to his fhare? Thirdly, Whether if the men had been pofted at proper places, they had not better have run from London to York twice, and back in the time, taking the meafure 180 miles?

Firft, for A's race, 4, firft term, 40, laft term, their fum 44.

N° of terms $10\times44=440$.

$\therefore$ 2)

$\therefore$ 2) 440 (220, A's first race.

Then $901 \times 4 = 3604$, first term of the last race.

Also $910 \times 4 = 3640$, last term.

$3604 + 3640 = 7244$, their sum.

$$\therefore \frac{7244 \times 10}{2} = 36220, \text{ A's last race.}$$

Then to find his whole part in this expedition, put 220, first term, 362220 last term, sum 36440.

$$\therefore \frac{36440 \times 10}{2} = 182200, \text{ sum of A's races.}$$

For B's part in the expedition:

First, $11 \times 4 = 44$; last term $20 \times 4 = 80$;

Also $44 + 80 = 124$; which $\times\ 10 = 1240$.

$\therefore$ 2) 1240 (620 yards, B's least race.

Last race $911 \times 4 = 3644$ } their sum 7324.
Also $920 \times 4 = 3680$ }

$$\therefore \frac{73240 \times 10}{2} = 36620, \text{ yards, B's greatest race.}$$

Then $620 + 36620 = 37240$, sum of the extremes.

$$\therefore \frac{37240 \times 10}{2} = 186200, \text{ sum of B's races.}$$

Again, $186200 - 182200 = 4000$ yards, common difference; which added continually to each of their shares, shews that

	yards.		miles.	furl.	poles.
A in all ran	182200	=	103	4	40
B - -	186200	=	105	6	80
C - -	190200	=	108	-	120
D - -	194200	=	110	2	160
E - -	198200	=	112	4	200
F - -	202200	=	114	7	20
G - -	206200	=	117	1	60
H - -	210200	=	119	3	100
I - -	214200	=	121	5	140
K - -	218200	=	123	7	180
	2002000	=	1137	4	— Q. E. F.

As $10010 : 911 :: 300 : 27$ 6 $-\frac{648}{1001}$, A's } part.
Also $10010 : 931 :: 300 : 27$ 18 $-\frac{504}{1001}$, B's }

Then 27 l. 18 s. $-\frac{504}{1001}$d. $-$ 27 l. 6s. $-\frac{648}{1001}$ d. $=$ 11 s. $11\frac{857}{1001}$ d. common difference; which added to each man's share,

 gives

		l.	s.	d.
gives	A	27	6	$-\frac{648}{1001}$
	B	27	18	$-\frac{504}{1001}$
	C	28	10	$-\frac{360}{1001}$
	D	29	2	$-\frac{216}{1001}$
	E	29	14	$-\frac{72}{1001}$
	F	30	5	$11\frac{929}{1001}$
	G	30	17	$11\frac{785}{1001}$
	H	31	9	$11\frac{641}{1001}$
	I	32	1	$11\frac{497}{1001}$
	K	32	13	$11\frac{353}{1001}$

Q. E. F.

From London to York, suppose 180 miles.
Miles in the whole expedition $1137\frac{1}{2}$
And $180 \times 4 = 720$

Short of the present undertaking $417\frac{1}{2}$ miles.

The following question I was favoured with by my esteemed friend, Major Watson, chief engineer in Lord Clive's expedition to the Indies.

14. Suppose a man to have a calf, which at the end of three years begins to breed (and afterwards) a female calf every year; and that each calf begins to breed in like manner at the end of three years, bringing forth a cow calf every year, and that these last breed in the same mannner, &c. &c. to determine the owners whole stock at the end of 20 years?

By nature of this question, the number of cows that calved at the end of these years will be as follows;

$$\text{viz.}\begin{cases} 3\cdot\ 4\cdot\ 5\cdot\ 6\cdot\ 7\cdot\ 8\cdot 9\cdot 10\cdot 11\cdot 12\cdot 13\cdot 14 \\ 1\cdot\ 1\cdot\ 1\cdot\ 2\cdot\ 3\cdot\ 4\cdot 6\cdot\ 9\cdot 13\cdot 19\cdot 28\cdot 41 \\ 15\cdot 16\cdot 17\cdot 18\cdot 19\cdot 20 \text{ years.} \\ 60\cdot 88\cdot 129\cdot 189\cdot 277, \text{ and } 406 \end{cases}$$

respectively, which are found by adding the last to the last but two.

Then of the whole series $1+1+1+2+3+4+6+9+13+19\ldots+D+E+F+G$ be represented by S, when D, E, F and G denote the four last terms, we shall then have $1+1+1+2+3+4+6+9+13+19+28\ldots+D=S-E-F-G$, which being taken from the above, we have $1+1+1+1+2+3+4+6+9+13\ldots+F=E+F+G$; and by adding

adding G to both sides of the equation we then get $1 + 1 + 1 + 1 + 2 + 3 + 4 + 6 + 9 + 13 \ldots + F + G = E + F + 2G$; which consequently will be the man's stock of cows and calves at the end of any number of years, which, in this case, $E = 189 + F = 277 + 2G = 812$, will be 1278. Q. E. F.

From the above solution it appears, that the whole stock of cows and calves, at the end of any number of years, will be equal to the number of cows that would calve at the end of three years after the given time.

SECT. II.

GEOMETRICAL PROGRESSION.

GEOMETRICAL PROGRESSION is when any rank or series of numbers increase by one common multiplier, or decrease by one common divisor;

As 2 . 4 . 8 . 16 . 32 . 64; here 2 is the common multiplier.

And 1215 . 405 . 135 . 45 . 15 . 5; here 3 is the common divisor.

Note, The common multiplier, or divisor, is called the ratio.

Here note, that if three numbers are in geometrical progression, the product of the two extremes will be equal to the square of the mean or middle term, as in these, 2 . 4 . 8.

$$\text{Here } 2 \times 8 = 4 \times 4, \text{ each being} = 16.$$

Also if four numbers are in geometrical progression, the product of the two means will be equal to the product of the two extremes, as in these, 135 . 45 . 15 . 5.

$$\text{Here } 135 \times 5 = 45 \times 15, \text{ each being } 675.$$

Hence, if ever so many numbers are in geometrical progression, the product of the two extremes are equal to the product of any two means that are equally distant from the extremes.

As in these, 3 . 9 . 27 . 81 . 243 . 729.

$$\text{Here } 3 \times 729 = 9 \times 243 = 27 \times 81 = 2187.$$

Or

Or if the number of terms be odd, as in theſe:

$$3 \cdot 9 \cdot 27 \cdot 81 \cdot 243, \&c.$$
$$3 \times 243 = 9 \times 81 = 27 \times 27 = 729.$$

In any geometrical progreſſion, the ſame things are to be taken notice of, as in arithmetical progreſſion;

viz. Firſt, The firſt term, commonly the leaſt.

Secondly, The laſt term, commonly the greateſt.

Thirdly, The number of terms.

Fourthly, The ratio, or common multiplier, or diviſor.

Fifthly, the ſum of all the ſeries.

Any three of theſe being known, the reſt may be found.

If to any ſeries of numbers in geometrical progreſſion not proceeding from unity, there be aſſigned a ſeries of numbers in arithmetical progreſſion, beginning with an unit or 1, whoſe common difference is 1, called indices, or exponents;

$$\text{thus} \begin{cases} 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7, \text{ indices.} \\ 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32 \cdot 64 \cdot 128, \&c. \end{cases}$$

then will the addition or ſubtraction of thoſe indices (or numbers in arithmetic progreſſion) directly correſpond with the product or quotient of their reſpective terms or ſeries in geometric progreſſion;

$$\text{that is,} \begin{cases} \text{as } 3 + 4 = 7, \\ \text{ſo } 8 \times 16 = 128, \text{ the ſeventh term in } \div\!\!\div. \end{cases}$$

$$\text{Again,} \begin{cases} \text{alſo } 7 + 7 = 14, \\ \text{ſo } 128 \times 128 = 16384, \text{ the 14th term in } \div\!\!\div. \end{cases}$$

$$\text{Or,} \begin{cases} \text{as } 7 - 3 = 4, \\ \text{ſo } 128 \div 8 = 16, \end{cases}$$

$$\text{Or,} \begin{cases} \text{as } 6 - 1 = 5, \\ \text{ſo } 64 \div 2 = 32, \&c. \end{cases}$$

But if the ſeries begin with unity, the indices muſt begin with a cypher.

$$\text{Thus,} \begin{cases} 0 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7, \&c. \\ 1 \cdot 2 \cdot 4 \cdot 8 \cdot 16 \cdot 32 \cdot 64 \cdot 128. \end{cases}$$

Now by theſe indices, and a few leading terms, the laſt term, or any diſtant one may be ſpeedily found.

PROPOSITION I.

The firſt term being unity, the ratio and number of terms being known, to find the laſt or any remote term.

RULE.

RULE.

Find a few of the leading terms, over which place their indices, as before directed; then multiply the last found term by itself, which will produce a term double thereto; and so proceed, till you either arrive at the term sought, or one that falls a little short of it; if so, multiply the term last found by that term, answering the difference of the indices of the last found term, and that sought; which last product will be the term required.

1. A country gentleman going to a fair, meets with a crafty youth who had a drove of 25 very good oxen; upon asking their price, was answered, he should have them for 16 pounds each, one with another; the gentleman offers him 15 pounds each, and take all: the young spark tells him it would not be taken, but if he would give him what the 20th ox would come to by beginning at the first with a single farthing, and doubling only to the 20th, he should have them all; what did they come to a head?

First, $\begin{cases} 0 \,.\, 1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5, \text{ indices.} \\ 1 \,.\, 2 \,.\, 4 \,.\, 8 \,.\, 16 \,.\, 32, \text{ terms.} \end{cases}$

Then, $\begin{cases} 5 + 5 = 10, \\ 32 \times 32 = 1024; \end{cases}$ also, $\begin{cases} 4 + 5 = 9, \\ 16 \times 32 = 512. \end{cases}$

$10 + 9 = 19.$

$1024 \times 512 = 524288$, which is the 20th term, as the indices are less than the term by one.

And 524288 farth. = 546 l. 2 s. 8 d. price of the whole.

$\therefore$ 25) 546 l. 2 s. 8 d. (21 l. 16 s. $10\frac{3}{4}$ d. Q. E. F.

But if the first term of any series be greater than unity, that and the ratio being known, to find any remote term without producing the rest,

RULE,

Find a few of the leading terms, as before directed; then multiply the last term so found by itself, and divide the product by the first term, and this again multiplied by the term as is wanting, and divided by the first, gives the term required.

2. A nobleman dying left ten sons, to whom and to his executor he bequeathed his estate in the manner following; viz. to his executor for seeing his will performed 1024 pounds;

pounds; the youngeſt ſon to have as much, and half as much, and every ſon to exceed the next younger in the ſame ratio of $1\frac{1}{2}$: what is the ſhare of the eldeſt?

$$\begin{array}{l} 0 \;.\; 1 \;.\; 2 \;.\; 3 \;.\; 4 \;.\; 5, \text{ indices.} \\ 1024 \;.\; 1536 \;.\; 2304 \;.\; 3456 \;.\; 5184 \;.\; 7776, \text{ terms.} \end{array}$$

$$\frac{7776 \times 7776}{1024} = 59049 \text{ l. eldeſt ſon's fortune.} \quad \text{Q. E. F.}$$

PROPOSITION II.

The firſt term, ratio, and laſt term given, to find the ſum of all the ſeries.

RULE.

Multiply the laſt term into the ratio, and from the product ſubtract the firſt term; divide the remainder by the ratio leſs unity, the quotient will be the ſum of all the ſeries.

3. On New-year's day a gentleman married, and received of his father-in-law a guinea, on condition that he was to have a preſent on the firſt day of every month for the firſt year, which ſhould be double ſtill to what he had the month before; what was the lady's portion?

$$\text{Firſt, } \begin{cases} 0 \;.\; 1 \;.\; 2 \;.\; 3 \;.\; 4 \;.\; 5, \text{ indices.} \\ 1 \;.\; 2 \;.\; 4 \;.\; 8 \;.\; 16 \;.\; 32, \text{ terms.} \end{cases}$$

$$\text{Then, } \begin{cases} 3 + 3 = 6, \\ 8 \times 8 = 64; \end{cases} \quad \text{alſo } \begin{cases} 6 + 5 = 11 \\ 64 \times 32 = 2048, \end{cases}$$ laſt term.

Again, $2048 \times 2 = 4096$; alſo $4066 - 1 = 4095$.

$$\begin{array}{r} 20)\,4095 \\ 204\;\;15 \\ \hline \pounds\,4299\;\;15\text{s.} \end{array}$$

£ 4299 15s. the lady's fortune. Q. E. F.

4. One at a country fair had a mind to a ſtring of 20 fine horſes; but not caring to take them at 20 guineas per head, the jockey conſented that he ſhould, if he thought good, pay but a ſingle farthing for the firſt, doubling it only to the 19th, and he would give the 20th into the bargain; this being preſently accepted, how were they ſold?

$$\text{Firſt, } \begin{cases} 0 \;.\; 1 \;.\; 2 \;.\; 3 \;.\; 4 \;.\; 5 \\ 1 \;.\; 2 \;.\; 4 \;.\; 8 \;.\; 19 \;.\; 32, \text{ \&c.} \end{cases}$$

Then

Then $\begin{cases} 5 + 5 = 10, \\ 32 \times 32 = 1024; \end{cases}$ also $\begin{cases} 5 + 3 = 8 \\ 32 \times 8 = 252 \end{cases}$

Again, $\begin{cases} 10 + 8 = 18 \\ 1024 \times 252 = 262144, \text{ the 19th term.} \end{cases}$

Then $262144 \times 2 = 524288$;

also, $524288 - 1 = 524287$ farth. $= 546$ l. 2 s. $7\frac{3}{4}$ d.

$\therefore$ 20) 546 l. 2 s. $7\frac{3}{4}$ d. (27 l. 6 s. $1\frac{47}{80}$ d. each. Q. E. F.

5. A cunning servant agreed with a master (unskilled in numbers) to serve him eleven years without any reward for his service but the produce of a wheat-corn for the first year; and that product to be sown the second, and so on from year to year, until the end of the time, allowing the increase to be in a tenfold proportion: it is required to find the sum of the whole produce?

First, $\begin{cases} 1 \,.\, 2 \,.\, 3 \,.\, 4 \,.\, 5 \text{ years.} \\ 10 \,.\, 100 \,.\, 1000 \,.\, 1000 \,.\, 100000 \text{ corns of wheat.} \end{cases}$

Then $\begin{cases} 4 + 2 = 6 \\ 10000 \times 100 = 1000000, \text{ the 6th year's produce.} \end{cases}$

And $\begin{cases} 6 + 5 = 11 \\ 1000000 \times 100000 = 100000000000, \text{ the 11th} \end{cases}$ or last year's produce.

Then ratio $10 \times 100000000000 = 1000000000000$;

Also $1000000000000 - 10 = 999999999990$.

Ratio $10 - 1 = 9$) 999999999990 (111111111110 corns in all.

As hath been before observed, 7680 wheat-corns will fill a statute pint;

Then 7680) 111111111110 (14467591 pints.

In a bushel 64) 14467591 ($226056\frac{1}{9}$ bushels, which suppose at 3s. 4d. per bushel.

$\frac{1}{6}$) $226056\frac{1}{9}$

s. d.

Answer £ 37676 – $4\frac{1}{3}$, a very ample reward. Q. E. F.

6. It is reported that one Sessa, in India, having first invented the game at chess, shewed it to his prince Shekram; the king, who being highly pleased with it, bid him ask what he would for the reward of his invention; whereupon he asked, that for the first little square of the chess-board he might have one grain of wheat given him; for the second two, and so on, doubling continually according to the number of squares on the chess-board, which were 64: the king, who intending him a noble reward, was displeased that he had asked so trifling a one; but Sessa declaring

claring he would be contented with this, it was ordered to be given him; but the king was astonish'ed when he found that this would raise so vast a quantity, that even the whole earth could not produce it! so you are desired to repeat the operation.

First, $\begin{cases} 0 \, . \, 1 \, . \, 2 \, . \, 3 \, . \, 4 \, . \, 5, \text{ indices.} \\ 1 \, . \, 2 \, . \, 4 \, . \, 8 \, . \, 16 \, . \, 32, \text{ terms, \&c.} \end{cases}$

Then $\begin{cases} 5 + 5 = 10, \\ 32 \times 32 = 1024; \end{cases}$ also $\begin{cases} 10 + 10 = 20 \\ 1024 \times 1204 = 1048576 \end{cases}$

Again, $\begin{cases} 20 + 20 = 40 \\ 1048576 \times 1048576 = 1099511627776. \end{cases}$

Also

$40 + 20 = 60$
$1099511627776 \times 1048576 = 1152506950068846976$

Lastly,

$60 + 3 = 63$
$1152506950068846976 \times 8 = 9220055600547775808$

Now, $9220055600547775808 \times 2 = 18440111201095551616$.

$\because$ $18440111201095551616 - 1 = 18440111201095551615$ wheat-corns.

7680) 18440111201095551615 (2401056145975997 pints.

64) 2401056145975997 (37516502280874 bushels.

8) 37516502280874 (4689562785109.25 quarters, which, at 1l. 7s. 6d. per quarter, amounts to 6448148829612l. which is more than would pay one year's rent of all the dry land on the face of the earth, at 1 l. 10 s. per acre, which may thus be proved:

The circumference of the earth 360 degrees ($69\frac{1}{2}$ English miles to a degree) = 25020 English miles, circumference of the earth.

Also, $25020 \times 25020 \times .31832 = 199268447.328$, area in square miles of a perfect globe.

Also, $199268447.328 \times 640 = 127531806289.92$ acres of land and water, $\frac{2}{3}$ of which is supposed to be water.

$\because$ 3) 127531806289.92 (42510602096.64 acres of dry land, which, at 1 l. 10 s. per acre, is 63765903144.96 pounds a year; which, compared with the valuation of the wheat, as above, will be found 715585151 l. less.

PROPOSITION. III.

Of any decreasing series in $\div$, whose last term is a cypher, to find the sum of those series.

RULE.

RULE.

Divide the square of the first term by the difference between the said first term and the second term in the series, the quotient will be the sum of all the series.

7. To find the sum of $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$, &c. *ad infinitum*

Thus, $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$; also, $\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$.

$\because$ $\frac{1}{4})\frac{1}{4}(\frac{4}{4}=1$, the sum of the series required.

8. To find the sum of $\frac{1}{3}\times\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$, &c. *ad infinitum*.

First, $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$; also, $\frac{1}{3}=\frac{3}{9}$; and $\frac{3}{9}-\frac{1}{9}=\frac{2}{9}$.

$\because$ $\frac{2}{9})\frac{1}{9}(\frac{9}{18}=\frac{1}{2}$. Q. E. F.

9. Suppose a ball to be put in motion by a force which drives it 12 miles the first hour, 10 miles the second, and so on continually, decreasing in proportion of 12 to 10, to infinity; what space would it move through?

First, $12\times12=144$; also $12-10=2$.

$\because$ 2) 144 (72 miles. Q. E. F.

It may appear strange to some, that it should be possible to give the sum of an infinite progression in numbers; whereas, if the terms were continued, it would, after a thousand years labour, and after producing thousands of millions of terms, be never the nearer finishing.

SECT. III.

VARIATIONS.

BY variation is meant the different ways any number of things may be altered or changed, in respect to their places.

To find the number of different changes that may be rung on any proposed number of bells.

RULE,

Multiply all the terms in a series of arithmetical progressionals continually, whose first term and common difference is

is unity or 1, and the laſt term the number of things propoſed to be varied; the laſt product will be the number of variations required.

The changes on any number of bells not exceeding 12, are exhibited in the following Table.

The number of things propoſed to be varied.	The manner how their ſeveral variations are produced.	The different changes or variations every one of the propoſed numbs. can admit of.
1	1 × 1	= 1
2	1 × 2	= 2
3	2 × 3	= 6
4	6 × 4	= 24
5	24 × 5	= 120
6	120 × 6	= 720
7	720 × 7	= 5040
8	5040 × 8	= 40320
9	40320 × 9	= 362880
10	362880 × 10	= 3628800
11	3628800 × 11	= 39916800
12	39916800 × 12	= 479001600

Let it be propoſed to find the number of changes that may be rung on 12 bells, and to compute how long all theſe changes would take ringing once over.

Firſt, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$, the number of changes.

And ſuppoſe 10 changes to be rung in a minute; viz. $12 \times 10 = 120$ ſtrokes in one minute, or two ſtrokes in a ſecond:

Then 10) 479001600 (47900160 minutes.

Alſo 1 year = 365 days, 5 hours, 49′ = 525949 minutes.

525949) 47900160 (91 years, 26 days, 22 hours, 41 minutes; ſo long would 12 bells be ringing, without any intermiſſion, to ring their different changes but once over.

564750

In 1 day are 1440) 38801

1000

60) 1361

2. A young scholar, but an arithmetician, coming into a town for the convenience of a good library, demands of a gentleman, with whom he lodged, what his diet would cost for a year? The gentleman asks him 10 l. the scholar answered, he was not certain what time he might stay, and would know what he must give him for his diet so long as he could place his family (consisting of six persons besides himself) every day at dinner in a contrary position? The gentleman considering of it, and thinking it could not be long, tells him, he would allow him his diet so long for five pounds? to which the scholar assents: what did he give him for his table per annum?

First, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$ variations, or days.

Then 365.26) 5040,00 (13 years, 291 days, the answer.

13.79725) 5.000000 (.36239 = 7s. 3d. per annum nearly the answer.

SECT. IV.

COMBINATIONS.

COMBINATIONS are the various conjunctions which several things may receive without any regard to order, being taken 2 and 2, 3 and 3, &c.

To find the different combinations in any number of quantities.

RULE.

Having placed the given quantity by itself, decrease it gradually by an unit, so often as there are quantities in the combination; placing them one above another, with a sign of multiplication between them, which number must be multiplied into one another for a dividend: then placing an unit with the like number of places, increasing by unity till you arrive at the number to be combined; which multiply continually for a divisor, and the quotient will be the number of combinations sought.

1. A famous general having serv'd his king
Long time in wars, and had victorious been;

For which his service (with a pleasant smile)
Ask'd of his king one farthing for each file
Of ten men in a file, which he could then
Make with a body of one hundred men.
The king, considering his brave actions past,
And seeming modesty of his request,
Gave his consent.—To what will it amount
In sterling money? take your pen and count.

$$\frac{100}{1}\times\frac{99}{2}\times\frac{98}{3}\times\frac{97}{4}\times\frac{96}{5}\times\frac{95}{6}\times\frac{94}{7}\times\frac{93}{8}\times\frac{92}{9}\times\frac{91}{10}=$$

$$\frac{62815650955529472000}{3628800}=17310309456440 \text{ farthings.}$$

$\therefore$ 17310309456440 farth. $=$ 18031572350 l. 9s. 2d. Q. E. F.

The number of combinations of 2 in any number of things, are 2 raised to the power of the number of things to be combined; for instance, if it was required to find the different ways 11 halfpence, husled in a hat, may turn up; as a halfpenny has two faces, $\because 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$, or $2^{11}=2048$, the different combinations required.

Now to find the different chances for any number of heads or tails, put a for the heads, and b for the tails; then,

$$a^{11}+a^{10}b+a^9b^2+a^8b^3+a^7b^4+a^6b^5+a^5b^6+a^4b^7+a^3b^8+a^2b^9+ab^{10}+b^{11}.$$

Here it is to be observed, that a^{11}, or all heads, hath but one way of turning up; the same may be observed for 11 tails.

But 10 heads and 1 tail, and the contrary, may come up 11 different ways each.

Also nine heads and two tails, or the contrary, may each come $\frac{11\times10}{2}=55$ different ways.

Again, eight heads and three tails, or the contrary, may each come $\frac{55\times9}{3}=165$ different ways; which numbers so found are called unciæ, or coefficients.

From hence may be deduced this general rule,

RULE.

If the index of the first letter of any term be multiplied into its own unciæ, and that product be divided by the number of terms

terms to that place, the quotient will be the unciæ of the next succeeding term forward.

Let us proceed to find the reſt of the coefficients, or chances, by the rule above;

viz. ſeven heads and four tails, or the contrary, may each come $\frac{165 \times 1}{4} = 330$ different ways

Again, for ſix heads and five tails, or the contrary, each may come $\frac{330 \times 7}{5} = 462$.

It may be obſerved, by proceeding as above, that the unciæs, or coefficents, do only increaſe until the indices of the two letters become equal, or change places, and then the reſt will decreaſe in the ſame order.

Thus $a^{11} + 11a^{10}b + 55a^9b^2 + 165a^8b^3 + 330a^7b^4 + 462a^6b^5 + 462a^5b^6 + 330a^4b^7 + 165a^3b^8 + 55a^2b^9 + 11ab^{10} + b^{11}$, are all the different combinations, or ways, 11 halfpence can turn up;

viz. $1 + 11 + 55 + 165 + 330 + 462 + 462 + 330 + 165 + 55 + 11 + 1 = 2^{11} = 2048$, as before.

Alſo a die, having ſix faces or ſides, the number of combinations or different ways 11 dice may turn up, are $6^{11} = 362797056$.

From conſiderations like theſe I compoſed the following queſtion, which was publiſhed in the Palladium 1753, and the next year was anſwered by three ingenious gentlemen; but they not conſidering, that in the ſame web of a gooſe's foot the punch mark and ſlit mark cannot exiſt together, the laſt mark naturally deſtroying the firſt, which cauſed a miſtake in their calculations; for both may combine together as well as with all the reſt, and bring out the ſame number of combinations as eight halfpence, and four triangular dice, with three faces each, ſhaken together, could produce.

2. In Lincolnſhire, where bounteous nature yields
Fat ſheep and oxen, and luxuriant fields;
Our generous clime, replete with roſy health,
Choice friends afford, bright, fair, and plenteous wealth.
Some fenny ground have we, with flocks of geeſe,
Yielding five times a year their feather'd fleece;
On which, devoid of care, ſwains ſleeping lie,
After repaſt of ſavoury giblet-pye.

 One

One day, at Boſton, o'er a jug of ale,
A goſſard offer'd all his flock to ſale,
At fifteen pence a-piece; but I propos'd
A different price, with which he quickly clos'd.
(The geeſe are mark'd, by cutting toe or heel,
The webs are pierc'd or ſlit with ſharpen'd ſteel)
An hundred pounds for juſt as many geeſe,
As may be different mark'd: what's that a-piece?

Each gooſe having three toes, two webs, and one heel on each foot, in all 12 different things to be marked.

But as the four webs may be either ſlit or pinched, but not both together, ∵ $2^8 \times 3^4 = 20736$, the number of combinations.

Alſo, $20736 - 1 = 20735 =$ number of marks.

And 100 l. $=$ 24000 pence.

∴ 20735) 24000 ($1\frac{653}{4147}$d. the price of a gooſe. Q. E. F.

3. A perſon, P, bets ſix pounds with another perſon, Q, that in throwing up three halfpence, they ſhall all come up the ſame way, viz. all heads, or all tails, once at leaſt in three trials: at the ſame time Q bets 10 guineas with R, that in throwing up four halfpence, they ſhall not all come up the ſame (*i. e.* all heads, or all tails) once in four trials: required each perſon's advantage and diſadvantage, with the odds in each caſe, by an arithmetical computation only?

Since there are but two chances for three halfpence coming all one way at a ſingle throw, and ſix for the contrary, it is evident the probability of miſſing all the three throws is $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$, and that of the contrary $1 - \frac{27}{64} = \frac{37}{64}$.

∴ The odds in favour of P, are as 37 to 27.

In the ſame manner $\frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = \frac{2041}{4096} =$ probability of miſſing all the four throws with four halfpence, and that of the contrary $\frac{1695}{4096}$. The odds in favour of Q, are as 2401 to 1695; hence P's advantage in the firſt caſe is 18 s. 9 d. and Q's in the ſecond caſe is equal 2 l. 6 s. $2\frac{1}{4}$ d.

4. Two deſperate gameſters, A and B, agreed to throw each of them two guineas at a particular point or mark; and then to toſs up the four guineas fairly, and each perſon to take up the heads in the following manner as they ariſe; that is, A's two guineas happened to light the firſt and ſecond, or the two neareſt to the ſaid point; ſo that if either one or two heads ariſe, they muſt fall to A's lot; but B's two guineas being the third or fourth, or the two fartheſt from the point, ſo that if either three or four heads ariſe, they are to be B's property: moreover, if two heads happen

pen to arise in the first toss, then A gets all the four guineas; and if three heads come up first to B, then will the other guinea be A's of course. Quere, what is each person's probability of winning, and how much is the value of A's chance before they begin to toss?

Martin's Magazine.

First, let a represent heads, and b tails; then in the binomial $a+b$ raised to the 4th power, the powers of b being rejected (as no winning chances on either side) will remain $a^4 + 4a^3 + 6aa + 4a$; the indices representing the chances, and coefficients the number of different ways those chances may happen; the two last terms being A's, and the two first B's; in all, 15.

$\frac{6}{15} + \frac{4}{15}$ of $\frac{1}{4} = \frac{7}{15}$, value of A's } first winning toss;
$\frac{1}{15} + \frac{4}{15}$ of $\frac{1}{4} = \frac{4}{15}$ = value of B's } sum $\frac{11}{15}$.

Then $\frac{15}{15} - \frac{11}{15} = \frac{4}{15}$, whole value of the second toss. Now B's chance of getting four heads must cease, so there can only remain seven winning chances, viz. $a^3 + 3a^2 + 3a$; whereof only one, viz. the first, belongs to B.

Also, $\frac{6}{7}$ of $\frac{4}{15} = \frac{24}{105}$ } = value of { A's } second winning
And $\frac{1}{7}$ of $\frac{4}{15} = \frac{4}{105}$ } { B's } toss.

Also, $\frac{7}{15} + \frac{24}{105} = \frac{73}{105}$; and $\frac{4}{15} + \frac{4}{105} = \frac{32}{105}$.

Consequently, A's chance to that of B's, is as 73 to 32, viz. more than 9 to 4.

$\therefore \frac{7}{15} + \frac{24}{105} = \frac{73}{105} =$ 2l. 8s. $4\frac{4}{5}$d. = value of A's } chance.
and $\frac{4}{15} + \frac{4}{105} = \frac{32}{105} =$ 1l. 5s. $7\frac{1}{5}$d. = value of B's }

5. Two gamesters met the other day,
The one call'd B, the other A;
But having neither cards nor dice,
They get to hotch-cap in a trice
With sixteen halfpence fair and flat,
All which they husled in a hat.
Says A to B, all these are mine,
And I will lay a pint of wine,
That in two trials there will be
Nine heads or tails, as here you see.
No matter which, but on they play'd,
Till silver, brass, and gold were laid;
But as to B, his chance was bad,
For he got broke of all he had.
What were the odds, I pray declare,
Groom-porters sly, and ladies fair?

By the late Mr. Joseph Smith, of Fleet.

$2^{16} = 65536$, number of different chances on 16 halfpence.

Let a represent the heads, and b the tails.

Then $a^{16} + 16a^{15}b + 120a^{14}b^2 + 560a^{13}b^3 + 1820a^{12}b^4 + 4368a^{11}b^5 + 8008a^{10}b^6 + 11440a^9b^7$.

Then $11440 \times 2 = 22880$, chances for nine heads or tails to come.

$\therefore 65536 - 22880 = 42656$, not to come the first time; viz. 22880 to 42656, that they come nine heads or tails the first toss.

As $65536 : 22880 :: 42656 : 14892\frac{7}{64}$.

Then $22880 + 14892\frac{7}{64} = 37772\frac{7}{64}$, for $\Big\}$ nine heads
Also $65536 - 37772\frac{7}{64} = 36763\frac{57}{64}$ against $\Big\}$ or 9 tails
turning up once in two trials.

From hence A's chance to that of B,
Is something more than four to three.

6 Two gamesters one day at dice they would play,
And being full merry in wine,
Says B unto A, what odds will you lay,
I cast not all the six faces this time?
Says A then to B, ten to one I'll lay thee,
With six dice the six faces you cast not.
Pray gentlemen shew, and next year let them know,
For the odds on the cast, sirs, they do not.

$6^6 = 46656$ different combinations.

And $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ variations.

Then $46656 - 720 = 45936$ chances against A.

But as A laid 10 to 1 $\therefore$ 7200 chances for B.

$\therefore$ A's chance to that of B, is as 45936 to 7200, or as 6.38 to 1.

A TABLE *shewing the probability of winning or losing any number of games together, when the gamesters are equal.*

Powers.			Odds.			
$2 \cdot\cdot 2$	viz.	$2 - 1 =$	1	to 1, not	1	games running.
$2^2 = 4$		$4 - 1 =$	3		2	
$2^3 = 8$		$8 - 1 =$	7		3	
$2^4 = 16$		$16 - 1 =$	15		4	
$2^5 = 32$		$32 - 1 =$	31		5	
$2^6 = 64$		$64 - 1 =$	63		6	
$2^7 = 128$		$128 - 1 =$	127		7	
$2^8 = 256$		$256 - 1 =$	255		8	

7. To find how many holes a person can make at cribbage, that has the whole pack in his hand.

First for the sequences:

As

As 4 is the number of ways an ace can be ſhewn, it follows that 4^2 will be the number of different ways that one ace and one duce can be ſhewn; and 4^3 the number of ſequences with an ace, duce and tray, &c. &c.

Whence it appears, that 4^{13} will be the number of ſequences of thirteen in each; which multiplied by 13,

viz. $4^{13} \times 13 = 872415232$, the number of holes to which all the ſequences amount.

Secondly, for the number of fifteens.

The determimation of theſe depends upon the following caſes, according to the ſeveral ways by which 15 can be made by 2, 3, 4, 5, &c. cards, the number correſponding to which caſes are found from the following

Theorem.

Let $a =$ number of cards, of one ſort, b of a ſecond, and c of a third (all the cards together making 15) then will $\frac{4}{1} \times \frac{3}{2}\,(a) \times \frac{4}{1} \times \frac{2}{2}\,(b) \times \frac{4}{1} \times \frac{2}{3}\,(c)$ &c. be the number of 15's correſponding.

For example, Let there be two 5's, one (tray) or 3, and one (duce) or 2, then will the number of 15's correſponding be $= \frac{4}{1} \times \frac{3}{2} \times \frac{4}{1} \times \frac{4}{1} = 96$.

		Nº of ways.
10 +	5 - - - - - - -	64
	4 + 1 - - -	256
	3 + 2 - - -	256
	3 + 1 + 1 -	384
	2 + 2 + 1 -	384
	2 + 1 + 1 + 1	256
		Sum 1600
9 +	6 - - - - - - -	16
	5 + 1 - - -	64
	4 + 2 - - -	64
	3 + 3 - - -	24
	4 + 1 + 1 -	96
	3 + 2 + 1 -	256
	2 + 2 + 2 -	16
	3 + 1 + 1 + 1	64
	2 + 2 + 1 + 1	144
	2 + 1 + 1 + 1 + 1	16
		Sum 760

		Nº of ways
7 +	7 + 1 - - - -	24
	6 + 2 - - - -	64
	5 + 3 - - - -	64
	4 + 4 - - - -	24
	6 + 1 + 1 - -	96
	5 + 2 + 1 - -	256
	4 + 3 + 1 - -	256
	4 + 2 + 2 - -	96
	3 + 3 + 2 - -	96
	5 + 1 + 1 + 1	64
	4 + 2 + 1 + 1	384
	3 + 3 + 1 + 1	144
	3 + 2 + 2 + 1	384
	2 + 2 + 2 + 2	4
	4 + 1 + 1 + 1 + 1	16
	3 + 2 + 1 + 1 + 1	256
	2 + 2 + 2 + 1 + 1	96
	2 + 2 + 1 + 1 + 1 + 1	24
		Sum 2348

		N° of ways.
8+	7 - - - - - - -	16
	6 + 1 - - - -	64
	5 + 2 - - - -	64
	4 + 3 - - - -	64
	15 + + 1 - -	96
	4 + 2 + 1 - -	256
	3 + 3 + 1 - -	96
	3 + 2 + 2 - -	96
	4 + 1 + 1 + 1	64
	3 + 2 + 1 + 1	384
	2 + 2 + 2 + 1	64
	3 + 1 + 1 + 1 + 1	16
	2 + 2 + 1 + 1 + 1	96
		Sum 1376

		N° of ways.
6+	6 + 3 - - - -	24
	5 + 4 - - - -	84
	6 + 2 + 1 - -	96
	5 + 3 + 1 - -	256
	5 + 2 + 2 - -	96
	4 + 4 + 1 - -	96
	4 + 3 + 2 - -	256
	3 + 3 + 3 - -	16
	6 + 1 + 1 + 1	24
	5 + 2 + 1 + 1	384
	4 + 3 + 1 + 1	384
	4 + 2 + 2 + 1	384
	3 + 3 + 2 + 1	384
	3 + 2 + 2 + 2	64
	5 + 1 + 1 + 1 + 1	16
	4 + 2 + 1 + 1 + 1	256
	3 + 3 + 1 + 1 + 1	96
	3 + 2 + 2 + 1 + 1	576
	2 + 2 + 2 + 2 + 1	16
	3 + 2 + 1 + 1 + 1 +	64
	2 + 2 + 2 + 1 + 1 +	64
		Sum 3616

		N° of ways.
5+	5 + 5 - - - -	4
	5 + 4 + 1 - -	96
	5 + 3 + 2 - -	96
	4 + 4 + 2 - -	96
	4 + 3 + 3 - -	96
	5 + 3 + 1 + 1	144
	5 + 2 + 2 + 1	144
	4 + 4 + 1 + 1	144
	4 + 3 + 2 + 1	1024
	4 + 2 + 2 + 2	64
	3 + 3 + 3 + 1	64
	3 + 3 + 2 + 2	144
	5 + 2 + 1 + 1 + 1	96
	4 + 3 + 1 + 1 + 1	256
	4 + 2 + 2 + 1 + 1	576
	3 + 3 + 2 + 1 + 1	576
	3 + 2 + 2 + 2 + 1	256
	4 + 2 + 1 + 1 + 1 + 1	64
	3 + 3 + 1 + 1 + 1 + 1	24
	3 + 2 + 2 + 1 + 1 + 1	384
	2 + 2 + 2 + 2 + 1 + 1	24
	2 + 2 + 2 + 1 + 1 + 1 + 1	16
		Sum 4388

		N° of ways.
4+	4 + 4 + 3 - -	16
	4 + 4 + 2 + 1	64
	4 + 3 + 3 + 1	144
	4 + 3 + 2 + 2	144
	3 + 3 + 3 + 2	64
	4 + 4 + 1 + 1 + 1	16
	4 + 3 + 2 + 1 + 1	576
	4 + 2 + 2 + 2 + 1	96
	3 + 3 + 3 + 1 + 1	96
	3 + 3 + 2 + 2 + 1	576
	3 + 2 + 2 + 2 + 2	16
	4 + 3 + 1 + 1 + 1 + 1	24
	4 + 2 + 2 + 1 + 1 + 1	144
	3 + 3 + 2 + 1 + 1 + 1	384
	3 + 2 + 2 + 2 + 1 + 1	384
	3 + 2 + 2 + 1 + 1 + 1 + 1	96
	2 + 2 + 2 + 2 + 1 + 1 + 1	16
		Sum 2856

		No. of ways.
3 +	3 + 3 + 3 + 2 + 1 - - - - - - - - -	16
	3 + 3 + 2 + 2 + 2 - - - - - - - - -	16
	3 + 3 + 3 + 1 + 1 + 1 - - - - -	4
	3 + 3 + 2 + 2 + 1 + 1 - - - - -	144
	3 + 2 + 2 + 2 + 2 + 1 - - - - -	24
	3 + 3 + 2 + 1 + 1 + 1 + 1 - -	16
	3 + 2 + 2 + 2 + 1 + 1 + 1 - -	96
	Sum	320

Then 1600 + 760 + 1376 + 2348 + 3616 + 4388 + 2856 + 320 = 17264, different ways to count 15; and 17264 × 2 = 34528, the number of holes.

Lastly, The number of prials will be 13; and 13 × 12 = 156, number of holes.

872415232 + 34528 + 156 = 872449916 holes the pack will make. Q. E. F.

SECT. V.

MAGIC SQUARES.

A MAGIC SQUARE is a square figure composed of a series of numbers in arithmetical proportion, so disposed in parallel and equal ranks, as that the sum of each row taken either perpendicularly, horizontally, or diagonally, are equal.

In ignorant ages, when mathematics passed for magic, these squares were made use of by conjurers, for the construction of talismans.

However, they have since become the serious research among mathematicians; not that they are of any real or solid use, or advantage, but only as a kind of play, where the difficulty makes the merit, as it may chance to produce some new views of numbers, which mathematicians will not lose the occasion of.

1. The numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9, being given to form them in a magic square, viz. counting each rank perpendicularly, horizontally, or diagonally, that those ranks may be equal to each other.

Suppose

Suppose it done and represented in its proper form, by the following symbols thus placed, viz.

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array}$$

First, the sum of the progessional numbers ~~are 45~~.

Then $3 =$ number of rows.

Also $\frac{15}{3} = 5 =$ sum of each side or rank.

And $\frac{45}{3} = 5 = e$, the middle number.

Again, to find the corner figures, and first to find the figure represented by a.

Beginning with 1, I find the corner letter a, or any other corner letter, cannot be 1; for if a was $= 1$, then i must be 9; and $b + c = 15 - 1 = 14$; as also $d + g = 15 - 1 = 14$. But there remains no two numbers after 5, 1, and 9, whose sum is 14, but 6 and 8: $\because$ if any of those figures were b, the other would be c; and then no figures would remain for the value of either d or g; wherefore a is not equal 1, nor any corner letter equal 1 or 9.

3 cannot be $= a$; for if it were, then i should be $= 7$; and $b + c = 15 - 3 = 12$; as also $d + g = 12$: but there remains no two numbers after 5, 3, and 7, whose sum is 12, but $8 + 4$, which cannot answer to b and c, and d and g; wherefore a, or any other corner letter, is not $=$ 3; neither is i, nor any other corner letter, $= 7$.

From what hath been said, it is plain, that (if the question proposed is capable of being solved) the corner letters are all even numbers; wherefore, if $a = 2$, i will be $=$ 8, and c must be either 4, or 6. Let $c = 4$; then $g = 6$, $b = 9$, $d = 7$, $f = 3$, and $h = 1$; and so the square is completed as required.

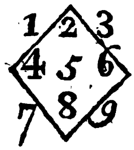
$$\begin{array}{ccc} 2 & 9 & 4 \\ 7 & 5 & 3 \\ 6 & 1 & 8 \end{array}$$

But if c were equal 6 (a being $= 2$); then $g = 4$, $b =$ 7, $d = 9$, $f = 1$, and $h = 3$, and then the squares will stand thus:

A

2	7	6
9	5	1
4	3	8

Or they may be found mechanically thus: set them all down progressively, about which draw a square cornerways.



Then set the four angular figures at the corners, and put the outermost alternately.

2	.	6
.	5	.
4	.	8

2	7	6
9	5	1
4	3	8

2. Let it be required to form the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and 16, into a magic square, viz. so that counting each rank from one hand to the other, as also up, down, and diagonal-wise, those ranks may be equal to each other.

Suppose it done, and represented in its proper form by the following symbols, viz.

a	b	c	d
e	f	g	h
i	k	l	m
n	o	p	q

The sum of the said progressional figures are 136.

Also 4 = number of rows.

Then $\frac{136}{4} = 34 =$ sum of each side, or rank.

Now beginning with the least of those numbers, put $a = 1$.

Then the other corner letter n cannot be $= 2$; for if it was, $h + m = a + n = 1 + 2$ would be 3; but there are no remaining two numbers of the given one, whose sum is 3, therefore n cannot be $= 2$, a being $= 1$.

Neither can $n = 3$; for supposing $n = 3$, then $h + m = a + n\ 1 + 3 = 4$; but there are no remaining two numbers whose sum is 4.

Now putting $n = 4$; then $h + m = a + n = 1 + 4 = 5$; that is, $h, m = 2, 3$, which are the only two numbers

bers remaining, whose sum is 5: $\therefore d + q = 34 - 5 = 29$; that is, d, q are 13, 16, or 14, 15, for no other couple amounts to 29.

Suppose again $q = 13$; then must $d = 16$, when we design the square in part, viz.

1	b	c	16
	f	g	
14, 15			2, 3
	k	l	
4	o	p	13

As the four corner figures are fixed, and e, $i = 14, 15$; also h, m 2, 3; it is plain f cannot be 5, 6, or 7, for if it was, l would be 15, 14 or 13, which numbers are already disposed of; therefore, supposing it 8, and then $l = 12$.

Again, $g + k = 14$, and as there remain no two numbers whose sum is 14, but only $5 + 9$; but $k + 12 = 2 + 14 = e + h$. $17 = 2 + 15$, or $3 + 14$, or $18 = 3 + 15$; consequently $k = 4$, 5 or 6 (not equal to 9). $\therefore k = 5$. And $k + l = 5 + 12 = 17$, must likewise be $= e + h$; which may be effected two different ways, either by putting $e = 15$ or 14, and then $h = 2$ or 3, by chusing the former; $i = 14$, and $m = 3$, and then the square will be farther designable, viz.

1	b	c	16
15	8	9	2
14	5	12	3
4	o	p	13

It remains to dispose of four numbers, 6, 7, 10 and 11, instead of b c, o and p, so as $b + c$ may be $= 17$; as also $o + p = 17$; which may be done by coupling 6, 11; as also 7, 10: but $c + p$ must be $= k + f = 13$, which will be effected by $6 + 7$; from whence p being $= 6$, c will be $= 7$; and then $o = 11$, and consequently $b = 10$; and then the square will be fully completed, thus:

1	10	7	16
15	8	9	2
14	5	12	3
4	11	6	13

Or putting $p = 7$; then $c = 6$, $o = 10$, and $b = 11$; and then the square will stand thus:

I

1	11	6	16
15	8	9	2
14	5	12	3
4	10	7	13

Or by ſetting down the numbers progreſſionally, reſerving the diagonal numbers, the ſquare may be filled up by an eaſy tranſpoſition of the reſt, as follows:

1	.	.	4		1	15	14	4
.	6	7	.		12	6	7	9
.	10	11	.		8	10	11	5
13	.	.	16		13	3	2	16

3. Suppoſe a ſquare form of ſet numbers there be,
In their natural order (as 1, 2, and 3)
Amount to the ſum, when they're added together,
Of 62 juſt, in rank and file either:
If alſo from corner to corner you count,
Yet ſtill 62 ſhall be their amount;
What numbers are they, and how muſt they be put,
When ſixteen there be that completely will do't?

The ſum of one row 62 × 4 (number of rows) = 248 = ſum of all thoſe progreſſional numbers.

And their common difference is = 1.

16) 248 (15.5. Then 15.5 — 7.5 = 8, the firſt term
And 8 + 15 = 23 = laſt term.

Then obſerving the directions given in the foregoing queſtion, this magic ſquare may be filled as follows;

8	.	.	11		8	22	21	11
.	13	14	.		19	13	14	16
.	17	18	.		15	17	18	12
20	.	.	23		20	10	9	23

4. 'Tis to you, lovely ladies, I ſue and ſubmit,
(Who outvie Sidrophel in magic and wit)
For ſolution of this notty problem propos'd,
By which undertaking my ſenſes are doz'd;
To find by what method thoſe ſquares you may fill,
Which are magical call'd, and by that try your ſkill;
8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 26, 27, 28, 29.

To

To place all these numbers, so that they amount,
Just half a score ways, seventy four you may count.
If you'll answer but this, now yourself do assure,
I will meddle with what you call magic no more.

8	.	.	11		8	28	27	11
.	15	16	.		23	15	16	20
.	21	22	.		17	21	22	14
26	.	.	29		26	10	9	29

5. To form a magic square of the numbers 1, 2, 4, 5, &c. to 25, inclusive.

First, 5 are the number of rows
also 325, the sum of those progressional numbers ;
and 5) 325 (65 = sum of each side or rank

16	14	8	2	25
3	22	20	11	9
15	6	4	23	17
24	18	12	10	1
7	5	21	19	13

6. To form the progressional numbers from 1 to 49, both inclusive (their common difference being 1) into a magic square.

First $1 + 49 = 50$ which $\times 49 = 2450$.
Also, 2) 2450 (1225 = sum of those progressional numbers.
∴ 7) 1225 (175 = sum of each row.

1	9	17	25	33	41	49
24	32	40	48	7	8	16
47	6	14	15	23	31	39
21	22	30	38	46	5	13
37	45	4	12	20	28	29
11	19	27	35	36	44	3
34	42	43	2	10	18	26

7. You that delight in figures, try your skill,
A magic square with numbers for to fill ;
One to a hundred numbers just must be,
Which to the numbers of the squares agree:
But farther, you must them so just contrive,
Twenty two ways, make five hundred and five.

No

No two squares alike in numbers must be,
But ten in breadth, and ten in length, lets see.

11	92	12	88	14	15	16	84	83	90
100	82	26	27	67	35	59	58	50	1
99	19	75	74	33	66	42	43	51	3
2	20	76	73	34	36	60	57	49	98
4	81	25	28	68	65	41	44	52	97
94	21	77	72	32	37	61	56	48	7
5	80	24	29	69	64	40	45	53	96
6	79	23	30	70	38	62	55	47	95
93	22	78	71	31	63	39	46	54	8
91	9	89	13	87	86	85	17	18	10

Any one of the foregoing squares may be disposed many other different ways, as may be tried by those who have time and inclination for such operations.

CHAPTER VI.

SECT. I.

COMPOUND INTEREST.

COMPOUND INTEREST is that which ariseth not only from the use of the principal, but also from the use of the interest as it becomes due; the interest being added to the principal at the end of every year, making a new principal for the succeeding year; so that the principal and interest are continually increasing.

CASE I.

The principal, rate, and time given, to find the interest.

RULE. I.

To the principal add the interest for the first year, which will be a new principal; to which add the interest for another year, for a fresh principal for the second year; and so proceed for any number of years.

RULE. II.

Multiply the principal by the amount of one pound for one year continually for all the proposed years; the last product will be the amount as before.

1. What

1. What is the compound intereſt of 500 l. for four year at five per cent.?

l. l.

$5 = \frac{1}{20}$) 500, principal - - - - - - - - - } add.
25, intereſt for the firſt year }

20) 525, amount for the 1 ſt, or principal for 2d year.
26 5, intereſt for the ſecond year.

20) 551 5, amount for 2d. principal for the 3d.
27 11 3, intereſt for the 3d year.

20) 578 16 3, amount for 3d. principal for the 4th.
28 18 9¾, intereſt for the 4th year.

607 15 –¾, amount for the 4th year.
500 – –, principal.

£ 107 15 –¾, intereſt.

By RULE II.

The amount of 1 l. for one year, is 1.05.

Then 500
1.05

525, amunt for the firſt year.
1.05

2625
525

551.25, amount for the ſecond year.
1.05

275625
55125

578.8125, amount for the third year.
1.05

28940625
5788125

607.753125, amount for the fourth year.
500. principal.

107.753125, intereſt.

2. What

2. What is the compound interest of 760 l. 10 s. for four years, at four per cent. per annum?

	l.	s.	d.	
20 = $\frac{1}{5}$	760	10,		principal.
$\frac{1}{5}$	152	2		
	30	8	$4\frac{3}{4}$,	interest the first year.
$\frac{1}{5}$	790	18	$4\frac{3}{4}$,	amount; principal for the 2d year.
$\frac{1}{5}$	158	3	8	
	31	12	$8\frac{3}{4}$,	interest the 2d year.
$\frac{1}{5}$	822	11	$1\frac{1}{2}$,	amount; principal for the 3d year.
$\frac{1}{5}$	164	10	$2\frac{1}{2}$	
	32	18	$-\frac{1}{2}$,	interest for 3d year.
$\frac{1}{5}$	855	9	2,	amount; principal for the 4th year.
$\frac{1}{5}$	171	1	10	
	34	4	$4\frac{1}{4}$,	interest for the 4th year.
	889	13	$6\frac{1}{4}$,	amount for the 4th year.
	760	10	–,	principal.
£	129	3	$6\frac{1}{4}$,	interest.

By RULE II.

The amount of 1 l. for a year, at four per cent. is 1.04 l.

```
Then 760.5
       1.04
     ------
      30420
     7605
     ------
     790.92, amount for the first year.
       1.04
     ------
     316368
    79092
    --------
    822.5568, amount for the 2d year.
        1.04
    --------
     32902272
    8225568
   -----------
   855 459072
       1.04
   -----------
    3421836
   855459
  ------------
  889.67736, amount for the 4th year.
  760.5      principal.
  ------------
  129.17736, interest = 129 l. 3 s. 6½d.
  ------------
```

CASE II.

The amount, rate per cent. and time given, to find the principal or present worth.

RULE.

As the amount of 1 l. compound interest, at the rate and for the time given: is to 1 l. :: so is the amount given: to the present worth required.

3. What

3. What is the preſent worth of 889 l. 13 s. 6½d. due four years hence, at four per cent. per annum, compound intereſt?

Firſt, $1.04 \times 1.04 \times 1.04 \times 1.04 = 1.16985856$:
Therefore $1.16985856 : 1 :: 889.67736 : 760.5$.
Anſwer, 760 l. 10 s. preſent worth.

CASE III.

The principal, rate, and amount given, to find the time.

RULE.

Divide the amount by the principal, and that quotient by the amount of 1 l. for a year, and the next quotient by the ſame; and ſo on continually, till the laſt quotient be unity; the number of which diviſions will be the time required.

4. In what time will 760l. 10s. amount to 889l. 13s. 6½d compound intereſt, being allowed at four per cent.?

760.5) 889.677083 (1.16

1.04	1.16 - - - - 1ſt	diviſion.
1.04	1.12 - - - - 2d	
1.04	1.08 - - - - 3d	
1.04	1.04 - - - - 4th	

Hence the term is four years.

The 4th Caſe is to find the rate per cent. the principal amount, and time given; but this requires the extraction of the roots of very high powers, or the uſe of logarithms; which (as my book is ſwelled to a greater bigneſs than at firſt intended) I am obliged to omit.

And for the ſolving queſtions in compound intereſt with more facility, I have inſerted the following tables.

The conſtruction of the firſt table following, ſhewing the amount of 1 l. for years, is only by the involution of the amount of 1 l. for years, to the power of the number of years.

Thus the amount of 1 l. for two years, at five per cent. compound intereſt, will be 1.05 x 1.05 = 1.025.

Alſo, 1.05 × 1.05 x 1.05 = 1.157625 = the amount of 1 l. for three years, at five per cent.

And the conſtruction of the ſecond table is by the continual multiplication of the amount of 1 l. for a day; the amount of 1 l. for a day being the root of its amount for a year, extracted to the 365th power.

The amount of 1 l. for a day, at five per cent. being 1,0001336, its amount for two days will be 1.0001336 x 1.0001336 = 1.0002672, &c. and 1.0001336 × 1.0001336 × 1.0001336 = 1.0004011, the amount of 1 l. at compound intereſt, for three days, at five per cent.

And thus by continually multiplying by the amount for a day, at each rate per cent. the ſecond table is conſtructed and the 364th product will be the amount.

DECIMAL TABLES *of* COMPOUND INTEREST.

At the rates of 3, 3½, 4. 4½, and 5 per cent. per annum.

TABLE I.

The amount of one pound for YEARS.

ys.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1.0300000	1.0350000	1.0400000	1.0450000	1.0500000
2	1.0609000	1.0712250	1.0816000	1.0920250	1.1025000
3	1.0927270	1.087178	1.1248640	1.1411661	1.1576250
4	1.1255088	1.1475230	1.1698586	1.1925186	1.2155063
5	1.1592740	1.1876863	1.2166529	1.2461819	1.2762816
6	1.1940523	1.2292553	1.2653190	1.3022601	1.3400956
7	1.2298738	1.2722792	1.3159318	1.3608618	1.4071004
8	1.2667700	1.3168090	1.3685691	1.4221006	1.4774554
9	1.3047731	1.3628973	1.4233118	1.4860951	1.5513282
10	1.3439163	1.4105987	1.4802443	1.5529694	1.6288946
11	1.3842338	1.4599697	1.5394541	1.6228530	1.7103393
12	1.4257608	1.5110686	1.6010322	1.6958814	1.7958563
13	1.4685337	1.5639560	1.6650735	1.7721961	1.8856491
14	1.5125897	1.6186945	1.7316764	1.8519449	1.9799316
15	1.5579674	1.6753488	1.8009435	1.9352824	2.0789282
16	1.6047064	1.7339860	1.8729812	2.0223701	2.1828746
17	1.6528476	1.7946755	1.9479005	2.1133768	2.2920183
18	1.7024330	1.8574892	2.0258165	2.2084787	2.4066192
19	1.7535060	1.9225013	2.1068492	2.3078603	2.5269502
20	1.8061112	1.9897888	2.1911231	2.4117140	2.6532977
21	1.8602945	2.0594314	2.2787681	2.5202411	2.7859626
22	1.9161034	2.1315115	2.3699188	2.6336520	2.9252607
23	1.9735865	2.2061144	2.4647155	2.7521663	3.0715238
24	2.0327941	2.2833284	2.5633042	2.8760138	3.2251000
25	2.0937779	2.3632449	2.6658363	3.0054344	3.3863549
26	2.1565912	2.4459585	2.7724697	3.1406790	3.5556727
27	2.2212890	2.5315671	2.8833685	3.2820095	3.7334563
28	2.2879276	2.6201719	2.9987033	3.4296999	3.9201291
29	2.3565655	2.7118779	3.1186514	3.5840364	4.1161356
30	2.4272624	2.8067937	3.2433975	3.7453181	4.3219424
31	2.5000803	2.9050314	3.3731334	3.9138574	4.5380395
32	2.5750827	3.0067075	3.5080587	4.0899810	4.7649415
33	2.6523352	3.1119423	3.6483811	4.2740301	5.0031885
34	2.7319053	3.2208603	3.7943163	4.4663615	5.2533480
35	2.8138624	3.3335904	3.9460889	4.6673478	5.5160154
36	2.8982783	3.4502661	4.1039325	4.8773784	5.7918161
37	2.9852266	3.5710254	4.2680898	5.0968604	6.0814069
38	3.0747834	3.6960113	4.4388134	5.3262192	6.3854773
39	3.1670269	3.8253717	4.6163659	5.5658990	6.7047511
40	3.2620377	3.9592597	4.8010206	5.8163645	7.0399887

Decimal Tables of Compound Interest.

TABLE II.

The amount of one pound for days.

days	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
1	1.0000809	1.0000942	1.0001074	1.0001206	1.0001336
2	1.0001619	1.0001885	1.0002149	1.0002412	1.0002673
3	1.0002429	1.0002827	1.0003224	1.0003618	1.0004011
4	1.0003240	1.0003770	1.0004299	1.0004824	1.0005348
5	1.0004050	1.0004713	1.0005374	1.0006031	1.0006685
6	1.0004860	1.0005656	1.0006449	1.0007238	1.0008023
7	1.0005670	1.0006600	1.0007524	1.0008445	1.0009361
8	1.0006480	1.0007542	1.0008600	1.0009652	1.0010699
9	1.0007291	1.0008486	1.0009675	1.0010859	1.0012037
10	1.0008101	1.0009429	1.0010751	1.0012066	1.0013376
20	1.0016209	1.0018867	1.0021513	1.0024148	1.0026770
30	1.0024324	1.0028315	1.0032288	1.0036243	1.0040182
40	1.0032445	1.0037771	1.0043074	1.0048354	1.0053611
50	1.0040573	1.0047236	1.0053871	1.0060479	1.0067059
60	1.0048708	1.0056710	1.0064680	1.0072618	1.0080525
70	1.0056849	1.0066193	1.0075501	1.0084773	1.0094009
80	1.0064996	1.0075685	1.0086333	1.0096942	1.0107511
90	1.0073151	1.0085186	1.0097177	1.0109125	1.0121031
100	1.0081311	1.0094696	1.0108033	1.0121324	1.0144569
110	1.0089479	1.0104214	1.0118900	1.0133537	1.0148125
120	1.0097653	1.0113742	1.0129779	1.0145765	1.0161699
130	1.0105834	1.0123279	1.0140670	1.0158007	1.0175291
140	1.0114021	1.0132825	1.0151572	1.0170265	1.0188902
150	1.0122215	1.0142379	1.0162487	1.0182537	1.0202531
160	1.0130415	1.0151943	1.0173412	1.0194824	1.0216178
170	1.0138623	1.0161516	1.0184350	1.0207126	1.0229843
180	1.0146837	1.0171098	1.0195299	1.0219442	1.0243527
190	1.0155057	1.0180689	1.0206261	1.0231774	1.0257228
200	1.0163284	1.0190288	1.0217233	1.0244120	1.0270949
210	1.0171518	1.0199897	1.0228218	1.025[illegible]81	1.0284687
220	1.0179759	1.0209515	1.0239215	1.0268858	1.0298444
230	1.0188006	1.0219142	1.0250223	1.0281249	1.0312219
240	1.0196260	1.0228778	1.0261243	1.0293655	1.0326013
250	1.0204520	1.0238424	1.0272275	1.0306076	1.0339825
260	1.0212788	1.0248078	1.0283319	1.0318512	1.0353656
270	1.0221062	1.0257741	1.0294375	1.0330963	1.0367[illegible]05
280	1.0229342	1.0267414	1.0305443	1.0343429	1.0381373
290	1.0237630	1.0277096	1.0316522	1.0355910	1.0395259
300	1.0245924	1.0286786	1.0327614	1.0368406	1.0409164
310	1.0254225	1.0296486	1.0338717	1.0380917	1.0422087
320	1.0262532	1.0306195	1.0349832	1.0393444	1.043[illegible]029
330	1.0270847	1.0315914	1.0365960	1.040[illegible]985	1.0459900
340	1.0279168	1.032[illegible]641	1.0372099	1.0418542	1.0464969
350	1.0287495	1.03353[illegible]8	1.038325[illegible]	1.0431114	1.047896[illegible]
360	1.0295830	1.334512[illegible]	1.0394413	1.0443700	1.0492984

The uſe of the foregoing tables.

CASE I.

Principal, rate, and time given, to find the amount.

RULE.

Multiply the amount of 1 l. found in the firſt table, at the rate and for the time given, by the propoſed principal, and the product gives the anſwer.

5. What will 721 l. amount to in 21 years, at four per cent. per annum?

The tabular number againſt 21 years at four per cent. 2.2787681. Then $721 \times 2.2787681 = 1642.9918 =$ 1642 l. 19 s. 10 d. the amount required.

6. What will 358 l. amount to in 40 days, at five per cent. per annum, compound intereſt?

In the ſecond table againſt 40 days, at five per cent. is 1.0053611.

$\therefore 358 \times 1.0053611 = 359.9192738 =$ 359 l. 18 s. $4\frac{1}{2}$d. the anſwer.

If the amount be required for any number of years exceeding thoſe in the table, divide the given number of years into two or more ſuch numbers as are in the table, and multiply the amounts anſwering thereto into one another continually, and the laſt product by the principal, which will be the amount required.

7. What is the amount of 82 l. 10 s. for 75 years, at five per cent. per annum, compound intereſt?

First, $40 + 35 = 75$.

The amount of 1 l. for 40 years at 5 per cent. is 7.0399887.

Ditto for 35 years. - - - - 5.5160154.

Then $7.0399887 \times 5.5160154 = 38.8326861$.

Alſo $82.5 \times 38.8326861 = 3203.6966$.

Anſwer, 3203 l. 13 s. 11 d.

If the amount be required for any number of days which are not in the tables, proceed as with the years in the laſt example.

 8. What

8. What is the amount of 523l. in 275 days, at four per cent. compound intereſt?

The amount of 1 l. for 270 days at 4 per cent. is 1.0330963.
Ditto for 5 days, - - - 1.0006031.
Then 1.0330963 × 1.0006031 = 1.0337194.
And 1.0337194 × 523 = 540.6352462.
Anſwer, 540 l. 12s. 8¼d.

To find the amount for years and days, obſerve the following example.

9. What will 357 l. 15 s. amount to in four years and 274 days, at 3½ per cent. per annum, compound intereſt?

The amt. of 1l. for 4 years, at 3½ per cent. is 1.1475230.
Ditto for 270 days, - - - - - 1.0257741.
Ditto for 4 days, - - - - - 1.0003770.
Then 1.147523 × 1.0257741 × 1.0003770 = 1.1775431.
And 357.75 × 1.1775431 = 421.266044 = 421 l. 5 s. 3¾d. the anſwer.

CASE II.

Amount, rate, and time given; to find the principal.

RULE.

Divide the amount given by the amount of 1l. found in the firſt table, and the quotient will be the anſwer.

10. What is the preſent worth of 1642l. 19s. 10¼d. due 21 years hence, at 4 per cent. per annum, compound intereſt?

The amt. of 1l. in 21 years, at 4 per cent. is 2.2787681.
Then 2.2787681) 1642.9918 (721, the anſwer.

11. What is the preſent worth of 3203l. 13s. 11d due 75 years hence, at 5 per cent. compound intereſt?

The amt of 1l. for 40 years, at 5 per cent. is 7.0399887
Ditto for 35 years, - - - - 5.5160154
Then 7.0399887 × 5.5160154 = 38.8326864.
∴ 38.8326864) 3203.6966 (82.5 = 82l. 10s. the anſwer.

12. What is the preſent worth of 421 l. 5 s. 4d. due four years and 274 days hence, at 3½ per cent. per annum

B?

By TABLE first and second.

The amount of 1 l. for four years, is - - 1.147523.
Ditto for 270 days, - - - 1.0257741.
Ditto for four days, - - - 1.000377.
Then 1.147523 × 1.0257741 × 1.000377 = 1.1775431.
1.1775431) 421.2666 (357.75 = 357 l. 15 s. the anſwer.

CASE III.

Any principal, rate, and amount being given, to find the time.

RULE.

Divide the amount by the principal, and the quotient will be the amount of 1 l. at the given rate, which will be found in the firſt table under that rate, even with the time required.

13. In what time will 721 l. amount to 1642 l. 19s. 10d. at 4 per cent. per annum, compound intereſt?

721) 1642.9918 (2.278768, the amount of 1 l. for the time; oppoſite to which, under 4 per cent. in the ſecond table, is 21 years, the anſwer required.

But if the quotient cannot be truly found in the table, take out the next number, and make it a diviſor, by which divide the firſt quotient, and ſeek the ſecond quotient in table the ſecond; but if it cannot be truly found in that table, take out the next leaſt number there, and divide the ſecond quotient by it, and then ſeek again for the third quotient, and the number thus found in the table is the number of days.

14. In what time will 357 l. 15 s. amount to 421 l. 5 s. 4 d. at $3\frac{1}{2}$ per cent. compound intereſt?

357.75) 421.26 (1.177543, the number next to which, under $3\frac{1}{2}$ per cent. ſtands againſt four years, and is 1.147523.

Then 1.147523) 1.177543 (1.0261608, the next leſs number to which, under $3\frac{1}{2}$ per cent. ſtands againſt 270 days, and is as follows:

viz.

viz. 1.025741) 1.0261608 (1.00377 ſtands againſt four days.
Anſwer, four years and 274 days.

CASE IV.

Principal, time, and amount given, to find the rate of intereſt.

RULE.

Divide the amount by the principal, and the quotient will be the amount of 1 l. which being found in the firſt table, even with the given time, is under the rate required.

15. At what rate per cent. per annum will 721 l. become 1642 l. 19 s. 10 d. in 21 years.

721) 1642.9918 (2.278768, the amount of 1 l. for 21 years, which will be found under 4 per cent. the anſwer to the queſtion.

✱ ✱

SECT. II.

Purchaſing FREEHOLD *or* REAL ESTATES *at* COMPOUND INTEREST.

FReehold, or real eſtates, are ſuch as are purchaſed to continue for ever; queſtions relating to which (except in reverſion) are ſolved in the moſt eaſy manner only by the rule of three.

CASE I.

When the yearly income is required.

RULE.

As 100l. : is to the propoſed rate per cent. : : ſo is the ſum to be laid out : to the yearly income.

1. A perſon deſirous to lay out 1760 l. in the purchaſe of a freehold eſtate, ſo as to get $4\frac{1}{2}$ per cent. for the money, compound intereſt, what muſt be the annual income of ſuch an eſtate?

$$
\begin{array}{r}
100 : 4.5 : : 1760 \\
4.5 \\
\hline
880 \\
704 \\
\hline
\end{array}
$$

100) 7920.0 (79.2 = 79 l. 4 s. the anſwer.

CASE

CASE II.

If the value of the eſtate is required.

RULE.

As the rate per cent. : is to 100 l. : : ſo is the yearly rent : to the value required.

2. An eſtate brings in yearly 79l. 4s. what would it ſell for, allowing the purchaſer $4\frac{1}{2}$ per cent. for his money?

4.5 : 100 : : 79.2 : 1760l. the anſwer.

CASE III.

To find the rate per cent. on money laid out on the purchaſe of freehold eſtates.

RULE.

As the money laid out on the purchaſe : is to the yearly rent, : : ſo is 100 l. : to the rate per cent.

3. Suppoſe 1760 l. to be paid for a freehold eſtate, which yields yearly 79l. 4s. what rate of intereſt hath the purchaſer for his money?

1760 : 79.2 : : 100 : 4.5 = $4\frac{1}{2}$ per cent. anſwer.

4. Suppoſe an eſtate of 79 l. 4s. per annum be ſold at $22\frac{2}{9}$ years purchaſe; how much per cent, hath the purchaſer for his money?

22.$\dot{2}$ x 79.2 = 1760 : 79.2 : : 100 : 4.5 per cent.

SECT. III.

Purchaſing FREEHOLD ESTATES *in* REVERSION.

CASE I.

THE yearly rent of a freehold eſtate being known, to find the preſent worth of the reverſion of the ſaid eſtate, after the expiration of a certain number of years.

RULE.

RULE.

Find the full value of the estate by the second case of the last section. Then, by case the second of compound interest, find what principal or sum will amount to the full value of the estate at the time and rate given.

Suppose the reversion of a freehold estate 79 l. 4 s. per annum, to commence seven years hence, is to be sold; what is it worth in ready money, allowing the purchaser $4\frac{1}{2}$ per cent. for his money?

First as 4.5 : 100 : : 79.2 : 1760.

By the first table, against seven years, the amount of 1 l. is found 1.3608618.

∵ 1.3608618 : 1 : : 1760 : 1293.2981147.

Answer, 1293 l. 5 s. $11\frac{1}{2}$d. nearly.

CASE II.

The sum given for the reversion of a freehold estate, to commence after a certain number of years, being known, to find the yearly income, allowing the purchaser so much per cent. for his money.

RULE.

Find the amount of the purchase money to the time when the reversion is to commence, by the first case of compound interest, then find the yearly income which that amount will purchase.

Suppose the reversion of a freehold estate to commence seven years hence, is sold for 1293 l. 5 s. $11\frac{1}{2}$d. allowing the purchaser $4\frac{1}{2}$ per cent. compound interest for his money; what ought the yearly rent to be?

The amount of 1 l. for seven years, at $4\frac{1}{2}$ per cent. is 1.3608618.

Then 1293.2981147 × 1.3608618 = 1760 l. amount.

And 100 : 4.5 : : 1760 : 79.2.

Answer, 79 l. 4 s. per annum.

SECT.

SECT. IV.

PURCHASING ANNUITIES.

ANnuities, penſions, ſalaries, &c. as rents, profits, and payments, made yearly or half yearly, &c. and they are ſaid to be in areas when they are due and unpaid for any number of payments.

In order to ſolve queſtions in annuities, I have inſerted four tables more of compound intereſt, the conſtruction whereof follow.

Conſtruction of the firſt TABLE *of* ANNUITIES.

This table ſhews the preſent worth or value of 1 l. payable at any period, from one to forty years incluſive, and is conſtructed, by dividing 1 l. by its amount found in the ſecond table of compound intereſt for the time and rate aſſigned.

Againſt the firſt table of compound intereſt, the preſent worth of 1 l. for three years, at 3 per cent. is 1.092727.

1.092727) 1.0000000 (.9151417, for the preſent worth of 1 l. three years hence, compound intereſt, at three per cent.

Conſtruction of the ſecond TABLE.

This table ſhews the amount of 1 l. per annum, and is conſtructed from the firſt table of compound intereſt, thus:

To 1 l. the firſt year of this table, and the firſt year of the table for years in compound intereſt, and the amount will be the ſecond year in this table; to which add the ſecond year in the table of compound intereſt, and the amount of it will be the third year in this table, &c.

Thus 1.000000
add 1.030000 the amt. of 1 l. for 1 year, at 3 per cent.

2.030000 the amount 1 l. for 2 years.
1.060930 amount of the ſecond year.

3.090900 third year of the ſecond table.

Conſtruction

Construction of the third TABLE.

The third table ſhews the preſent value of 1 l. per annum, and is conſtructed as follows, viz. the preſent value of the firſt year in the firſt table, is the ſame as the firſt year in the third table; the firſt and ſecond years in the firſt table, added together, make the ſecond year in the third table; and the third year in the firſt table, added to the ſecond year in the third table, make the third year in the third.

Thus, 1ſt year, tables 1ſt and 3d, at 3 per cent. is	.9708738
The 2d year, in table 1ſt. - - - - - - - -	.9425959
Their ſum, 2d year in the third table, is - -	1.9134697
Third year, in the firſt table, - - - - - - -	.9151417
Third year, in the third table, - - - - - - -	2.8286114

Construction of the fourth TABLE.

This table ſhews what annuity 1 l. will purchaſe &c. and is conſtructed, by finding the preſent worth of 1 l. per annum in the third table at the aſſigned rate and time, and dividing unity thereby, and the quotient will be the annuity that 1 l. will purchaſe at the ſame rate for the ſame time.

EXAMPLE. What annuity will 1 l. purchaſe, to continue three years, at 3 per cent?

In the third table, under 3 per cent. oppoſite to three years, is 2.8286114.

2.8286114) 1.00000000 (.3535304, the annuity for three years.

DECIMAL

DECIMAL TABLES *of* COMPOUND INTEREST.

TABLE I.

The present worth of one pound for years.

yrs	3 per Cent.	3 ¼ per Cent	4 per cent.	4 ½ per cent.	5 per cent.
1	.9738738	.9661836	.9615385	.9569378	.9523809
2	.9425959	9335107	.9245562	.9157299	.9070295
3	.9151417	.9019427	.8889964	.8762966	.8638376
4	.8884870	.8714422	.8548042	.8385613	.8227025
5	.8626088	.8419732	.8219271	.8024511	.7835262
6	.8374843	.8135006	.7903145	.7678957	.7462154
7	.8130915	7859910	.7599178	.7348285	.7106813
8	.7894092	.7594116	.7306902	.7031851	.6768394
9	.7664167	7337310	.7025867	.6729044	.6446089
10	.7440939	.7089188	6755642	.6439277	.6139133
11	7224213	.6849457	.6495809	.6161988	5846793
12	7013799	.6617833	.6245971	.5896639	.5568374
13	.6809513	.6394041	.6005741	.5642716	5303214
14	6611178	.6177818	.5774751	.5399729	.5050679
15	.6418619	5968906	.5552645	.5167204	.4810171
16	.6231[illegible]6[illegible]	.5767059	5339082	4944693	.4581115
17	.6050164	.[illegible]572038	5133733	.4731764	.4362967
18	.5[illegible]73946	5383611	.4936281	.4528004	.4155207
19	.5702860	.5201557	4746424	4433018	3957340
20	.5536758	.5025059	4563870	4146429	3768895
21	.5375493	.5855709	.4388336	.3967874	.3589424
22	.5218925	.4691506	.4219554	.3797009	.3418499
23	.5066917	.4532856	.4057263	36335[illegible]1	.3255713
24	.4919337	.4379571	.3901215	3477035	.3100679
25	.4770056	.4231470	.3751168	.3327306	.2953028
26	.4636947	.4088378	360689[illegible]	3184025	.2812407
27	.4501891	.3950123	3468166	3046914	2.678483
28	.4370768	.3816543	.3334775	.2915707	.25[illegible]0936
29	.4243464	.3687482	.3206514	.2790150	.2429463
30	.4119868	.3562784	.3083187	.2670000	.2313775
31	.3999871	.3442304	.2964603	2555024	.2203595
32	.3883370	.3325897	.2850579	2444999	.2098662
33	3770263	.3213427	2740942	.2339712	.1998726
34	.3660449	.3104761	.2635521	.2238959	.1903548
35	.3553834	.2999765	.2534155	.2142544	.1812903
36	.3450324	.2898327	.2436687	2050282	.1726574
37	.3349829	.2800316	.2342969	.1961992	.1644356
38	.3252262	.2705619	.2252854	1877504	.1566054
39	.3157536	.2614125	.2166206	.1796655	.1491479
40	.3065568	.2525725	.2082890	.1719287	.1420457

DECIMAL TABLES of COMPOUND INTERSET.

TABLE II.

The amount of one pound per annum, or annuities for years.

Yrs	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
2	2.0300000	2.0350000	2.0400000	2.0450000	2.0500000
3	3.0909000	3.1062250	3.1216000	3.1370250	3.1525000
4	4.1836270	4.2149429	4.2464640	4.2781911	4.3101250
5	5.3091358	5.3624659	5.4163226	5.4707097	5.5256312
6	6.4684099	6.5501522	6.6329755	6.7168917	6.8019128
7	7.6624622	7.7794075	7.8982945	8.0191581	8.1420084
8	8.8923360	9.0516866	9.2142263	9.3800136	9.5491089
9	10.1591061	10.3684958	10.5827953	10.8021142	11.0265643
10	11.4638793	11.7313931	12.0061071	12.2882094	12.5778925
11	12.8077957	13.1419919	13.4863514	13.8411788	14.2067871
12	14.1920296	14.6019616	15.0258055	15.4640318	15.9171265
13	15.6177904	16.1130303	16.6268377	17.1599133	17.7129828
14	17.0863242	17.6769864	18.2919112	18.9321094	19.5986320
15	18.5989139	19.2956809	20.0235876	20.7840543	21.5785636
16	20.1568813	20.9710297	21.8245311	22.7193367	23.6574918
17	21.7615877	22.7050158	23.6975124	24.7417069	25.8403664
18	23.4144354	24.4996913	24.6454129	26.8550837	28.1323847
19	25.1168684	26.3571805	27.6712294	29.0635625	30.5390039
20	26.8703745	28.2796818	29.7780786	31.3714228	33.0659541
21	28.6764857	30.2694707	31.9692017	33.7831368	35.7192518
22	30.5367803	32.3289022	34.2479698	36.3033779	38.5052144
23	32.4528837	34.4604137	36.6178886	38.9370299	41.4304751
24	34.4264702	36.6665282	39.0826041	41.6891963	44.5019989
25	36.4592643	38.9498567	41.6459083	44.5652101	47.7270988
26	38.5530422	41.3131017	44.3117446	47.5706446	51.1134538
27	40.7096335	43.7590602	47.0842144	50.7113236	54.6691265
28	42.9309225	45.2906273	49.9675830	53.9933332	58.4025828
29	45.2188502	48.9107993	52.9662863	57.4230332	62.3227119
30	47.5754157	51.622[illegible]773	56.0849377	61.0070697	66.4388475
31	50.0026782	54.4294710	59.3283352	64.7523878	70.7607899
32	52.5027585	57.3345025	62.7014687	68.6662452	75.2988294
33	55.0778413	60.3412101	66.2095274	72.7562263	80.0637708
34	57.7301765	63.4531524	69.8579085	77.0302565	85.0669594
35	60.4620818	66.6740127	73.6522248	81.4966180	90.3203073
36	63.2759443	70.0076032	77.5983138	86.1639658	95.8363227
37	66.1742226	73.4578693	81.7022464	91.0413443	101.6281388
38	69.1594493	77.0288947	85.9703362	96.1382048	107.7095458
39	72.2342327	80.7249060	90.4091497	101.4644240	114.0950231
40	75.4012597	84.5502778	95.0255157	107.0303231	120.7997742

DECIMAL TABLES of COMPOUND INTEREST.

TABLE III.

The present worth of one pound per annum, or annuity for years.

ys.	3 per Cent.	3½ per Cent	4 per Cent.	4½ per Cent.	5 per Cent.
1	0.9708738	0.9661836	0.9615385	0.9569378	0.9523809
2	1.9134697	1.8996943	1.8860947	1.8726678	1.8594104
3	2.8286114	2.8016370	2.7750910	2.7489644	2.7232480
4	3.7170984	3.6730792	3.6298952	3.5875257	3.5459505
5	4.5797072	4.5150524	4.4518223	4.3899767	4.3294767
6	5.4171914	5.3285530	5.2421369	5.1578725	5.0756921
7	6.2302829	6.1145439	6.0020547	5.8927009	5.7863734
8	7.0196922	6.8739555	6.7327448	6.5958861	6.4632128
9	7.7861089	7.6076865	7.4353314	7.2687905	7.1078217
10	8.5302028	8.316605[illegible]	8.1108955	7.9127182	7.7217349
11	9.2526241	9.0015510	8.7604763	8.5289169	8.3064142
12	9.9540040	9.6633343	9.3850733	9.1185808	8.8632516
13	10.6349553	10.3027385	9.9856473	9.6828524	9.3935730
14	11.2960731	10.9205203	10.5631223	10.2228253	9.8986409
15	11.9379351	11.5174109	11.1183868	10.7395457	10.3796580
16	12.5611020	12.0941168	11.6522949	11.2340151	10.8377695
17	13.1661185	12.6513206	12.1656680	11.7071914	11.2740622
18	13.7535131	13.1896817	12.6592961	12.1599918	11.6895869
19	14.3237991	13.7098374	13.1339385	12.5932936	12.0853208
20	14.8774748	14.2124033	13.5903253	13.0079365	12.4622103
21	15.4150241	14.6979742	14.0291589	13.4047239	12.8211527
22	15.9369166	15.1671248	14.4511142	13.7844248	13.1630026
23	16.4436084	15.6204105	14.8568405	14.1477749	13.4885739
24	16.9355421	16.0583676	15.2469619	14.4954784	13.7986418
25	17.4131477	16.4815146	15.6220787	14.8282089	14.0939445
26	17.8768424	16.8903523	15.9827678	15.1466115	14.3751853
27	18.3270315	17.2853645	16.3295844	15.4513028	14.6430336
28	18.7641082	17.6670188	16.6630618	15.7428735	14.8981272
29	19.1884546	18.0357670	16.9837132	16.0218885	15.1410735
30	19.6004413	18.3920454	17.2920318	16.2888885	15.3724510
31	20.0004285	18.7362758	17.5884921	16.5443909	15.5928104
32	20.3887655	19.0688656	17.8735500	16.7888900	15.8026766
33	20.7657918	19.3902082	18.1476441	17.0228621	16.0025491
34	21.1318367	19.7006842	18.4111962	17.2467580	16.1929039
35	21.4872200	20.0006512	18.6646116	17.461012[illegible]	16.374194[illegible]
36	21.8322525	20.2904938	18.9082803	17.6660400	16.5468516
37	22.1672354	20.5705254	19.1425771	17.8622398	16.7112872
38	22.4924616	20.8410874	19.367862[illegible]	18.04309[illegible]	16.[illegible]678926
39	22.8082151	21.1024939	19.58448[illegible]	18.22965[illegible]	17.0170406
40	23.1147719	21.3550723	19.79277[illegible]	18.4015844	[illegible].159086[illegible]

DE-

Decimal Tables of Compound Interest.

TABLE IV.

The annuity which one pound will purchase for any number of years.

yrs	3 per Cent.	$3\frac{1}{2}$ per Cent	4 per Cent	$4\frac{1}{2}$ per Cent.	5 per Cent.
1	1.0300000	1.0350000	1.0400000	1.0450000	1.0500000
2	.5226108	.5264005	.5301961	.5339976	.5378049
3	.3535304	.3569342	.3603485	.3637734	.3672086
4	.2690271	.2722511	.2754901	.2787437	.2820118
5	.2183546	.2214814	.2246271	.2277916	.2309748
6	.1845975	.1876682	.1907619	.1938784	.1970175
7	.1605064	.1635445	.1666096	.1697015	.1728198
8	.1424564	.1454767	.1485279	.1516097	.1547218
9	.1284339	.1314460	.1344930	.1375745	.1406901
10	.1172305	.1202414	.1232909	.1263788	.1295046
11	.1080775	.1110920	.1141490	.1172482	.1203889
12	.1004621	.1034840	.1065522	.1096662	.1128254
13	.0940295	.0970616	.1001437	.1032754	.1064558
14	.0885263	.0915707	.0946690	.0978203	.1010240
15	.0837666	.0868251	.0899411	.0931138	.0963423
16	.0796109	.0826848	.0858200	.0890154	.0922699
17	.0759525	.0790431	.0821985	.0854176	.0886991
18	.0727087	.0758168	.0789933	.0822369	.0855462
19	.0698139	.0729403	.0761386	.0794073	.0827450
20	.0672157	.0703611	.0735818	.0768761	.0802426
21	.0648718	.0680366	.0712801	.0746006	.0779961
22	.0627474	.0659321	.0691988	.0725457	.0759705
23	.0608139	.0640188	.0673091	.0706825	.0741368
24	.0590474	.0622728	.0655868	.0689870	.0724709
25	.0574279	.0606740	.0640120	.0674390	.0709525
26	.0559383	.0592054	.0625674	.0660214	.0695643
27	.0545642	.0578524	.0612385	.0647195	.0682919
28	.0532932	.0566027	.0600130	.0635208	.0671225
29	.0521147	.0554454	.0588799	.0624146	.0660455
30	.0510193	.0543713	.0578301	0613915	.0650514
31	.0499989	.0533724	.0568554	.0604435	.0641321
32	.0490466	.0524415	.0559486	.0595632	.0632804
33	.0481561	.0515724	.0551036	.0587445	.0624900
34	.0473220	.0507597	.0543148	.0579819	.0617554
35	.0465393	.0499984	.0535773	.0572705	.0610717
36	.0458038	.0492842	.0528869	.0566058	.0604345
37	.0451116	.0486133	.0522396	.0559840	.0558398
38	.0444593	.0479821	.0516319	.0554017	.0592842
39	.0438439	.0473878	.0510608	.0548557	.0587646
40	.0432624	.0468273	.0505235	.0543431	.0582782

CASE I.

Principal, rate, and time being given, to find the annuity.

RULE.

Multiply the annuity which 1 l. will purchase, at the rate and for the time given (found in the fourth table) and the quotient will be the answer.

1. A gentleman hath 1760l. which he would sell for an annuity, to continue 21 years, at 5 per cent. compound interest; I demand what will be his income per annum?

$1760\text{l.} \times .0779961 = 137.273136.$
Answer, 137 l. 5s. $5\frac{1}{2}$d.

2. A fine for the lease of a tenement is settled at 153 l. under a reserved rent of 16 l. a year: now the tenant cannot conveniently pay more than 50 l. but for twelve years to come of the term is willing rather to pay an adequate rent, computing 5 per cent. compound interest; what ought that rent to be?

First, $153 - 50 = 103$.

Then, by the fourth table, 1 l. will purchase for 12 years, at 5 per cent. an annuity of .1128254 per annum.

Then $103 \times .1128254 = 11.6210162 =$ 11 l. 12 s. 5d. advance rent.

$\therefore$ 16l. + 11l. 12s. 5d. = 27l. 12s. 5d. Q. E. F.

3. A son, previous to his marriage, is minded to have 50 l. a-year, freehold estate, settled on his family; and to have immediate possession of it, offers his father in lieu, an annuity for his life, valued at twelve years purchase, discounting 4 per cent. thereon; whereas he is content the estate should be valued at a discount of 3 per cent. which is $33\frac{1}{3}$ years purchase; pray what had the father for his life?

First, $33.\dot{3} \times 50 = 1666.\dot{6} =$ 1666 l. 13 s. 4 d. value of the annuity.

Then 1 l. by the fourth table, for 12 years, at 4 per cent. will purchase .1065522 per annum.

$\therefore$ $1666.\dot{6} \times .1065522 = 177.58699 =$ 177 l. 11 s. $8\frac{3}{4}$d.

CASE II.

Principal, annuity, and rate given, to find the time.

RULE.

Divide the annuity by the principal, and the quotient will be the annuity which 1 l. will purchase at the given rate, which will be found in the fifth table under that rate, and even with the time required.

4. If an annuity of 137 l. 5s $5\frac{1}{2}$d. is purchased for 1760 l. at 5 per cent. compound interest, what time ought it to continue?

1760) 137.273136 (.0779961;

which under 5 per cent. in the 4th table, is opposite to 21 years.

CASE III.

Principal, annuity, and time given, to find the rate.

RULE.

Divide the annuity by the principal, and the quotient will be the annuity which 1 l. will purchase for the given time, which will stand even with the time, and under the rate required.

5. If an annuity of 80 l. 4s. $10\frac{1}{4}$d. to continue 20 years, be purchased for 1000 l. what rate of interest hath the purchaser for his money?

1000) 80.2425 (.0802425, under 5 per cent. which is the answer.

CASE IV.

Annuity, rate, and time given, to find the amount.

RULE.

Find the amount of 1 l. per annum, at the rate for the time given, by the first table; by which multiply the annuity, and the product will be the amount required.

6. A minor of 14 had an annuity left him of 70 l. a-year, the proceed of which, by will, was to be put out, both principal and interest, yearly, as it fell due, at 5 per cent. till he

he should attain to 21 years of age; the utmost improvement being made of this part of his fortune, what had he then to receive?

The amount of 1 l. annuity, at 5 per cent. forborn seven years, by the second table, is 8.1420084.

Then 8.1420084 × 70 = 569.940588.

Answer, 569 l. 18 s. 9¾d.

CASE V.

Annuity, rate, and amount, being given, to find the time.

RULE.

Divide the amount by the annuity, and the quotient will be the amount of 1 l. at the given rate, which will be found in the second table, under that rate, even with the time required.

7. In what time will an annuity of 70 l. amount to 569 l. 18 s. 9¾d. compound interest, at 5 per cent.?

70) 569.940588 (8.1420084, even with seven years in the second table, under 5 per cent.

CASE VI.

Annuity, time, and amount given, to find the rate.

RULE

Divide the amount by the annuity, the quotient will be the amount of 1 l. per annum, for the given rate; which will be found in the second table, below the required rate.

8. At what rate per cent. per annum will an annuity of 137 l. 5 s. 5½d. amount to 1760 l. in 21 years.

137.273136) 1760.000000 (12.8211523, in the second table, below 5 per cent.

CASE VII.

Amount, rate, and time being given, to find the annuity.

RULE.

Divide the amount given, by the amount of 1 l. found in the second table, at the rate and time given, the quotient will be the annuity required.

9. What

9. What annuity will amount to 569l. 18s. 9¼d. in ſeven years, at 5 per cent.?

8.1420084) 569.940588 (70l. the annuity required.

CASE VIII.

Annuity, time in reverſion, and rate being given, to find the preſent worth.

RULE.

In the third table find the preſent value of 1l. per annum, at the given rate, both for the time being, and alſo for that and the time in reverſion added together, then ſubtract the time in being from the other, and multiply the remainder by the annuity, the product will anſwer the queſtion.

10. What ought a man to give down in ready money, for the reverſion of 1000l. a year, to continue 20 years on a leaſe which cannot commence till five years are at an end, allowing the purchaſer compound intereſt at 5 per cent.?

The preſent value of 1l. per annum, by table 3d, for 25 years, 14.0939445
for five years, 4.3294767

9.7644678 × 1000 = 9764.4678.
Anſwer, 9764l. 9s. 4¼d.

11. Suppoſe I would add five years to a running leaſe of 15 years to come, the improved rent being 186l. 7s. 6d. per annum; what ought I to pay down for this favour, diſcounting 4 per cent. compound intereſt?

Firſt, 15 + 5 = 20 years, 1l. is worth -	13.5903253
Alſo 15 years is worth - - - - - - -	11.1183868
	2.4719385

Then 186.375 × 2.4719385 = 460.70753.
∴ 460l. 14s. 1¾d. the fine required.

12. Held of a college 486l. 10s. a year, on a reſerved rent of 94l. money being at 5 per cent. intereſt; what fine ought ſeverally to be paid on a 7, a 14, and a 21 years leaſe?

486l. 10s. — 94 = 392l. 10s. annuity.
The preſent worth of 1l. for the time and rate is 5.7863734.
Then

Then 392.5 × 5.7863734 = 2271.15057.
∴ 2271 l. 3s. its worth for seven years.

Also the present worth of 1 l. for 14 years, at 5 per cent. is 9.8986409.

Again, 392.5 × 9.8986409 = 3885.21655.
∴ 3885 l. 4s. 4d. its worth for 14 years.

The present worth of 1l. for 21 ys. at 5 per ct. is 12.8211527.

Also 392.5. × 12.8211527 = 5032.30243.
Answer, 5032 l. 6s. for 21 years.

CASE IX.

An annuity, several times in reversion, and rate given, to find the present value.

RULE.

In the third table find the present value of 1 l. per annum, at the given rate, for the several given times, which being severally multiplied by the annuity, the products will be the several present values of that annuity for the several times given: then subtract the several present values one from another, and the several remainders answer the question.

13. A has a term of seven years in an estate of 50 l. per annum; B hath a term of 14 years in the same estate; and C hath a further term of 10 years after B in the same estate; what is the present value of their several interests in the said estate?

First, 7 + 14 + 10 = 31.

The present worth of 1 l. at 5 per ct. for 31 ys, is 15.5928104
For 21 years - - - - - - - - - - - 12.8211527
And 7 years - - - - - - - - - - - 5.7863734

			l.	s.	d.
Then	50 × 15.5928104 = 779.64052	=	779	12	9½
Also	50 × 12.8211527 = 641.057635	=	641	1	1¾
And	50 × 5.7863734 = 289.31867	=	289	6	4¼

	l.	s.	d.		
∴	289	6	4½	value of	A's term.
Also 641 1 1¾ − 289 6 4½ =	351	14	9¼	value of	B's term.
And 779 12 9½ − 641 1 1¾ =	138	11	7¾	value of	C's term.

14. Which is most advantageous, a term of 19 years of an estate of 100 l. per annum, or the reversion of such an

estate for ever, at the expiration of the said 19 years, computing at the rate of 4 per cent. compound interest?

First, 4 : 100 :: 100 : 2500 l, value of the estate for ever.

		l.	s.	d.
And by the third table, the present worth of 100l. annuity for 19 years, at 4 per cent. is	1313.3938 =	1313	7	$10\frac{1}{2}$
Value of the reversion is - -	1186.6062 =	1186	12	$1\frac{1}{2}$
The first 19 years better than the reversion by		£ 126	15	9

15 For a lease of certain profits for seven years, A offers to pay 150 l. gratuity, and 300 l. per annum; B offers 400 l. gratuity, and 250 l. per annum; C bids 650 l. gratuity, and 200 l. per annum; and D offers 1800 l. for the whole purchase, without any yearly rent: query, which is the best offer, and what the difference, computing at 4 per cent.?

By the third table, the present worth of 300l. per annum, for seven years, at 4 per cent. viz. 6.002547 × 300 is - - - - - } 1800.61641.

	l.	s.	d.	
Then 1800.61641 + 150 =	1950	12	4,	val. of A's offer.
Also 6.0020547×250+400 =	1900	10	$3\frac{1}{4}$,	val. of B's offer.
Again, 6.0020547×200+650 =	1850	8	$2\frac{1}{2}$,	val. of C's offer.
	1800			value of D's offer.

Hence it appears, that A's offer is better by above { 50 than B's; 100 than C's; 150 than D's }

SECT. V.

The VALUATION of ANNUITIES upon LIVES.

THE value of an annuity for life, depends not only on the interest that money bears, but also on the probability of the continuance of life, as it is evident that there must be a great difference in the value of an annuity for the life of a man of 20, and a like annuity for the life of a man of 60.

The

The late Mr. Demoivre and Mr. Simpson have both handled this subject in a very skilful manner; from the latter of which I have extracted the following tables and problems, whereby an annuity on any life or lives may be valued according to the probability of the continuation thereof.

The Use of the Table of Lives.

If it was required the probability that a person of 36 lives 30 years longer:

Look in the table against 36 years, and opposite thereto is the number 331.

Also against 66 is the number 93, which shews, that out of 331 persons living of 36, only 93 arrived at 66.

$\therefore \frac{93}{331}$ is the measure of the probability required.

Let it be required to find the value of an annuity of 100 l. for a life of 20, interest at 4 per cent.

By the second table in the foregoing section, the present worth of 100 l. discount 4 per cent. due at the expiration of one year, was it sure to be paid, is 96.15385.

But the probability of the continuance of the said life one year, by this table, appears to be only $\frac{455}{462}$.

$\therefore 96.15385 \times \frac{455}{462} = 94.697$, the value of the first year's rent.

In like manner the value of the second year's rent may be calculated; the probability of his living two years is $\frac{448}{462} = \frac{32}{33}$; and 92.45562, the present worth of 100 l. at the end of two years.

$\therefore 92.45562 \times \frac{32}{33} = 89\ 65393$, value of the second year's rent.

And by a like way of proceeding, the values of the third, fourth, fifth, &c. years rents, to the utmost extent of life, may be determined; and the sum of all these will be the required value of the annuity; which will be found to come out 1480 l. very near.

A TABLE

A TABLE *ſhewing the Probabilities of* LIFE, *&c.*

Num. of perſons.	Ages curr.	Num. of perſons.	Ages curr.	Num. of perſons.	Ages curr.	Num. of perſons.	Ages curr.
1280	born	462	20	294	40	130	60
— 410		— 7		— 10		— 7	
870	1	455	21	284	41	123	61
— 170		— 7		— 10		— 6	
700	2	448	22	274	42	117	62
— 65		— 7		— 10		— 6	
635	3	441	23	264	43	111	63
— 35		— 7		— 9		— 6	
600	4	434	24	255	44	105	64
— 20		— 8		— 9		— 6	
580	5	426	25	246	45	99	65
— 16		— 8		— 9		— 6	
564	6	418	26	237	46	93	66
— 13		— 8		— 9		— 6	
551	7	410	27	228	47	87	67
— 10		— 8		— 8		— 6	
541	8	402	28	220	48	81	68
— 9		— 8		— 8		— 6	
532	9	394	29	212	49	75	69
— 8		— 9		— 8		— 6	
524	10	385	30	204	50	69	70
— 7		— 9		— 8		— 5	
517	11	376	31	196	51	64	71
— 7		— 9		— 8		— 5	
510	12	367	32	188	52	59	72
— 6		— 9		— 8		— 5	
504	13	358	33	180	53	54	73
— 6		— 9		— 8		— 5	
498	14	349	34	172	54	49	74
— 6		— 9		— 7		— 4	
492	15	340	35	165	55	45	75
— 6		— 9		— 7		— 4	
486	16	331	36	158	56	41	76
— 6		— 9		— 7		— 3	
480	17	322	37	151	57	38	77
— 6		— 9		— 7		— 3	
474	18	313	38	144	58	35	78
— 6		— 9		— 7		— 3	
468	19	304	39	137	59	32	79
— 6		— 10		— 7		— 3	
462	20	294	40	130	60	29	80

N. B. Thoſe marked with the ſign — are ſuppoſed to die off yearly.

PROBLEM I.

To find the value of an annuity for an aſſigned life.

RULE.

Look for the given age in Table I. and againſt it, under the aſſigned rate of intereſt, will ſtand the number of years purchaſe.

1. Suppoſe one of 18 years of age would ſell an annuity of 100l. during his life, what ready money would the annuity be worth, allowing a diſcount of 4 per cent. compound intereſt?

Firſt, oppoſite to 18 years, Table I. under 4 per cent. is 15.2 years purchaſe. ∴ $100 \times 15.2 = 1520$ l. the preſent worth.

2. A widow lady with 200l. a year jointure, aged 30 years, marries a young merchant, who, to enlarge his capital, propoſes to ſell the jointure; what ready money ſhould he receive, diſcounting intereſt at 5 per cent.?

Oppoſite to 30, under 5 per cent. Table I. is 11.6.
∴ $200 \times 11.6 = 2320$ l. the anſwer required.

PROBLEM II.

To find the value of an annuity upon two aſſigned joint lives.

CASE I.

If the lives are equal.

RULE.

Againſt the given age, Table II. under the given rate per cent. will ſtand the number of years purchaſe.

3. Let the two given ages be each 18, and the intereſt 5 l. per cent. and annuity 50l.

Table II. againſt 18, under 5 per cent. is 10.5.
∴ $50 \times 10.5 = 525$ l. the anſwer required.

CASE II.

If the given ages be unqual, but neither of them leſs than 25, or greater than 50.

RULE.

RULE.

Take half the sum of the two for a mean age, and proceed as in Case I.

4. There are two joint lives upon an annuity of 250l. one of 34, the other of 48; what is the present worth of that annuity, compound interest, at 3 per cent.?

First, $\frac{34+48}{2}=41$, half sum of the ages.

Table II. against 41, under 3 per cent. is 8.9 years purchase.

$\therefore$ 250l. $\times 8.9 = 2225$ l. the answer.

CASE III.

If one or both ages be within the limits, but so that the difference of the values corresponding to those ages be not more than $\frac{1}{2}$ of the lesser.

RULE.

Add $\frac{4}{10}$ of that difference to the said lesser value, and the sum will be the value sought.

5. Let one age be 15, and the other 29, annuity 150 l. interest 3 per cent. the present value is required.

Against 15, under 3 per cent. per Table II. is	13.9
And against 29 - - - - -	11.0
Difference	2.9

Also $2.9 \times .4 = 1.16$ and $11 + 1.16 = 12.16$, the years purchase.

$\therefore 12.16 \times 150 = 1824$.

A general RULE, *be the difference of the values what they will.*

Multiply the difference of the values by half of the lesser of the two values, and divide the product by the greater; then to the lesser add the quotient, which will give the true answer very near.

6. Let one age be 11 years, and the other 68, annuity 160l. and interest at 4 per cent. the present value is required?

Against 11 years, under 4 per cent. is	12.9
Also against 68 - - - -	$4.6 \div 2 = 2.3$
Difference	8.3

Then $8.3 \times 2.3 = 19.09$; also $19.09 \div 12.9 = 1.48$.

And $4.6 + 1.48 = 6.08$ years purchase.

$\therefore 160 \times 6.08 = 972$ l 16 s. the answer required.

PROBLEM

PROBLEM III.

To find the value of an annuity upon two lives that is to continue as long as either of them is in being.

CASE I.

If the lives be equal.

RULE.

Find the given age in Table III. and against it, under the proposed rate of interest, is the number of years purchase.

7. Let the given ages be each 50 years, and the rate of interest 4 per cent. required the value of an annuity of 30l.?

In Table III. against 50, under 4 per cent. is 13.3 years purchase

$\therefore$ $30 \times 13.3 = 399$, the value required.

CASE II.

If both ages be between 25 and 50.

RULE.

Take half their sum for a mean age, which proceed with as in the last case.

8. Suppose one age to be 30 years, and the other 46, rate 3 per cent. and annuity 70l. required the present value?

Then $46 + 30 = 76$, and $\frac{76}{2} = 38$, half their sum.

Answering to which, under 3 per cent. stands 17.7 years purchase.

$\therefore$ £ $70 + 17.7 = 1239$, the answer.

CASE III.

If one or both ages be without the limits mentioned in the last case, but the difference of the values corresponding to those ages, as found in Table III. be no more than $\frac{1}{6}$ of the lesser.

RULE.

RULE.

Take half the ſum of thoſe values for the value required.

9. If the two propoſed ages be 6 and 21 years, the annuity 25 l. and intereſt 4 per cent. its preſent value is required?

Againſt 6 years is - 19.7
And againſt 21 years is 18.8

2) 37.9 (18.95 years purchaſe.

$\because$ $25 \times 18.95 = 473.75 = 473$ l. 15 s. the anſwer.

CASE IV.

Let the given ages be what they will.

RULE.

Find the value of the two joint lives, by caſe IV. Prob. II. which ſubtract from the ſum of the values of the two ſingle lives, and the remainder will be the required value upon the longeſt life.

10. Let the propoſed ages be 10 and 66, the rate of intereſt 4 per cent. and the annuity 70 l. required its preſent worth?

Table II. againſt 10 years, under 4 per cent. is - - - } 13.0
Alſo againſt 66 - - - $4.9 \div 2 = 2.45$

8.1 difference.

Then $8.1 \times 2.45 = 19.845$; which $\div 13 = 1.5$; alſo $1.5 + 4.9 = 6.4$.

Againſt the two ſingle lives, per Table I. viz. $16.4 + 7.3 = 23.7$.

Laſtly, $23.7 - 6.4 = 17.3$ years purchaſe.

$\because$ 70 l. $\times 17.3 = 1211$ l. the anſwer required.

PROBLEM IV.

To find the value of an annuity upon three joint lives.

CASE I.

If all the lives be equal.

RULE.

RULE.

Find out the given age in Table IV. and againſt it, under the propoſed rate of intereſt, will be the number of years purchaſe.

11. Let each age be 27, the rate of intereſt 3 per cent. and the annuity 65 l. its value is required?

Table IV. againſt 27, under 3 per cent. is 8.8 years purchaſe.

$\because\ 65 \times 8.8 = 572$ l. the anſwer required.

CASE II.

If all the three ages be between 15 and 55 years, and the difference between the greateſt and leaſt not more than 15 years.

RULE.

Take $\frac{1}{3}$ of their ſum for the mean age, and proceed as in Caſe I.

12. Let the propoſed ages be 21, 27, and 33, intereſt 5 per cent. and annuity 50l. its value is required?

First, $\frac{21+27+33}{3} = 27$, mean age.

Alſo Table IV. againſt 27 years, under 5 per cent. is 7.3 years purchaſe.

$\because\ 7.3 \times 50 = 365$l. the anſwer required.

CASE III.

If one or more of the propoſed ages be without the limits mentioned in Caſe II. but the difference of the values, anſwering to the greateſt and leaſt of them, be not greater than half the leaſt.

RULE.

To the ſum of the two greateſt values add twice the leaſt and take $\frac{1}{4}$ of the ſum for the mean value required.

13. Let the three ages be 7, 15, and 33, the annuity 50 l. and intereſt 3 per cent. the preſent value is required?

In

In Table IV: against $\left\{\begin{array}{c}7\\15\\33\end{array}\right\}$ stands $\left\{\begin{array}{c}11.9\\11.2\\7.9\end{array}\right\}$

Also $11.9+11.2=23.1$; and $79\times2=15.8$.

Then $23.1+15.8=38.9$; and $\frac{38.9}{4}=9.725$ years val.

$\therefore 50\times9.725=486.25$, the value required.

CASE IV.

Let the ages be what they will.

RULE.

Multiply the sum of the three corresponding values by the square of the least of them, reserving the product; multiply the two greater values into each other, and to the double of the product add the square of the lesser value; divide the reserved product by this sum, and subtract the quotient from twice the lesser value, the difference will be the value sought.

14. Let three ages be 13, $31\frac{1}{2}$, and 53 years, annuity 60l. and interest 4 per cent.

Against these in Table IV. $\left\{\begin{array}{c}13\\31\frac{1}{2}\\53\end{array}\right\}$ stand $\left\{\begin{array}{c}10.5.\\7.3.\\5.0.\end{array}\right\}$

Then $10.5+7.3+5.0=22.8$; also $5\times5=25$.
Which $22.8\times25=570$, to be reserved.
Again, $10.5\times7.3=76.65$; which $\times2=153.3$.
Also $153.3+25=178.3$) 570 (3.2, nearly.
Then $5\times2=10$, double the least value.
Lastly, $10-3.2=6.8$ years purchase.
$\therefore 60\times6.8=408$ l. the value required.

PROBLEM V.

To find the value of an annuity upon the longest of three lives

CASE I.

If the lives be all equal.

RULE.

Seek the common age in Table V. and against it, under the common rate of interest, will be the number of years purchase required.

15. Let

15. Let the three ages be each 45 years, the annuity 275 l. and the intereſt 4 per cent. required its value?

In Table V. againſt 45, under 4 per cent. ſtands 15.9 years purchaſe.

$\because$ 275 l. $\times$ 15.9 = 4372 l. 10 s. the value required.

CASE II.

If none of the ages be leſs than 10, nor greater than 60 years, and the difference of the greateſt and leaſt of them not more than 15 years.

RULE.

To twice the ſum of the two leaſt add the greateſt, and take $\frac{1}{5}$ of the ſum for a mean age.

16. Let the propoſed ages be 16, 24, and 30 years, the annuity 170 l. and intereſt 4 per cent. the value is required?

Firſt, $\overline{16 + 24} \times 2 = 80$; alſo $80 + 20 = 110$.

Then $\frac{110}{5} = 22$, mean age, againſt which, Table V. is 19.4 years purchaſe.

$\because 170 \times 19.4 = 3298$ l. the value ſought.

CASE III.

If the difference of the greateſt and leaſt values found againſt the propoſed ages, in Table V. be no more than $\frac{1}{4}$ of the leaſt.

RULE.

To twice the ſum of the two greateſt values add the leaſt, taking $\frac{1}{5}$ of the ſum for a mean value.

17. Suppoſe the three ages be 28, 35, and 44, the rate 4 per cent. and the annuity 60 l.

By Table V. the value of the three ages viz. $\left.\begin{matrix}28\\35\\44\end{matrix}\right\}$ are $\left\{\begin{matrix}18.3,\\17.3,\\16.\end{matrix}\right.$

Then $\overline{18.3 + 17.3} \times 2 = 71.2$; alſo $71.2 + 16 = 87.2$

And 5) 87.2 (17.44 years purchaſe.

$\because 17.44 \times 60 = 1046.4 = 1046$ l. 8 s. the anſwer.

CASE IV.

Let the given ages be what they will.

RULE

Find the value anſwering to the greateſt of the given ages in Table III. and the values correſponding to all the three ſeveral ages in Table V. and let the difference of the values, anſwering to the greateſt age, be taken and reſerved; let the ſquare of the greater of theſe two be divided by the product of the other two remaining values, and multiply the ſquare of the quotient by the reſerved difference; then this laſt product added to the value of the annuity for the two youngeſt lives, will be the value required.

18. Suppoſe the given ages 20, 36, and 60, the intereſt 4 per cent. and annuity 75l. the preſent value is required?

By Table III. the value found againſt 60 years is 11.2.

By Table V. thoſe againſt $\left\{\begin{matrix}20\\36\\60\end{matrix}\right\}$ are $\left\{\begin{matrix}19.7,\\17.2,\\12.7.\end{matrix}\right.$

Then $12.7 - 11.2 = 1.5$, the reſerved difference.

Again, $12.7 \times 12.7 = 161.29$; alſo $19.7 \times 17.2 = 338.84$.

Then 338.84) 161.290 (.5 nearly; and $.5 \times .5 = .25$.

And the difference reſerved $1.5 \times .25 = .375$, nearly .4.

Alſo $\frac{20 + 36}{2} = 28$, mean age, by Caſe 4. Problem III. the value of which, by Table III. is 16.9, or nearly 17 years.

And $17 + .4 = 17.4$ years purchaſe.

∴ $17.4 \times 75 = 1305$l. value of the annuity required.

Table I. *For the valuation of annuities upon one life.*

Age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.	Age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.
6	14.1	16.2	18.8	41	10.2	11.4	13.0
7	14.2	16.3	18.9	42	10.1	11.2	12.8
8	14.3	16.4	19.0	43	10.0	11.1	12.6
9	14.3	16.4	19.0	44	9.9	11.0	12.5
10	14.4	16.4	19.0	45	9.8	10.8	12.3
11	14.3	16.4	19.0	46	9.7	10.7	12.1
12	14.2	16.3	18.9	47	9.5	10.5	11.9
13	14.1	16.2	18.7	48	9.4	10.4	11.8
14	14.0	16.0	18.5	49	9.3	10.2	11.6
15	13.9	15.8	18.3	50	9.2	10.1	11.4
16	13.7	15.6	18.1	51	9.0	9.9	11.2
17	13.5	15.4	17.9	52	8.9	9.8	11.0
18	13.4	15.2	17.0	53	8.8	9.6	10.7
19	13.2	15.0	17.4	54	8.6	9.4	10.5
20	13.0	14.8	17.2	55	8.5	9.3	10.3
21	12.9	14.7	17.0	56	8.4	9.1	10.1
22	12.7	14.5	16.8	57	8.2	8.9	9.9
23	12.6	14.3	16.5	58	8.1	8.7	9.6
24	12.4	14.1	16.3	59	8.0	8.6	9.4
25	12.3	14.0	16.1	60	7.9	8.4	9.2
26	12.1	13.8	15.9	61	7.7	8.2	8.9
27	12.0	13.6	15.6	62	7.6	8.1	8.7
28	11.8	13.4	15.4	63	7.4	7.9	8.5
29	11.7	13.2	15.2	64	7.3	7.7	8.3
30	11.6	13.1	15.0	65	7.1	7.5	8.0
31	11.4	12.9	14.8	66	6.9	7.3	7.8
32	11.3	12.7	14.6	67	6.7	7.1	7.6
33	11.2	12.6	14.4	68	6.6	6.7	7.4
34	11.0	12.4	14.2	69	6.4	6.7	7.1
35	10.9	12.3	14.1	70	6.2	6.5	6.9
36	10.8	12.1	13.9	71	6.0	6.3	6.7
37	10.6	11.9	13.7	72	5.8	6.1	6.5
38	10.5	11.8	13.5	73	5.6	5.9	6.2
39	10.4	11.6	13.3	74	5.4	5.6	5.9
40	10.3	11.5	13.2	75	5.2	5.4	5.6

Table II. *For the valuation of annuities upon two joint lives.*

Mean age.	Years purchas. at 5 per cent.	Years purchas at 4 per cent.	Years purchas at 3 per cent.	Mean age.	Years purchas at 5 per cent.	Years purchas at 4 per cent.	Years purchas at 3 per cent.
6	11.3	12.7	14.4	41	7.2	8.0	8.9
7	11.5	12.9	14.6	42	7.1	7.8	8.7
8	11.6	13.0	14.7	43	7.0	7.7	8.6
9	11.6	13.0	14.7	44	6.9	7.6	8.5
10	11.6	13.0	14.7	45	6.7	7.4	8.3
11	11.5	12.9	14.6	46	6.6	7.3	8.2
12	11.4	12.8	14.5	47	6.5	7.2	8.1
13	11.3	12.7	14.3	48	6.4	7.1	7.9
14	11.2	12.5	14.1	49	6.3	7.0	7.8
15	11.0	12.3	13.9	50	6.2	6.8	7.6
16	10.8	12.1	13.7	51	6.1	6.7	7.4
17	10.7	11.9	13.5	52	6.0	6.6	7.3
18	10.5	11.7	13.2	53	5.9	6.5	7.2
19	10.3	11.5	13.0	54	5.8	6.3	7.0
20	10.1	11.3	12.8	55	5.7	6.2	6.9
21	10.0	11.2	12.6	56	5.6	6.1	6.7
22	9.8	11.0	12.4	57	5.5	6.0	6.6
23	9.7	10.8	11.2	58	5.4	5.8	6.4
24	9.5	10.6	12.0	59	5.3	5.7	6.3
25	9.4	10.5	11.8	60	5.2	5.6	6.1
26	9.2	10.3	11.6	61	5.1	5.5	6.0
27	9.1	10.1	11.4	62	5.0	5.4	5.9
28	8.9	9.9	11.2	63	4.9	5.3	5.7
29	8.8	9.8	11.0	64	4.8	5.1	5.5
30	8.6	9.6	10.8	65	4.7	5.0	5.4
31	8.5	9.4	10.6	66	4.6	4.9	5.3
32	8.3	9.2	10.4	67	4.5	4.8	5.1
33	8.2	9.1	10.2	68	4.4	4.6	4.9
34	8.1	8.9	10.0	69	4.3	4.5	4.8
35	8.0	8.8	9.9	70	4.2	4.4	4.6
36	7.8	8.6	9.7	71	4.1	4.3	4.5
37	7.6	8.4	9.5	72	3.9	4.1	4.3
38	7.5	8.3	9.3	73	3.8	4.0	4.2
39	7.4	8.2	9.2	74	3.7	3.8	4.0
40	7.3	8.1	9.1	75	3.6	3.7	3.8

TABLE III. *For the valuation of annuities upon the longest of two lives.*

Mean age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.	Mean age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.
6	16.9	19.7	23.3	41	13.2	14.9	17.0
7	17.0	19.8	23.4	42	13.1	14.7	16.8
8	17.1	19.9	23.5	43	13.0	14.5	16.5
9	17.1	19.9	23.5	44	12.9	14.3	16.3
10	17.1	19.9	23.5	45	12.8	14.2	16.1
11	17.1	19.9	23.5	46	12.6	14.0	15.8
12	17.0	19.8	23.4	47	12.5	13.8	15.6
13	16.9	19.7	23.3	48	12.4	13.6	15.3
14	16.7	19.5	23.1	49	12.2	13.4	15.1
15	16.6	19.3	22.9	50	12.1	13.3	14.9
16	16.4	19.1	22.6	51	11.9	13.1	14.6
17	16.2	18.9	22.4	52	11.8	12.9	14.4
18	16.1	18.7	22.1	53	11.6	12.7	14.1
19	15.9	18.5	21.9	54	11.5	12.5	13.9
20	15.7	18.3	21.6	55	11.3	12.3	13.6
21	15.6	18.2	21.3	56	11.2	12.1	13.4
22	15.4	18.0	21.1	57	11.0	11.9	13.1
23	15.3	17.8	20.8	58	10.9	11.7	12.8
24	15.1	17.6	20.6	59	10.7	11.5	12.5
25	15.0	17.4	20.3	60	10.5	11.2	12.2
26	14.9	17.3	20.1	61	10.3	11.0	12.0
27	14.7	17.1	19.9	62	10.1	10.8	11.7
28	14.6	16.9	19.7	63	9.9	10.5	11.4
29	14.5	16.8	19.5	64	9.7	10.3	11.1
30	14.4	16.6	19.3	65	9.4	10.0	10.8
31	14.2	16.4	19.1	66	9.2	9.7	10.5
32	14.1	16.2	18.9	67	8.9	9.4	10.2
33	14.0	16.1	18.7	68	8.7	9.2	9.9
34	13.9	15.9	18.5	69	8.5	8.9	9.5
35	13.8	15.8	18.3	70	8.2	8.6	9.2
36	13.7	15.6	18.1	71	8.0	8.4	8.9
37	13.6	15.5	17.9	72	7.7	8.1	8.6
38	13.5	15.3	17.7	73	7.5	7.8	8.2
39	13.4	15.2	17.5	74	7.2	7.5	7.9
40	13.3	15.0	17.3	75	6.9	7.2	7.6

TABLE IV. *For the valuation upon three joint lives.*

Mean age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.	Mean age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.
6	9.7	10.6	11.7	41	5.5	6.1	6.8
7	9.9	10.8	11.9	42	5.4	6.0	6.7
8	10.0	10.9	12.0	43	5.4	5.9	6.5
9	10.0	10.9	12.0	44	5.3	5.8	6.4
10	10.0	10.9	12.0	45	5.2	5.7	6.3
11	9.9	10.8	11.9	46	5.1	5.6	6.2
12	9.8	10.7	11.8	47	5.0	5.5	6.1
13	9.6	10.5	11.6	48	5.0	5.4	5.9
14	9.5	10.4	11.4	49	4.9	5.3	5.8
15	3.3	10.2	11.2	50	4.8	5.2	5.7
16	9.2	10.0	11.0	51	4.7	5.1	5.6
17	9.0	9.8	10.8	52	4.7	5.1	5.5
18	8.8	9.6	10.6	53	4.6	5.0	5.4
19	8.6	9.4	10.4	54	4.5	4.9	5.3
20	8.4	9.2	10.2	55	4.4	4.8	5.2
21	8.2	9.0	10.0	56	4.4	4.7	5.1
22	8.1	8.9	9.8	57	4.3	4.6	5.0
23	7.9	8.7	9.6	58	4.2	4.5	4.9
24	7.7	8.5	9.4	59	4.1	4.4	4.8
25	7.6	8.3	9.2	60	4.0	4.3	4.6
26	7.4	8.1	9.0	61	3.9	4.2	4.5
27	7.3	8.0	8.8	62	3.8	4.1	4.4
28	7.1	7.8	8.6	63	3.7	4.0	4.3
29	7.0	7.7	8.5	64	3.7	3.9	4.2
30	6.8	7.5	8.3	65	3.6	3.8	4.1
31	6.7	7.4	8.2	66	3.5	3.7	3.9
32	6.5	7.2	8.0	67	3.4	3.6	3.8
33	6.4	7.1	7.9	68	3.3	3.5	3.7
34	6.2	6.9	7.7	69	3.2	3.4	3.6
35	6.1	6.8	7.6	70	3.1	3.2	3.4
36	6.0	6.7	7.4	71	3.0	3.1	3.3
37	5.9	6.5	7.2	72	2.9	3.0	3.1
38	5.8	6.4	7.1	73	2.8	2.9	3.0
39	5.7	6.3	7.0	74	2.6	2.7	2.8
40	5.6	6.2	6.9	75	2.5	2.6	2.7

TABLE

TABLE V. *For the valuation of annuities upon the longest of three lives.*

Mean age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.	Mean age.	Years purchas. at 5 per cent.	Years purchas. at 4 per cent.	Years purchas. at 3 per cent.
6	18.0	21.0	25.0	41	14.6	16.4	18.9
7	18.1	21.1	25.1	42	14.5	16.3	18.7
8	18.2	21.2	25.2	43	14.4	16.2	18.5
9	18.2	21.2	25.2	44	14.3	16.0	18.2
10	18.2	21.2	25.2	45	14.2	15.9	18.0
11	18.2	21.2	25.2	46	14.1	15.7	17.7
12	18.1	21.1	25.1	47	13.9	15.5	17.5
13	18.0	21.0	25.0	48	13.8	15.2	17.2
14	17.9	20.9	24.8	49	13.7	15.1	17.0
15	17.8	20.7	24.6	50	13.5	14.9	16.7
16	17.6	20.5	24.3	51	13.4	14.7	16.5
17	17.5	20.3	24.1	52	13.2	14.5	16.2
18	17.3	20.1	23.8	53	13.1	14.3	15.9
19	17.2	19.9	23.5	54	12.9	14.1	15.7
20	17.0	19.7	23.3	55	12.8	13.9	15.4
21	16.9	19.5	23.1	56	12.6	13.7	15.1
22	16.8	19.4	22.8	57	12.5	13.5	14.8
23	16.6	19.2	22.6	58	12.3	13.2	14.5
24	16.5	19.0	22.3	59	12.1	12.9	14.1
25	16.4	18.8	22.1	60	11.9	12.7	13.8
26	16.3	18.7	21.9	61	11.7	12.5	13.5
27	16.1	18.5	21.6	62	11.5	12.2	13.1
28	16.0	18.3	21.4	63	11.3	11.9	12.8
29	15.9	18.2	21.2	64	11.0	11.6	12.5
30	15.8	18.0	21.0	65	11.8	11.4	12.2
31	15.6	17.8	20.8	66	10.5	11.1	11.8
32	15.5	17.7	20.6	67	10.2	10.8	11.5
33	15.4	17.6	20.4	68	9.9	10.5	11.2
34	15.3	17.4	20.2	69	9.6	10.2	10.9
35	15.2	17.3	20.0	70	9.3	9.9	10.5
36	15.1	17.2	19.9	71	9.0	9.6	10.2
37	15.0	17.0	19.7	72	8.7	9.2	9.8
38	14.9	16.9	19.5	73	8.4	8.9	9.5
39	14.8	16.7	19.3	74	8.1	8.6	9.1
40	14.7	16.6	19.1	75	7.8	8.2	8.7

PROBLEM VI.

To find the value of the reverſion of one life after another.

RULE.

From the value of the life in expectation take the value of the two joint lives, or from the value of the longeſt of two lives take the value of the life in poſſeſſion; the remainder in either caſe will be the value of the reverſion.

19. Suppoſe the life in poſſeſſion be 68 years, the life in expectation 11 years, and intereſt four per cent. and annuity 50 l. the value of the reverſion is required?

Againſt 11 years, under 4 per cent. 12.9, Table II.
Alſo againſt 68 years - - - $4.6 \div 2 = 2.3$.

Difference 8.3.

Then $8.3 \times 2.3 = 19.9$; alſo $\frac{19.9}{12.9} = 1.48$.
And $4.6 \times 1.48 = 6.08$, value of the two joint lives.
Alſo by Table I. againſt 11 years, is 16.4.
And $16.4 - 6.08 = 10.3$ years purchaſe.
$\therefore 50 \times 10.3 = 515$ l. value of the reverſion.
But if the youngeſt life be in poſſeſſion.
By Table I. againſt 68 years, at 4 per cent. is 6.9.
And $6.9 - 6.08 = .8$ years purchaſe.
$\therefore 50 \times .8 = 40$ l. value, if the youngeſt life be in poſſeſſion.

PROBLEM VII.

To find the value of the reverſion of two lives after one.

RULE.

From the value of the three lives ſubtract the value of the life in poſſeſſion, the remainder will be the value of the two lives in reverſion.

20. Let the age of the life in poſſeſſion be 50 years, and thoſe of the two lives in reverſion 45 and 56 years, the annuity 75 l. and intereſt at 4 per cent. the preſent value is required.

Firſt, $\overline{50 + 45} \times 2 = 190$; alſo $190 + 56 = 246$.
Then 5) 246 (49, mean age, againſt which, Table V. is 15.1.

Al,

Alſo, by Table I. the value of the life in poſſeſſion is 10.1.
Alſo $15.1 - 10.1 = 5$ years purchaſe.
$\therefore 75 \times 5 = 375$ l. value required.

PROBLEM VIII.

To find the value of a reverſion of one life after two.

RULE,

From the value of the three lives take the value of the two lives in poſſeſſion, the remainder will be the value of the life in reverſion.

21. Suppoſe 18 and 26 be the ages of the two lives in poſſeſſion; and 32 that of the life in expectation; the annuity 120 l. and intereſt 4 per cent.

Firſt, $\overline{18 + 26} \times 2 = 88$; alſo $88 + 32 = 120$.
Then 5) 120 (24, againſt which, under 4 per cent Table V. is 19 years.

Againſt $\left\{\begin{matrix} 18 \\ 26 \end{matrix}\right\}$ Table III. under 4 per ct. $\left\}\begin{matrix} 18.7 \\ 17.3 \end{matrix}\right.$

2) 36 (18 years.

And $19 - 18 = 1$ year's purchaſe, or 120 l. the anſwer.

What is above obſerved, hath regard to ſuch annuities as are paid yearly; but if the payments are made half yearly, which is moſt commonly the caſe, the above-mentioned Mr. Simpſon judiciouſly obſerves, that the value at which the annuity is eſtimated ought to be increaſed $\frac{1}{4}$ of a year's purchaſe; and if quarterly, $\frac{3}{8}$ of a year's purchaſe; as the life, upon whoſe failing the annuity ceaſes, has nearly the ſame chance to drop in the ſecond, third, or fourth quarter, as in that foregoing; in which caſe the purchaſer hath a chance to receive $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of a year's rent more than the annuity, when the annuity is paid yearly; and entirely loſes the laſt payments, if the death happens but one day before the annuity becomes due.

CHAP.

CHAPTER VII.

A COLLECTION of QUESTIONS.

SECT. I.

SUPERFICIAL MEASURE.

SURFACES, ſuch as land, flooring, painting, tyling, paving, plaiſtering, &c. if it be a four-ſided figure, whoſe oppoſite ſides are equal, by multiplying the length into the perpendicular height, gives the ſuperficial content, and either of the dimenſions being given, the other may be found by diviſion,

1. The biggeſt of the Egyptian pyramids, near Grand Cairo, being ſquare, and meaſuring, according to Mr. Greaves's account, 693 feet Engliſh on a ſide; how many acres then of ground doth it ſtand on?

Firſt, $693 \times 693 = 480249$ ſquare feet.
An acre $=$ □ feet $43560)\ 480249\ ($11 acres.
A perch $=$ □ feet $272.25)\ 1089\ ($4 perches.

2. What difference is there between a floor 28 feet long by 20 broad, and two others that meaſure 14 feet a-piece by 10; and what do all theſe come to at 45 s. per ſquare, viz. 10 feet by 10?

Firſt, $28 \times 20 = 560$; alſo $14 \times 10 \times 2 = 280$.
Then $560 - 280 = 280$, difference.
Alſo $560 + 280 = 840$; and 45 s. $= 2.25$ l.
$10 \times 10 = 100)\ 840\ ($8.4 ſquares.
$\because$ $8.4 \times 2.25 = 18.9 =$ 18 l. 18 s. amount.

3. A rectangular four-ſided room meaſures 129 feet 6 inches about, and is to be wainſcoted, at 3s. 6d. per yard ſquare: after the due allowance for girt of cornice and members, it is 16 feet 3 inches high; the door is 7 feet by 3 feet 9, the window-ſhutters, two pair, are 7 feet 3 by 4 feet 6; the check-boards round them come 15 inches below

low the ſhutters, and are 14 inches in breadth; the lining-boards round the door-way are 16 inches broad; the door and window-ſhutters, being wrought on both ſides, are reckoned as work and half, and paid for accordingly; the chimney 3 feet 9 by 3 feet, not being incloſed, is to be deducted from the ſuperficial content of the room; and the eſtimate of the charge is required?

	F. I.	F. I.		F. I. P.	
Firſt,	129 6 ×	16 3 –	=	2104 4 6,	room.
	$\frac{7\ – \times 3\ 9}{2}$	–	=	13 1 6,	door.
	7 3 ×	4 6 –	=	32 7 6,	ſhutters.
$\overline{8\ 6 + 4\ 6} \times 2 = 26 \times 1\ 2 \times 2$			=	60 8 –,	cheek-boards.
$\overline{14 + 3\ 9 =} 17\ 9 \times 1\ 4$			=	23 8 –,	door-linings.
				2234 5 6	
3 9 × 3, to be deducted			=	11 3 –,	chimney.
		Square feet		2223 2 6	

9) 2223 (247 ſquare yards.

$\frac{1}{20}$ $\frac{1}{8}$	247		
	30	17	6
	12	7	–
	£ 43	4	6, coſt.

4. When a roof is of a true pitch, the rafters are $\frac{3}{4}$ of the breadth of the building; now ſuppoſing the eave-boards to project 10 inches on a ſide, what will the new ripping an out-houſe coſt, that meaſures 32 feet 9 inches long, by 22 feet 9 inches broad upon the flat, at 15s. per ſquare?

F. I. F. I. P.
Breadth 22 9 - - - $\frac{3}{4}$ of which is 17 – 9
F. I P. I. F. I. P. F. I. P.
Alſo 17 – 9 + 10 = 17 10 9, which × 2 = 35 9 6
35 9 6 × 32 9 = 1172 feet 2 in. 1 6
100) 1172 (11.72 ſquares, and 15 s. = .75 l.
∴ 11.72 × .75 = 8.79 = 8 l. 15 s. 9½d. the anſwer.

5. If

5. If my court-yard be 47 feet 7 inches ſquare, and I have laid a foot-way of Purbeck-ſtone, four feet wide, along one ſide of it; what will paving the reſt with flints come to, at 6d. per yard ſquare?

Firſt, 47 f. 7 in — 4 f. = 43 f. 7 in. breadth.
Then 47 7 × 43 7 = 2073 10 1.
And 9) 2073 (230 yards 3 f. 10 inches.

$\frac{1}{3}$ | 6 d.

$\frac{1}{40}$ | 230 $\frac{1}{4}$ | $2\frac{1}{2}$ d. value of the 3 feet 10 inches.

s. d.
£ 5 15 $2\frac{1}{2}$, the anſwer required.

6. A ſquare cieling contains 114 yards 6 feet of plaiſtering, and the room 28 feet broad; what was the length of it?

Firſt, 114 yards 6 feet = 1032 ſquare feet.
Then 28) 1032 ($36\frac{6}{7}$ feet the anſwer.

7. An eml plank is 14 feet 3 inches long, and I would have juſt a yard ſquare ſlit off; at what diſtance from the edge muſt the line be ſtruck?

Firſt, 14 feet 3 inches = 171 inches; alſo 36 × 36 = 1296 inches in a ſquare yard. ∴ 171) 1296 ($7\frac{11}{19}$ inches, the anſwer. (99)

8. Having a rectangular marble ſlab, 58 inches by 27, I would have a foot ſquare cut off, parallel to the ſhorter edge; I would then have the like quantity divided from the remainder, parallel to the longer ſide; and this alternately repeated, till there ſhould not be the quantity of a foot left: what will the dimenſions of the remnant be?

Firſt, $12 \times 12 = 144$; alſo $\frac{144}{27} = 5.\dot{3}$, breadth of the firſt cut.

Then $58 - 5.\dot{3} = 52.\dot{6}$, the remaining length.

Alſo $\frac{144}{52.\dot{6}} = 2.734$, breadth of the ſecond cut.

Then $27 - 2.734 = 24.266$, the remaining breadth.

$\frac{144}{24.266} = 5.934$, breadth of the third cut.

$52.\dot{6}$

$52.6 - 5.934 = 46.732.$

$\frac{144}{46.732} = 3.0814$, breadth of the fourth cut.

$24.266 - 3.0814 = 21.1846.$

$\frac{144}{21.1846} = 6.7974$, breadth of the fifth section.

$46.732 - 6.7974 = 39.9346.$

$\frac{144}{39.9346} = 3.6059$, breadth of the sixth section.

$21.1846 - 3.6059 = 17.5787.$

$\frac{144}{17.5787} = 8.1917$, breadth of the seventh section.

$39.9346 - 8.1917 = 31.7429.$

$\frac{144}{31.7429} = 4.5364$, breadth of the eighth section.

$17.5787 - 4.5364 = 13.0423.$

$\frac{144}{13.0423} = 11.0411$, breadth of the ninth section.

Then $31.7429 - 11.0411 = 20.7018$, remaining length at last.

Also $\frac{144}{20.7018} = 6.956$, breadth of the tenth section.

$\because$ $13.0423 - 6.956 = 6.0863$, breadth remaining at the last. Q. E. F.

9. Being about to plant 10584 trees equally distant, the length of the grove must be six times the breadth; how many of the shorter rows will there be?

6) 10584 (1764; then $\sqrt{1764} = 42$ long rows.

$\because$ $42 \times 6 = 242$ short rows, 42 in a row.

10. A common joist is 7 inches deep, and $2\frac{1}{2}$ thick; but I want a scantling just as big again, that shall be 3 inches thick: what will the other dimension be?

First, $7 \times 2.5 = 17.5$ inches, area of an end.

Also $17.5 \times 2 = 35$, double area.

$\because$ 3) 35 ($11\frac{2}{3}$ inches. Q. E. F.

11. I have a square girder, 19 inches by 11, but one of a quarter of the timber in it, provided it be 9 inches deep, will serve; how broad will it be?

First, $19 \times 11 = 209$, area of an end.

Then 4) 209 ($52\frac{1}{4}$, area of an end of the piece wanted.

$\because$ 9) 52.25 (5.805. Q. E. F.

12. I

12. I have a wooden trough, that, at 6d. per yard, coſt me 3s. 2d. painting within; the length of it is 102 inches, the depth 21 inches; what is its breadth?

First, 36 × 36 = 1296, ſquare inches in a yard.
Alſo 3s. 2d. = 38d.

d. in. d.
As 6 : 1296 : : 38 : 8208, area of the whole trough.
Then 102 × 20 × 2 = 4284, area of the two ſides.

3924, area of the bottom and ends.
Then 102 + 42 = 144) 3924 (27¼ inches. Q. E. F.

13. My plumber has put 28 lb. per foot ſquare into a ciſtern, 74 inches and twice the thickneſs of the lead long, 26 inches broad, and 40 deep; he has put three ſtays within acroſs it, 16 inches deep, of the ſame ſtrength, and reckons 22s. per cwt. for work and materials: I being a maſon, have paved him a work-ſhop 22 feet 10 inches broad, with Purbeck-ſtone, at 7d. per foot, and upon the balance I find there is 3s. 6d. due to him; what was the length of his work-ſhop.

Firſt, 26 + 40 + 40 = 106, breadth of the bottom and ſides.

Alſo 106 × 74 = 7844, area of the bottom and ſides.
Then 40 × 26 × 2 = 2080, area of both ends.
And 26 × 16 × 3 = 1248, area of the ſtays.

11172, whole area in inches.

144) 11172 (77.583 ſquare feet.
4) 77.583 (19.3958 3 cwt.
$\frac{1}{10}$) 1.9395833
19.39583

£ 21.335416 = 21 l. 6 s. 8½d. value of the ciſtern.
3 s. 6 d. = .175; aſo 7d. = .029161.

.02916) 21.160216 (725.5 ſquare feet in his ſhop.
291) 2.116041

.02625) 19.044375 22 ft. 10 in. = 22.83.
22.83) 725.5 (31.776 = 31 feet 9¼ inches, the anſwer.
2.28) 72.5

20.55) 652.00

14. The

14. The area of a rectangular powdering-trough of a man of war measures 27 square feet, 112 inches, the depth is 20 inches, the breadth 16; the length is sought?

First, 27 feet 112 inches $=$ 4000 square inches.
Then $20 \times 16 \times 2$ $=$ 640, area of both ends.

3360, bottom and both sides.

$\because 20 + 20 + 16 = 56$) 3360 (60 inches, the answer.

15. In 110 acres of statute-measure, in which the pole is $16\frac{1}{2}$ feet long, how many Cheshire acres, where the customary pole is 6 yards long; and how many Yorkshire, where the pole in use is 7 yards in length?

$$\begin{array}{l} \text{sq. yds.} \\ \left.\begin{array}{l} \text{First, } 5.5 \times 5.5 = 30.25 \\ \text{Also } 6 \times 6 = 36 \\ \text{And } 7 \times 7 = 49 \end{array}\right\} = 1 \text{ perch} \left\{\begin{array}{l} \text{Statute} \\ \text{Cheshire} \\ \text{Yorkshire} \end{array}\right\} \text{measure} \end{array}$$

$\because$ Reciprocally, a. r. p.

$$\text{As } 30.25 : 110 :: \left\{\begin{array}{l} 36 : 92.4305 = 92 \;\; 1 \;\; 28, \text{ Cheshire} \\ 49 : 67.908 = 67 \;\; 3 \;\; 25, \text{ Yorksh.} \end{array}\right\} \text{mea.}$$

16. I would set 3584 plants in rows, each four feet asunder, and the plants 7 feet apart, in a rectangular plot of ground; what land will this take up?

First, $7 \times 4 = 28$, square feet (area) between the plants.
Then $3584 \times 28 = 100352$ square feet.
In an acre are 43560 square feet.

$\because$ 43560) 100352 (2 acres, 1 rod, $8\frac{1}{2}$ perches, the answer.
10890) 13232
272.25) 2342

.164

A triangle, or three-sided figure (being the half of a four-sided one of the same height and equal base) if you multiply the base, or longest side, by the shortest height, you have double the content.

17. A triangular field, 738 links in the base, and 583 in the perpendicular, brings in 12l. a year, what is it set at an acre?

First, $\frac{738}{2} \times .583 = 215127 = 2$ acres, 24 perches.

Also $2.15127 : 12 :: 1.00000 : 5.5781 = 5$ l. 11s. $6\frac{3}{4}$d. the answer.

18. A

18. A piece of garden-box lies in form of a regular pentagon, or figure of five equal ſides, each 48 feet; and from the center of the figure to the middle of one of theſe it meaſures 41.57 feet nearly: the area of the figure will be the content of theſe five triangles; pray what is that?

Firſt, $\frac{48}{2} \times 41.57 = 997.68$, area of one of the triangles.

Alſo $997.68 \times 5 = 4988.4$ ſquare feet, area of the pentagon.

19. The end-wall of an houſe is 24 feet 6 inches in breadth, and 40 feet to the roof; $\frac{1}{3}$ of which is two bricks thick, $\frac{1}{3}$ more $1\frac{1}{2}$ brick thick, and the reſt one brick thick: now the gable riſes 38 courſe of bricks (four of which uſually make a foot in depth) and this is but 4 inches, or half a brick thick; what will this piece of work come to at 5 l. 10 s. per ſtatute rod, the dimenſions of which are given?

4) 38 (9.5, height of the gable.

Alſo 3) 40 ($13.\dot{3}$, height of each floor.

Again, $24.5 \times 13\dot{3} = 326.\dot{6} = 435.\dot{5}$, ground-floor.

Alſo $326.\dot{6}$, firſt ſtory.

Then $326.\dot{6} = 217.7$, garret.

And $\frac{24.5}{2} \times 9.5 = 116.375 = 38.79$, gable.

1017 feet, ſtatute meaſ.

272.25) 1017.00 (3.7355 rods, ſtatute meaſure.

$\therefore 3.7355 \times 5.5 = 20.54525 =$ 20l. 10s. $10\frac{3}{4}$d. the anſ.

20. A four-ſided figure, whoſe ſides are equal, is called a trapeze: I have an orchard of that form, containing $3\frac{3}{4}$ acres, which being divided by a diagonal, or a line from corner to corner, the perpendicular of one of the triangles is 430 links, and the other 360: the length of the ſaid diagonal, or common baſe of thoſe triangles, is required?

Firſt, $430 + 360 = 790$; alſo $\frac{790}{2} = 395$.

And $3\frac{3}{4}$ acres $= 375000$ links.

$\therefore$ 395) 375000 ($949\frac{20}{75}$ links, the anſwer.

The

The areas of circles are found, either by multiplying half the circumference by half the diameter, or by multiplying the ſquare of the diameter by .7854, that being the area of the circle whoſe diameter is 1.

And if the diameter be 1, the circumference will be 3.1416 nearly.

21. Give the area of a circular bowling green, that is 16 poles-acroſs the middle, the circumference being 3.1416 times the diameter of a circle?

$16 \times 3.1416 = 50.2656$ poles circumference.

$\frac{16}{2} \times \frac{50.2656}{2} = 201.0625$ ſquare poles.

In an acre are 160) 201 (1 acre, 41 poles, the anſwer.

Or $16 \times 16 \times .7854 = 201.0624$, as before.

22. The ſurveying wheel is ſo contrived, as to turn juſt twice in the length of a pole, or $16\frac{1}{2}$ feet; what then is its diameter?

One round, per queſtion, is $8\frac{1}{4}$ feet.

3.1416) 8.2500 (2.626 feet = 2 ft. $7\frac{1}{4}$ in. the anſwer.

23. I would turf a round plat, meaſuring 130 feet about, and would know the charge at 4 d. per yard ſquare?

3.1416) 130.0000 (41.38, diameter.

$65 \times 20.69 = 1344.85$ ſquare feet.

9) 1344.85 (149.428 ſquare yards.

$\frac{1}{60}$) 149.428 (2.4906 = 2 l. 9 s. $9\frac{1}{4}$d. the anſwer.

In an $\frac{1}{2}$ acre are 2420 ſquare yards.

24. I want the length of a line, by which my gardiner may ſtrike a round orangery, that ſhall contain juſt half an acre of land?

Firſt, .7854) 2420.0000 (3081.23.

And $\sqrt{3081.23}$ (55.5, diameter.

∴ 2) 55.5 (27.75 = $27\frac{3}{4}$ yards, the anſwer.

25. Agreed for an oaken kerb to a round well, at 8 d. per foot ſquare; it is exactly 42 inches in diameter, within the brick-work, and the breadth of the kerb is to be $14\frac{1}{2}$ inches; what will it come to?

Firſt, $14.5 + 42 + 14.5 = 71$, greater diameter.

 Then

Then 71 × 71 = 5041; alſo 5041 × .7854 = 3959.2014
Then 42 × 42 = 1764; alſo .1764 × .7854 = 1385.4456

Difference of the areas are — — — — — — 2573.7558
Then 144) 2573.7558 (17.8733, area of the kerb.
Alſo 8d, = .6 ſhilling.
17.8733 × .6 = 11 s. 11 d. nearly, the anſwer.

26. It is obſerved, that the extreme end of the minute-hand of a public dial moves juſt five inches in the ſpace of $3\frac{1}{4}$ minutes; the queſtion is, what is the length of that index?

As 3.25 : 5 : : 60 : 92.307. circumference.
Alſo 3.1416 : 1 : : 92.307 : 29.38, diameter.
∴ 2) 29.38 (14.69 inches, the anſwer.

27. A, B, C join for a grindſtone 36 inches over, value 20s. towards which A paid 7s. B 8s. and C 5s. the waſte hole, through which the ſpindle paſſed, was 5 inches ſquare; to what diameter ought the ſtone to be worn, when B and C begin ſeverally to work with it?

Begin your calculations from the center.

Firſt 36 × 36 = 1296, which × .7854 = 1017.8784 the area of the whole ſtone.

5 × 5 = 25 + 25 = 50; alſo $\sqrt{50}$ = 7.071068, the diameter of the circle circumſcribing the ſpindle-hole

Then 50 × .7854 = 39.27, area of the circle circumſcribing the ſpindle-hole.

Alſo 1017.8784 — 39.27 = 978.6084, area to be divided.

$$\text{As } \overset{s.}{20} : 978.6084 :: \left\{\begin{array}{l} 7\text{ s.} : 324.5129 = \text{A's} \\ 8 \quad : 391.4433 = \text{B's} \\ 5 \quad : 244.6521 = \text{C's} \end{array}\right\} \text{area.}$$

Then 244.6521 + 39.27 = 283.9221.
.7854) 283.9221 (361.5.
Alſo $\sqrt{361.5}$ = 19.03, diameter where C begins to grind.
And 391.4433 + 283.9221 = 675.3654.
.7854) 675.3654 (859.9.
∴ $\sqrt{859.9}$ = 29.324 inches diameter, where B begins to grind. Q. E. F.

28. I

28. I demand what difference there is in the area of the ſection of a round tree, 20 inches over, and its inſcribed and circumſcribed ſquares?

Firſt, $10 \times 10 = 100$; alſo $100 + 100 = 200$.
Then $\sqrt{200} = 14.142135$, ſide of the inſcribed ſquare.
Alſo $14.142135 \times 14.142135 = 200$ its area.
Again, $20 \times 20 = 400$, area of the circumſcribed ſquare.
Laſtly, $400 \times .7854 = 314.16$. area of the circular ſection.
Hence the inſcribed ſquare is 114.16 } inches { too little.
And the circumſcribed - - 85.84 } inches { too much.

29. Having paved a ſemicircular alcove with black and white marble, at 2 s. 4 d. per foot, the maſons bill was juſt 10l. what then was this arch in front, conſidering that as .7854, the area of the circle, the ſquare of whoſe diameter is 1, ſo is the area of any other circle to the ſquare of its diameter?

Firſt, 2 s. 4 d. $= .11\dot{6}$l. : 1 :: 10l. : 85.7143 feet area.
Then $85.7143 \times 2 = 171.4286$.
Alſo .7854)171.4286 (218.269.
$\therefore \sqrt{218.269} = 14.7739 = 14$ ft. $9\frac{1}{4}$ in. the anſwer.

30. What proportion is there between the arpent of France, which contains 100 ſquare poles, of 18 feet each, and the Engliſh acre, containing 160 ſquare poles, of $16\frac{1}{2}$ feet each; conſidering that the length of the French foot is to that of the Engliſh, as 16 to 15?

Firſt. $18 \times 18 \times 100 = 32400$ French feet, the arpent.
Then $16\frac{1}{2} \times 16\frac{1}{2} \times 160 = 43560$ Engliſh feet in an acre.
Alſo $16 \times 16 = 256$; and $15 \times 15 = 225$.
Recip. 256 : 32400 :: 225 : 36864 Engliſh feet, an arpent.
So that the Engliſh acre is to the arpent of France, as 605 to 512, or nearly as 13 to 11; or as 1 to .84628, the anſwer.

31. In turning a one-horſe chair within a ring of a certain diameter, it was obſerved, that the outer wheel made two turns, while the inner made but one; the wheels were equally high, and ſuppoſing them fixed at the ſtatutable diſtance, or

5 feet asunder on the axletree; pray what was the circumference of the tract described by the outer wheel;

$3.1416 \times 4 = 12.5664$, the circumference of the wheel.
∴ $12.5664 \times 5 = 62.832$ by the greater.
And 31.416 by the lesser.

Multiply half the arch by half the diameters; also find the area of a sector; that is, any part of a circle cut through from the center to the circumference.

32. The area of a sector (suppose one of the divisions of a wilderness) which being struck from a center with a line 30 yards long, makes the sweep, or circular part, 63 feet, is required?

63 feet = 21 yards is half, being 10.5 yards.
Then $10.5 \times 30 = 315$ yards, the answer.

33. If the chord or line drawn through the two ends of the curve be 15 inches shorter than the arch line, I demand the segment;

First, 15 inches = to $.41\dot{6}$ yards.
Then $21 - .41\dot{6} = 20.583$, which $\div 2 = 10.291\dot{6}$.
$30 \times 30 = 900$.
The □ $10.291\dot{6} = 105.9184$

$\sqrt{794.0816} = 28.18$, perpendicular.
Then $10.291\dot{6} \times 28.18 = 290.0191\dot{6}$, area of the triangle.
∴ $315 - 1290.0191\dot{6} = 24.98$, the answer.

An ellipse, or oval, is measured, by multiplying the product of the long and short axes by .7854, as in the circle, and this will give the superficial content.

34. The ellipse in Grosvenor-square measures 840 links the longest way, and 612 across, within the rails; the walls are 14 inches thick; what ground do they stand upon?

$$\left.\begin{array}{l} \text{First, } 8.40 \times 66 = 554.4 \\ \text{Also } 6.12 \times 66 = 403.92 \\ \text{And } 12)28.0 = 2.\dot{3} \end{array}\right\} \text{feet.}$$

$$\left.\begin{array}{l} \text{Then } 554.4 + 2.\dot{3} = 556.7\dot{3} \\ \text{Also } 403.92 + 2.\dot{3} = 406.25\dot{3} \end{array}\right\} \text{diameters of the greater.}$$

Then $556.7\dot{3} \times 406.25\dot{3} \times .7854 = 177637.66$
And $554.4 \times 403.92 \times .7854 = 175877.17$

Area covered by the wall - - = 1760.49 square ft.

∴ 4840

∴ $4840 \times 9 = 43560$) 175877.17 (4 acres, 6 perches, its area. Q. E. F.

The dimenſions of all ſimilar figures are in proportion to their areas, as the ſquares of their reſpective ſides; *et contra.*

35. If a round pillar, 7 inches over, has 4 feet of ſtone in it; of what diameter is the column, of equal length, that meaſures ten times as much?

$$4 \times 10 = 40 \text{ feet.}$$
$$4 \text{ feet} : 49 :: 40 \text{ feet} : 490.$$
$$\sqrt{490} = 22.136 \text{ inches.} \quad \text{Q. E. F.}$$

36. A pipe of ſix inches bore will be 3 hours in running off a certain quantity of water; in what time will 4 pipes, each 3 inches bore, be in diſcharging double the quantity?

$$6 \times 6 = 36; \text{ alſo } 3 \times 3 \times 4 \times 2 = 72.$$
$$\therefore 36 : 3 :: 72 : 6. \quad \text{Q. E. F.}$$

37. A yard of rope 9 inches round weighs, ſuppoſe 22 lb. what will a fathom of that weigh, which meaſures a foot round?

$$9 \times 9 = 81; \text{ alſo } 12 \times 12 \times 2 = 288.$$
$$81 : 22 :: 288 : 78\tfrac{2}{9}. \quad \text{Q. E. F.}$$

38. If 20 feet of iron-railing ſhall weigh half a ton, when the bars are an inch and quarter ſquare; what will 50 feet of ditto come to, at $3\frac{1}{2}$d. per pound, the bars being but $\frac{7}{8}$ of an inch ſquare?

$$1.25 \times 1.25 \times 20 = 31.25.$$
$$\tfrac{7}{8} = .875 \times .875 \times 50 = 38.28125.$$
$$\text{As } 31.25 : 1120 :: 38.28125 : 1372.$$

$\frac{1}{80}$	1372		
$\frac{1}{8}$	17	3	
	2	17	2
£	20	–	2, the anſwer.

39. A looking-glaſs is 16 inches by 9, and contains a foot of glaſs; what will the content of the plate be, that has twice the length, and three times the breadth?

$$2 \times 16 = 32; \text{ and } 3 \times 9 = 27.$$
$$\text{Then } 32 \times 27 = 86 \text{ ſquare inches.}$$
$$\therefore 144)\ 864\ (6 \text{ ſquare feet, the anſwer.}$$

 40. A

40. A sack that holds three bushels of corn is $22\frac{1}{2}$ inches broad, when empty; what would the sack contain, that, being of the same length, had twice its circumference, or twice its breadth?

$$22.5 \times 22.5 = 506.25; \text{ also } 45 \times 45 = 2025.$$
$$506.25 : 3 \text{ bush.} :: 2025 : 12 \text{ bush. Q. E. F.}$$

41. My plumber has set me up a cistern, and his shop-book being burnt, he has no means of bringing in the charge, and I do not chuse to take it down to have it weighed; but by measure he finds it contains 64 square feet $\frac{3}{10}$, and that it was $\frac{3}{8}$ of an inch precisely in thickness. Lead was then wrought at 21 l. per fodder. Let the accomptant, from these items, make out the poor man's bill; considering farther, that 4 oz. $\frac{4}{11}$ is the weight of a cubic inch of lead.

First, $64.3 \times 144 = 9259.2$ square inches.
Also $9259.2 \times .375 = 3472.2$ solid inches.
And $3472.2 \times 4.\dot{3}\dot{6} = 15151.4\dot{1}\dot{8}$ ounces.
Likewise $15151.4\dot{1}\dot{8}$ oz. $= 8.455$ cwt.
$\therefore$ 19.5 cwt. : 21 l. :: 8.455 : 9.10538.
Answer, 9 l. 2 s. $1\frac{1}{4}$d.

SECT. II.

MEASUREMENT *of* SOLIDS.

MULTIPLY the area by the depth, to find the solidity of uniform bodies, or such as are equal from top to bottom.

1. What is the difference of a solid half foot, and half a foot solid?

First, $6 \times 6 \times 6 = 216$, solid inches in $\frac{1}{2}$ foot solid.
And 2) 1728 (864, solid inches in $\frac{1}{2}$ a solid foot.
$\therefore$ 216) 864 (4 times as much as the first.

2. What

2. What is the proportion, in point of ſpace, between a room 25½ feet long, 20 feet 2 inches broad, 14 feet high, and two others of juſt half the dimenſions?

	F. I.		F. I.					F. I.
Firſt,	25 6	×	20 2	×	14		=	7199 6
Alſo	12 9	×	10 1	×	7	× 2	=	1799 10 6, which

is evidently juſt $\frac{1}{4}$ of the firſt.

3. Another room is 17 feet 7 inches long within, 13 feet 10 inches broad, and 9 feet 6 inches high; it has a chimney carried up ſtraight in the angle, the plan whereof is juſt half of 5 feet 6 inches, by 4 feet 2: the queſtion is, how many cubic feet of air the ſame will contain, allowing the content of the fire-place and windows at four ſolid yards?

	F. I.		F. I.		F. I.		F. I.
Firſt,	17 7	×	13 10	×	9 6	=	2310 8 11
Then	5 6	×	2 1	×	9 6	=	108 10 3
					Rem.		2201 10 8
And	4 × 27	-	-	-	-	=	108 – –

Anſwer, feet 2309 10⅔ inches.

4. A ſhip's hold is 112 feet 6 inches long, 32 broad, and 5 feet 6 inches deep; how many bales of goods, 3 feet 4 inches long, 2 feet 4 inches broad, and 3 feet deep, may be ſtowed therein, leaving a gang-way the whole length of 4 feet and ½ broad?

Firſt $112.5 \times 32 \times 5.5 = 19800$

Gang-way $112.5 \times 4.5 \times 5.5 = 2784.375$

Remaining capacity 17015.625

$$\text{Alſo } 3.\dot{3} \times 2.\dot{3} \times 3 = 23\tfrac{1}{3} = \frac{70}{3}$$

$$\text{And } 17015.625 = 17015\tfrac{5}{8} = \frac{136125}{8}.$$

$$\therefore \frac{70}{3}\Big)\frac{136125}{8}\Big(\frac{408375}{560} = 729\tfrac{2}{112}, \text{ the anſwer.}$$

5. I want a rectangular ciftern, that, at 16 lb. to the foot fquare, fhall weigh juft a fodder of lead; it muft be 8 feet long, and $4\frac{1}{4}$ over; how many hogfheads, wine meafure, will this contain, taking it at $\frac{3}{4}$ of an inch from the top?

A fodder of lead weighs $19\frac{1}{2}$ cwt. = 2184 lb.
16) 2184 (136.5 fquare feet.
Then $8 \times 4.25 = 34$, area of the bottom.
Alfo $136.5 - 34 = 102.5$, fides and ends.
$8 + 8 + 8.5 = 24.5$, round.
24.5) 102.5 (4.183673 feet = 50.204 inches deep.
8 feet = 96 inches, and $4\frac{1}{4}$ feet = 51 inches.
Alfo $50.204 - .75 = 49.454$.
Then $96 \times 51 \times 49.454 = 242126.784$ cubic inches.
[282) 242126 (858 gal. = 16 hds. 42 gal. the anfwer.

6. A log of timber is 18 feet 6 inches long, 28 inches broad, and 14 thick, *die* fquare all through; now, if 2 folid feet and $\frac{1}{2}$ be fawed off the end, how long will the piece then be?

Firft, 18 inches = 1.5; alfo 14 inches = 1.1$\dot{6}$.
$1.5 \times 1.1\dot{6} = 1.75$) 2.5 (1.42857, length of the piece cut off.
Then $18.5 - 1.42857 = 17.07143$ feet, the anfwer.

7. The folid content of a fquare ftone is found to be $126\frac{1}{4}$ feet, its length is 8 feet 6 inches; what is the area of one end, and what the depth, if the breadth affigned be $38\frac{1}{2}$ inches?

Length 8.5) 126.25 (14.853 feet = 2138.8234 inches, area of an end.
38.5) 2138.8234 (55.55 inches deep, the anfwer.

8. The dimenfions of the circular Winchefter bufhel are $18\frac{1}{2}$ inches over, and 8 inches deep; how many quarters of grain then will the fquare bin hold, that meafures 7 feet 10 long, 3 feet 10 broad, and 4 feet 2 deep within?

Firft, $18.5 \times 18.5 \times .7854 = 268.80315$.
Then $268.80315 \times 8 = 2150.4$, cubic inches in a bufhel.
Alfo 7 ft. 10 in. = 94 in. 3 ft. 10 in = 46 in. and 4 ft. 2 in. = 50 inches.
Then $94 \times 46 \times 50 = 216200$ cubic inches, content of the bin.
2150.4) 216200.0 (100 bufh. 2 pecks = 12 qrs. 4 bufh. 2 pecks. Q. E. F.

9. Taking

9. Taking the dimenſions of the buſhel, as above, what muſt the diameter of the circular meaſure be, which at 12 inches deep will hold 9 buſhels of ſea-coal ſtruck?

Firſt, $2150.4 \times 9 = 19353.6$ inches, the content.
Then 12) 19353.6 (1612.8, area of the circle.
Alſo .7854) 1612.8000 (2053 47.
∵ $\sqrt{2053.47} = 45.3$ inches. Q. E. F.

10. A priſm of two equal baſes, and ſix equal ſides, that meaſures 28 inches acroſs the center, from corner to corner; the ſuperficial and the ſolid content is required, taking the length at 134 inches?

Radius $14 \times 14 = 196$; alſo $7 \times 7 = 49$.
Then $196 - 49 = 147$.
Alſo $\sqrt{147} = 12.1243557$, perpendicular.
Then $12.1243557 \times 7 = 84.8705$, area of one triangle.
Alſo $84.8705 \times 6 = 509.223$, area of the baſe.
And $14 \times 6 \times 134 = 11256$.
Other baſe $= 509.223$

Area of the priſm 12274.446 inches.
∵ 1296) 12274 (9 yards, 4 feet, 34 inches, its area, Q. E. F.
Again, $509.223 \times 134 = 68235.88$ ſolid inches.
∵ 1728) 68235 (39 ſolid feet, 843 cubic inches. Q. E. F.

11. I have a rolling-ſtone, 44 inches in circumference, and am to cut off three cubic feet from one end; whereabouts muſt the ſection be made?

Firſt, 3 cubic feet $= 5184$ cubic inches.
If the circumference be 44, the diameter is 14.
Then $22 \times 7 = 154$, area of an end.
∵ 154) 5184 (33.66 inches, the anſwer.

12. I would have a ſyringe, an inch and $\frac{1}{4}$ in the bore, to hold a pint, wine meaſure, of any fluid; what muſt the length of the piſton ſufficient to make an injection with it, be?

Firſt, $1.25 \times 1.25 \times .7854 = 1.2271875$, area of the circle.
In a pint are 28.875 cubic inches.
∵ 1.2271875) 28.8750000 (23.5294 inches. the anſwer.

13. I

13. I would have a cubic bin made capable of receiving just $13\frac{1}{2}$ quarters of wheat, Winchester measure; what will be the length of one of its sides?

In a bushel are 2150.4 cubic inches.
Then $2150.4 \times 8 \times 13.5 = 232243.2$ cubic in. $13\frac{1}{2}$ qrs.
$\because \sqrt[3]{232243.2} = 61.4678$ inches. Q. E. F.

14. A Bath-stone, 20 inches long, 15 over, and 8 deep, weighs 220 lb. how many cubic feet thereof will freight a ship of 290 tons?

First, $20 \times 15 \times 8 = 2400$ cubic inches.
Also 2400 inches : 220 lb. : : 1728 : 158.4.
Also 290 tons = 649600 pounds.
$\because$ 158.4) 649600.0 (4101 feet, the answer.

15. The common way of measuring timber being to girt a round straight tree in the middle, and to take $\frac{1}{4}$ of the girt for the side of a square, equal to the area of the section there; if this be not considered in the price appointed, pray on which side lies the advantage?

A piece of timber a foot long, and 4 feet round, is a foot customary measure.

Also if a circle be 4 feet round, 3.1416) 4 (1.2732 diameter.

And a circular piece of timber 1 foot in length will contain 1.2732 feet.

$\because 1.2732 \times 50$, the feet in a load, is 63.66.

So that, in a load of timber, there is gained by the buyer nearly $13\frac{2}{3}$ feet.

The circumscribing cylinder is in proportion to its greatest inscribed globe, and cone of the same base, and perpendicular altitude, as 3, 2, and 1.

Therefore the cube of the diameter of any cylinder, of equal height and breadth, multiplied by .7854, the area of a circle, whose diameter is 1, will be the solidity.

The cube of the diameter of a globe, multipled by $\frac{2}{3}$ of .7854, viz. .5236, gives its solid content.

And the said cube, multiplied by $\frac{1}{2}$ of .5236, or .2618, gives the solidity of any cone, whose breadth and height are equal.

Also

Alſo their ſuperficial content may be found, by conſidering the cylinder as a ſquare ſurface, multiplying the height by the circumference, and adding a double area for the two baſes; the globe, as a rectangle of the diameter and circumference; and the cone as a triangle, whoſe baſe is the circuit, and perpendicular the ſlope height, adding the area of the baſe.

16. The ſolid content of a globe 20 inches in diameter; a cylinder of the ſame diameter, 20 inches long; and a cone 20 inches diameter at the baſe, and 20 inches high, are ſeverally required; and alſo what they will coſt painting, at 8d. a yard?

$$20 \times 20 \times 20 \times \left\{ \begin{array}{l} .7854 = 6283.2, \text{ cylinder's} \\ .5236 = 4188.8, \text{ globe's} \\ .2618 = 2094.4, \text{ cone's} \end{array} \right\} \text{solidity.} \qquad \text{(cub. inches.)}$$

Alſo $20 \times 3.1416 = 62.832$, circumference.
Then $62.832 \times 10 = 628.32$, area of the two baſes,
And $62.832 \times 20 = 1256.64$.

Cylinders 1884.96, ſuperficial content,
Again, $62.832 \times 20 = 1256.64$, ditto of the globe.
1015.24, ditto of the cone.

4156.84, ſum of their areas.

Alſo $20 \times 20 = 400$
And $10 \times 10 = 100$

$\sqrt{500} = 22.31614$, ſlope height of the cone,
$31.416 \times 22.31614 = 701.08385$
Area of the baſe $\frac{628.32}{2} = 314.16$

Area of the cone, as above 1015.24385

As 1296 : 8 d. : : 4156.84 : 25.6 = 2s. $1\frac{1}{2}$d. the anſw.

17. Our ſatellite, the moon, is a globe in diameter 2170 miles; I require how many quarters of wheat ſhe would contain, if hollow, $2150\frac{4}{10}$ ſolid inches being the buſhel; and how much yard-wide ſtuff would make her a waiſtcoat, was ſhe to be cloathed?

Firſt, $2170 \times 2170 \times 2170 \times .5236 = 5350308686.8$, ſolid miles in the moon.

Then

Then 1760 × 1760 × 1760 = 5451776000, solid yards in a mile.

Also 5350308686.8 × 5451776000 = 29168684491287756800, solid yards in the moon.

In a solid yard are 46656 cubic inches.

29168684491287756800 × 46656 = 1360894143625521581260800 solid inches.

17203.2) 1360894143625521581260800.0 (79107034948470144000 quarters of wheat the moon would hold, if hollow. Q. E. F.

Again, 2170 × 3.1416 = 6817.272, circumference of the moon.

Also 6817.272 × 2170 = 14793480.25 square miles.

1760 × 1760 = 3097600 square yards in a square mile.

Then 14793480.24 × 3097600 = 45824284391424 square yards. Q. E. F.

18. Supposing the atmosphere, or body of the air and vapours, surrounds the globe of the earth and sea to 60 miles above the surface, and the earth is 7970 miles in diameter; how many cubic yards of air then hang about and revolve along with this planet?

First, 7970 + 120 = 8090, diameter of the earth and atmosphere.

Then 7970 × 7970 × 7970 × .5236 = 265078559622.8, solid miles in the globe of the earth.

Also 8090 × 8090 × 8090 × .5236 = 377233177544.4, miles solid in the earth and atmosphere.

And 277233177544.4 — 265078559622.8 = 12154617921.6, solid miles in the atmosphere.

Also 12154617921.6 × 545177600 = 6626425427414-8761600, solid yards in the atmosphere. Q. E. F.

19. A gentleman bargaineth with a mason for a piece of marble in the form of a tetraedron, on which he intends to have four sun-dials; the side of each triangle is $2\frac{1}{2}$ feet, or 30 inches; I demand its value, at 2d. a solid inch, and what it will cost polishing, at 1s. 3d. per foot superficial?

First, 30 × 30 = 900; also 15 × 15 × 15 = 225.

Then 900 — 225 = 675; also $\sqrt{675}$ = 25.98, perpendicular of each triangle.

Again,

Again, $25.98 \times 25.98 = 674.9604$; alſo $12.99 \times 12.99 = 168.7401$

Then $674.9604 - 168.7401 = 506.2203$.

$\sqrt{506.2203} = 22.4993$, perpendicular of the tetraedron.

Then $25.98 \times \frac{30}{2} = 389.7$, area of a triangle.

And $389.7 \times 4 = 1558.8$ inches $= 10.825$ ſquare feet.

Then $389.7 \times \frac{22.4993}{3} = 2922.659103$ inches.

	l.	s.	d.
Anſwer, the marble comes to - -	24	7	$1\frac{1}{4}$
And poliſhing to - - - - - - -	—	13	$6\frac{1}{4}$
In all, £	25	—	$7\frac{1}{2}$

To find the ſolidity of a pyramid, or cone: multiply the area of the baſe by $\frac{1}{3}$ of its perpendicular altitude.

20. A ſquare pyramid, whoſe ſides at the baſe meaſure 30 inches a-piece, and is 21 feet high by the ſlope in the middle of each ſide of the baſe, is to be ſold at 7 s. per ſolid foot; and if the poliſhing the ſurface of the ſides will be 8 d. per foot more, I would know the coſt of this ſtone when finiſhed;

Firſt. $21 \times 21 = 441$; alſo $1.25 \times 1.25 = 1.5625$.

Then $441 - 1.5625 = 439.4375$.

$\sqrt{439.4375} = 20.9627$, perpendicular height.

Alſo $2.5 \times 2.5 = 6.25$, area of the baſe.

Then 3) 2096276 (6.9876, nearly.

And $6.25 \times 6.9876 = 43.6725$ ſolid feet.

Then $21 \times 1.25 = 26.25$, area of one triangular ſide.

Alſo $26.25 \times 4 = 105$, area of the ſides.

	l.	s.	d.
Anſwer, at 7 s. per ſolid foot. - -	15	5	$8\frac{1}{4}$
And poliſhing, at 8 d. per foot -	3	10	—
£	18	15	$8\frac{1}{4}$

When figures run uniformly taper, but not to a point, they are to be conſidered as fruſtums, or portions of the cone or pyramid; by ſuppoſing, therefore, what is wanting to make the figure entire, and then deducting the part cut off, we find the ſolidity of the part propoſed.

In

In order to complete the cone, use this analogy; as half the difference of the top and bottom are to the depth, so is half the greater diameter to the altitude of the whole cone.

Or else, to the areas of the top and bottom add the square roots of the products of those areas, and this multiply by $\frac{1}{3}$ of the height of the frustum for the solidity.

21. A round mash-vat measures at the top 72 inches over, within, at the bottom 54, the perpendicular depth being 42 inches, the content in ale-gallons is required?

$$\text{As } 9 : 42 :: 36 : 168.$$

$72 \times 72 \times .7854 = 4071.5136$, area of the top of the tun.

$\frac{168}{3} = 56 = \frac{1}{3}$, altitude of the cone.

Also $54 \times 54 \times .7854 = 2290.2264$, area of its bottom.

$168 - 42 = 126$, which $\frac{126}{3} = 42$, altitude of the piece wanting.

Then $4071.5136 \times 56 = 228004.7616$, the whole pyramid.
Also $2290.2264 \times 42 = 96189.5088$, piece wanting.

131815.2528 cubic inches.

$\because$ 282) 131815. (467 gallons $3\frac{61}{141}$ pints, the answer.

Or,

$4071.5136 + 2290.2264 = 6361.74$, sum of the areas.
Also $4071.5136 \times 2290.2264 = 9324687.9346$.
And $\sqrt{9324687.9346} = 3053.6352$.
Then $6361.74 + 3053.6352 = 9415.3752$.

$\because 9415.3752 \times \frac{42}{3} = 131815.2528$, cub. in. as before.

22. The shaft of a round pillar, 16 inches in diameter at the top, is about eight of the bottom diameters in height, $\frac{1}{3}$ whereof is truly cylindrical, and the other $\frac{2}{3}$ swelling; but we will suppose it tapers strait; and that it is $\frac{1}{6}$ less at top than at bottom; the price of the stone and workmanship is sought, at 3s. 6d. per cubic foot; and farther, the superficial content, including both ends?

5) 16, top diameter

+ 3.2

19.2, bottom diameter.
× 8

3) 153.6, height.
51.2, cylindrical.

102.4, a conical fruſtum.

Firſt, 19.2 × 19.2 × .7854 = 289.525985. area of the greater.
Alſo 16 × 16 × .7854 = 201.0264, area of the leſſer baſe.

Sum of the areas 490.588385.

Then 289.526 × 201.0264 = 58212.7924.
$\sqrt{58212.7924} = 241.273$.
Then 490.588 + 241.273 = 731.861.

$\because$ $731.861 \times \frac{102.4}{3} = 24980.84546$, conical fruſt. } ſolidity.
And 201.0624 × 51.2 = 10294.39488, cylinder }

Solid content of pillar = 35275.25034 inches.

Then 1728) 35275.25 (20.414 ſolid feet.
Anſwer, 3l. 11s. $5\frac{1}{4}$d. coſt, at 3s. 6d. per foot.
16 × 3.1416 = 502656, circum. of the cylinder.
19.2 × 3.1416 = 60.3187, circum. of the baſe.

2) 110.5843 (55.29215.

Then $\frac{19.2}{2} - \frac{16}{2} = 1.6$; alſo 1.6 × 1.6 = 2.56
Alſo 102.4 × 102.4 = 10485.76

10488.32

$\sqrt{10488.32}$ = 102.412.4125, ſlope height.
Then 102.4125 × 55.2926 = 5662.6534, conical ſuperf.
Alſo 51.2 × 50.2656 = 2573.5987, cylinder.
289.526, bottom area.
201.0624, top area.

8726.8405

Superficial content 8726.84 inches = 60.8 feet. Q. E. F.

23. A stick of square timber tapers straight; the side at the greater end is $19\frac{1}{2}$ inches, at the less $13\frac{1}{2}$ inches; the length 16 feet 6 inches; the value, at 2s. 6d. per foot solid, is demanded?

First, $19\frac{1}{2} - 13\frac{1}{2} = 6$, difference of the sides.
3) 36 (12, the third part of the square of that difference.
Also 16 feet 6 inches = 198 inches, the length.

Then $\overline{19.5 \times 13.5 + 12} \times 198 = 54499.5$ cubic inches = 31.539 feet.

$\because$ $31.539 \times 1.25 = 3.942375 =$ 3l. 18s. 10d. the answer.

To measure a common cask: find the areas at head and bung; add $\frac{1}{3}$ of the less, and $\frac{2}{3}$ of the greater, for a mean area; this multiplied by the length of the cask is its solidity in inches which reduce. Or, to double the square of the bung diameter, add the square of the head; then multiply by the length of the cask, and divide by 1077.24 for beer, or by 882.42 for wine gallons.

24. What quantity of brandy will the distillers tun contain, that measures 40 inches within at the head, 52 at bung, and is 100 inches long; and how many barrels of London ale would fill it?

First, $40 \times 40 \times .7854 = 1256.64$.
Also $52 \times 52 \times .7854 = 2123.7216$.

Then $\frac{1256.64}{3} = 418.88$; also $2123.7216 \times \frac{2}{3} = 1415.8144$.

Then $1415.8144 + 418.88 = 1834.6944$.

$\because$ $1834.6944 \times 100 = 183469.44$ cubic inches, the content.

Also 231) 183469.44 (= 794 gallons of brandy.

And 282) 183469.44 (= $650\frac{1}{2}$ gallons = 20 barrels, $10\frac{1}{2}$ gallons of London ale. Q. E. F.

25. The famous tun of Heidelburgh, that being heretofore annually replenished with Rhenish, had in it some wine that was many ages old, before the French demolished it in the late war: it was 31 feet in length, and 21 feet in diameter, and pretty nearly cylindrical; pray how many tuns of wine would the same contain?

First, $21 \times 21 \times .7854 = 346.3614$, area of one end.
Then $346.3614 \times 31 = 10737.2034$ solid feet.

Also

Also $10737.2034 \times 1728 = 18553887.4752$ cubic inches. $231) 18553887 = 80319.8$ gallons $= 318$ tuns, 183.8 gallons, the answer.

SECT. III.

MISCELLANEOUS QUESTIONS.

1. A Detachment of four regiments consisted of 4600 men; Col. A's regiment exceeded Col. B's by 33, Col C's by 95 men, and Col. D's by 200 men, how many men were in each regiment;

$$\begin{array}{r} 4600 \\ 33 \\ 95 \\ 200 \\ \hline \end{array}$$

$$\left.\begin{array}{r} 4)4928(1232 \\ 1232 - 33 = 1199 \\ 1232 - 95 = 1137 \\ 1232 - 200 = 1032 \end{array}\right\} \text{Colonel} \left\{\begin{array}{l} \text{A's} \\ \text{B's} \\ \text{C's} \\ \text{D's} \end{array}\right.$$

2. There are 8000 men in garrison besieged, whose daily allowance is 24 ounces of bread for 7 weeks; but the governor finding the siege is likely to continue a longer time who can hold out 14 weeks at least, though he has by this time lost 1500 of his men, whereby he finds himself obliged to shorten that allowance of provisions; how much bread must each man's daily allowance be reduced to?

Recip. 7 weeks : 24 ounces : : 14 weeks : 12 ounces :
Then $8000 - 1500 = 6500$ men left.
Recip. 8000 men : 12 oz. : : 6500 : $14\frac{10}{13}$. Q. E. F.

3. Required to find the least three whole numbers, so that $\frac{3}{8}$ of one, $\frac{5}{14}$ of another, and $\frac{7}{18}$ of a third, shall be equal?

First, taking $\frac{3}{8}$ and $\frac{5}{14}$.
Then $3 \times 14 = 42$; also $5 \times 8 = 40$.
And $\frac{3}{8} : \frac{5}{14} : : 42 : 40$.

Qq

Then

Then taking $\frac{5}{14}$ and $\frac{7}{20}$; then $5 \times 20 = 100$; and $7 \times 14 = 98$.

$$\text{Alſo } \tfrac{5}{14} : \tfrac{7}{20} :: 100 : 98.$$

$$\text{And } 98 : 42 :: 100 : 42\tfrac{6}{7}.$$

$\because$ 40.42 and $42\frac{6}{7}$ are numbers in the ſame ratio, which $\times 7$ gives 280, 294, and 300, whole numbers; theſe numbers $\div 2$, gives 140, 147, and 150, the leaſt whole numbers. Q. E. F.

$$\text{For } \left.\begin{array}{l} 280 \times \frac{3}{8} \\ 294 \times \frac{5}{14} \\ 300 \times \frac{7}{20} \end{array}\right\} = 105. \qquad \text{Alſo } \left.\begin{array}{l} 140 \times \frac{3}{8} \\ 147 \times \frac{5}{14} \\ 150 \times \frac{7}{20} \end{array}\right\} = 52\tfrac{1}{2}.$$

4. An uſurer dying, had left the whole ſum of his fortune to be diſpoſed of in the following manner: To A $\frac{2}{5}$, to B $\frac{3}{10}$, to C $\frac{1}{8}$, to D $\frac{1}{20}$, to E $\frac{1}{40}$, and to F $\frac{1}{50}$; which ſums being all paid, the remainder he ordered to be paid to C, which was 800l. Quere the uſurer's whole ſum, and what each had to their ſhare?

$$\frac{2}{5} = \frac{80}{200},\ \frac{3}{10} = \frac{60}{200},\ \frac{1}{8} = \frac{25}{200},\ \frac{1}{20} = \frac{10}{200},\ \frac{1}{40} = \frac{5}{200},\ \text{and } \frac{1}{50} = \frac{4}{200} \text{ their ſum being } \frac{184}{200} = \frac{23}{25}.$$

$$\therefore \frac{25}{25} - \frac{23}{25} = \frac{2}{25} = 800\text{l.}$$

$$2 : 800 :: 25 : 10000\text{l. whole eſtate.}$$

$$200 : 10000 :: \begin{cases} 80 : 4000, \text{ A's.} \\ 60 : 3000, \text{ B's.} \\ 25 : 1250, \text{ C's} + 800 = 2050. \\ 10 : 500, \text{ D's.} \\ 5 : 250, \text{ E's.} \\ 4 : 200, \text{ F's.} \end{cases}$$

5. A worthleſs miſer, as I'm told,
Had hoarded up vaſt ſtore of gold,
Large ſums put out to uſury,
'Till aged fourſcore years and three,
When death depriv'd him of his pelf,
And took him from his ſecond ſelf;
Of wives it happen'd he had three,
Three ſons, and daughters two had he;
His third wife did ſurvive him ſtill,
But mark the tenor of his will:
Of ruſty gold, ten thouſand pound
Was in this miſer's coffers found;

Each

Each son must be paid down in store,
Each daughter's fortune three times o'er;
Each daughter's, as the will was made,
Must twice the widow's part be paid:
Now the old miser's in his grave,
Tell me the fortune each must have?

1 widow 1 share, 2 daughters 4 shares, and 3 sons 18 shares; $1 + 4 + 18 = 23$, divisor for the widow's part.

	l.	s.	d.	qrs.	
23) 10000 (	434	15	7	$3\frac{7}{23}$,	widow's part.
		× 2			
	869	11	3	$2\frac{14}{23}$,	each daughter's part.
		× 3			
	2608	13	10	$3\frac{19}{23}$,	each son's part.

6. A stone, weighing 40 pounds, is by accident broke into four pieces, by which may be weighed any quantity or number of pounds, from 1 to 40: Quere, the weight of each piece?

A general RULE *for the solution of* QUESTIONS *of this nature.*

To double the first or least weight, which always contains one pound; add 1, and it gives the second weight: again, to double the sum of these two weights, add 1, it produces the third weight; and again, to double the sum of these three weights, add 1, and we shall have the fourth weight.

Thus 1 lb. = first or least weight.
Then will $2 + 1 = 3$ = next least weight.
Also $\overline{3 + 1} \times 2 + 1 = 9$ = third.
And $\overline{9 + 3 + 1} \times 2 + 1 = 27$ = the fourth.
The sum of which, viz. $1 + 3 + 9 + 27 = 40$.

7. A lovely pair, delight of human race,
Collateral thus their sprightly lineage trace;
A thousand years are since their ancient stem,
Which branching forth, supply'd the branch to them;
Each male and female, as by what appears,
Liv'd to the age of threescore and ten years;
And each fair female brought forth children seven,
In seven successive years the gifts of heav'n;

From twenty-one to twenty-seven of age,
A boy and girl each year by turns engage;
The teeming mother views them with a smile,
Their pleasing innocence her cares beguile:
No jealousies the parents joys molest,
But each fond couple is with virtue blest.
Happy for those, who, to no vices blind,
Can virtue choose, and such relations find.
To what amount did all this kindred thrive,
How many dead, of each sex what alive?
And of the living, let it next be told,
How many virgins but just twenty old?

First 70) 1000 (14 generations.

Then if in 70 years 1 woman be increased to 3, in 70 more (viz. 140) 3 will be increased to 9; also, in 70 more (viz. 210) 9 will become twenty seven.

∵ The number of women after 1000 years, will be $3^{14} = 4782969$.

And the number of persons, men and women, then living, will be $\frac{7}{3} \times 3^{14}$, or $7 \times 3^{13} = 11160261$.

Lastly, supposing an equal number of all ages to be living at that time, then $\frac{20}{70} \times 3^{14} = 1366562$ women living under 20.

And $\frac{21}{70} \times 3^{14}, = 1434890$, woman living under 21.

∵ $\frac{21-20}{70}$, or $\frac{1}{70} \times 3^{14} = 68328$ women living between 20 and 21.

8. A petticoat of silk, 3 yards, 2 feet, and 1 inch long, and half a yard and 10 inches wide, is sent me to be quilted in equal squares of four-tenths of an inch to each square side, and 58 stitches to be taken in 9 inches length; it is required to find the exact number of stitches the petticoat will take, and what the work will come to, at 5s. per thousand stitches?

$$\left.\begin{array}{l}\text{First, 3 yards, 2 feet, 1 inch} = 133 \\ \text{Also } \tfrac{1}{2} \text{ yard, and 10 inches} = 28\end{array}\right\}\text{inches.}$$

Then $133 \times 28 = 3724$ square inches.

And $.4 \times .4 = .16$, area of one quilted square.

.16) 3724.00 (23275 quilted squares in all.

Now $.4 + .4 = .8$, inches of work in every square.

Also $23275 \times .8 = 18620$, inches of work in all the squares, besides $133 + 28 = 161$ inches for the half border; viz. $18620 + 161 = 18781$ wrought inches.

As 9 inches : 58 stitches : : 18781 : $121033\frac{1}{9}$ stitches.

1000 stitch. : .25 l. : : 121033.1 : 30 l. 5s. 2d. nearly.

Q. E. F.

APPEN-

APPENDIX;

Containing the Method of finding the Sums of certain Progressions, some Problems in Maxima and Minima, and the Investigation of the Sums of certain Infinite Series.

SECT. I.

Of finding the sums of any number of terms in certain progressions.

PROPOSITION I.

TO find the sum of any given number of terms of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5}, \text{ \&c or } \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}, \text{ \&c.}$$

Divide the given number of terms by the same more 1, the quotient will be the sum required.

The sum of three terms will therefore be $= \frac{3}{4}$, that of five $= \frac{5}{6}$, and that of ten $= \frac{10}{11}$, &c. &c.

PROPOSITION II.

To find the sum of any given number of terms of the series.

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6}, \text{ \&c.}$$

Multiply the number of terms more 1 by the same more 2; divide unity, or 1, by twice that product, and subtract the quotient from $\frac{1}{4}$, the remainder will be the sum required.

EXAMPLE. I.

Let the sum of five terms, viz. $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6} + \frac{1}{5.6.7}$ be sought.

Then $6 \times 7 = 42$, and $\frac{1}{4} - \frac{1}{84} = \frac{80}{336} = \frac{5}{21}$, the sum required.

EXAMPLE II.

Let the sum of eight terms of the above series be required.

Here $9 \times 10 = 90$, and $\frac{1}{4} - \frac{1}{180} = \frac{176}{720} = \frac{11}{15}$, the sum required.

PROPOSITION III.

To find the sum of any given number of terms of the series $\frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \frac{1}{3.4.5.6} + \frac{1}{4.5.6.7}$, &c.

Let the number of terms, added to 1, 2 and 3, respectively, be continually multiplied together; divide unity by 3 times that product, and subtract the quotient from $\frac{1}{18}$, the remainder will be the sum of the terms required.

EXAMPLE.

Let the sum of 20 terms of the above series be sought.

Then $21 \times 22 \times 23 = 10626$, and $\frac{1}{18} + \frac{1}{3 \times 10626} = \frac{31860}{573804} = \frac{295}{5313}$, equal the sum required.

PROPOSITION IV.

To find the sum of any given number of terms of the series $\frac{1}{1.2.3.4.5} + \frac{1}{2.3.4.5.6} + \frac{1}{3.4.5.6.7} + \frac{1}{4.5.6.7.8}$, &c.

Let the number of terms be increased by 1, 2, 3 and 4, respectively, multiply those sums continually together, divide unity by four times their product, and subtract the quotient from $\frac{1}{96}$, the remainder will be the sum required.

EXAMPLE.

Let the value of 96 terms of this series be sought.

Then $97 \times 98 \times 99 \times 100 \times 4 = 3764376$, and $\frac{1}{96} - \frac{1}{3764376} = \frac{3764280}{361380096} = \frac{176845}{15057504}$, equal the sum required.

It may not be improper to observe, that the sum of the series

$\frac{1}{1.2}$

$$\left.\begin{array}{l}\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}\ \&c.\\ \frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\frac{1}{4.5.6}\ \&c.\\ \frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}+\frac{1}{4.5.6.7}\ \&c.\\ \frac{1}{1.2.3.4.5}+\frac{1}{2.3.4.5.6}+\frac{1}{3.4.5.6.7}+\frac{1}{4.5.6.7.8}\ \&c.\end{array}\right\}\textit{ad infinitum}\text{, will be}\left\{\begin{array}{l}1\\ \frac{1}{4}\\ \frac{1}{18}\\ \frac{1}{96}\end{array}\right.$$

PROPOSITON V.

To find the ſum of any number of terms of the ſeries $1^2+2^2+3^2+4^2+5^2$ &c. or, $1+4+9+16+25$ &c.

Let $\frac{1}{6}$, half the number of terms, and $\frac{1}{3}$ of the ſquare of the ſaid number, be collected into one ſum, multiply that ſum by the number of terms, and the product will be the aggregate of the terms required.

EXAMPLE I.

Let the ſum of ſix terms of the abovesaid ſeries be required.

Then $\overline{\frac{1}{6}+3+\frac{36}{3}}\times 6=91$, the ſum required.

EXAMPLE II.

Conceive a pyramid to be conſtituted of geometrical ſquare ſlabs, each a foot thick, and ſuppoſe the baſe, or greateſt ſlab, to be 20 feet ſquare, the next 19, the next 18, the next 17, and ſo on, it is required to find the ſolid content of ſuch a pyramid?

Here $\overline{\frac{1}{6}+10+\frac{400}{3}}\times 20=\frac{861}{6}\times 20=2870$ feet, the ſolidity required.

PROPOSITION VI.

To find the ſum of any number of terms of the ſeries $1^3+2^3+3^3+4^3+5^3$ &c. or, $1+8+27+64+125$, &c.

Let the number of terms more than 1 be ſquared, and multiplied by the ſquare of the number of terms; $\frac{1}{4}$ of this product will be the ſum required.

EXAMPLE.

Let the value of 8 terms of this series be required.

Then $9 \times 9 \times 64 = 5184$, and $\frac{5184}{4} = 1296$, the value sought.

PROPOSITION VII.

To find the sum of any number of terms of the series $1^4 + 2^4 + 3^4 + 4^4 + 5^4$ &c. or, $1 + 16 + 81 + 256 + 625$ &c.

Let $\frac{1}{5}$ of the biquadrate, $\frac{1}{2}$ of the cube, and $\frac{1}{3}$ of the square of the number of terms, be collected into one sum, from which subtract $\frac{1}{30}$; multiply the remainder by the number of terms, and the product will be the sum required.

EXAMPLE.

Let the value of eight terms be required: then $\frac{8^4}{5} + \frac{8^3}{2} + \frac{8^2}{3} = \frac{32896}{30}$; and $\overline{\frac{32896}{30} - \frac{1}{30}} \times 8 = \frac{263160}{30} = 8772$, the sum required.

PROPOSITION VIII.

To find the sum of any number of terms of the series $1^5 + 2^5 + 3^5 + 4^5 + 5^5$ &c. or, $1 + 32 + 162 + 512 + 1250$ &c.

Let $\frac{1}{6}$ of the biquadrate, $\frac{1}{2}$ the cube, and $\frac{5}{12}$ the of square of the number of terms, be collected into one sum, from which subtract $\frac{1}{12}$; multiply the remainder by the square of the number of terms, and the product will be the sum of terms required.

EXAMPLE.

Let the sum of 10 terms be required: then $\frac{10^4}{6} + \frac{10^3}{2} + \frac{5 \times 10^2}{12} = \frac{159000}{72}$; and $\overline{\frac{159000}{72} - \frac{6}{72}} \times 100 = \frac{15899400}{72} = 220825$, the value required.

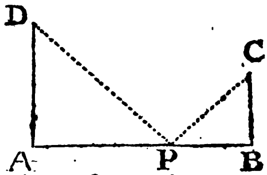
SECT.

SECT. II.

A Collection of PROBLEMS *concerning the Maxima and Minima of Quantities.*

PROBLEM I.

GIVEN the position of the points D and C, in respect to the given right line AB, to find the point P, so that DP + PC shall be a minimum.



THEOREM. $\frac{BC \times AB}{AD + BC} = BP$ when DP + PC is least possible.

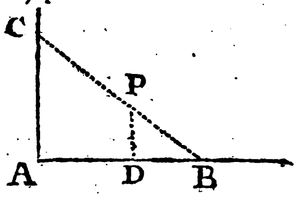
EXAMPLE.

Let AB = 50, AD = 40, and BC = 30, required PB in the above circumstances?

Then $\frac{30 \times 50}{40 \times 30} = \frac{1500}{70} = 21.42857 = PB$, and therefore AP = 28.57143.

PROBLEM II.

Through the given Point P, placed within the right angle CAB, to draw the shortest line (CB) possible.



THEOREM. $\overline{AD \times \overline{PD}^2}|^{\frac{1}{3}} =$ DB, where DP is perpendicular to AB.

EXAMPLE.

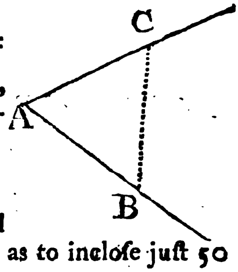
Suppose AD = 10, DP = 8, required the position of the line CB when a minimum?

Then $10 \times 64 = 640$, whose cube root is 8.618 nearly = DB, from which the position of the line is determined.

PROBLEM III.

Two right lines AC and AB, making the given angle A, it is required to cut off a given area ACB with the shortest line (CB) possible?

THEOREM.



THEOREM. $1.4142 \times \sqrt{\frac{a}{s}} =$ AB $=$ AC, where $a =$ the given area, and $s =$ the natural sine of the given angle A.

EXAMPLE.

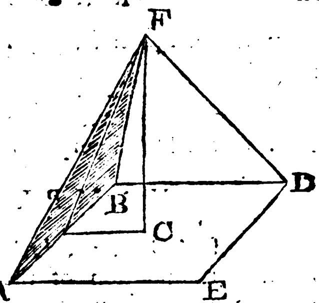
Let the angle A be $= 54^\circ\ 20'$, to find the length of the shortest fence BC, so as to inclose just 50 acres.

Then will $s = .812423$ to radius 1, and $a = 500$ square chains. Therefore $\sqrt{\frac{500}{.812423}} \times 1.4142 = 35.07216 =$ AB $=$ AC; wherefore, by Trigonometry,

As sine $\angle$ B - -	62°	50′	Co-ar.	0.050765
To log. of AC $=$ BC	35	07.		1.544934
So is sine $\angle$ A - -	54	20		9.909782
To log. of BC - -	31	02	chains	1.505481 required.

PROBLEM IV.

Of all the pyramids AFBDE of a given solidity, to find that of the least superficies, excluding the square base ABDE.



THEOREM. Height CF $= \left|\frac{3 \times \text{solidity}}{2}\right|^{\frac{1}{3}}$ And AB $=$ BD $=$ DE $=$ EA $= \sqrt{\frac{3 \times \text{solidity}}{\text{CF}}}$

EXAMPLE.

To find the dimensions of a square pyramid, made with the least surface, to contain just one malt bushel, or 2150.4 cubic inches.

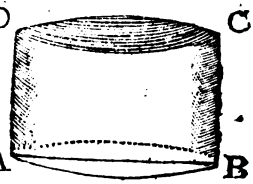
Then $\frac{3 \times 2150.4}{2} = 3225.6$, whose cube root is 14.7 5 inches, the depth CF; and therefore $\frac{3 \times 2150.4}{14.775} = 436.629$ whose square root is $= 20.89 =$ AB, each side of the square base.

PROBLEM

PROBLEM V.

To determine the dimenſions of a cylinder ABCD, open at the top, ſo as to contain any quantity of liquor, grain, &c. and to have the leaſt internal ſuperficies, or, which is the ſame thing, to be made of the leaſt metal of a given thickneſs.

THEOREM. $AB = 2 \times \overline{\frac{\text{ſolidity}}{3.1416}}\Big|^{\frac{1}{3}}$ and $AD\ (= BC) = \frac{1}{2}\ AB$.



EXAMPLE.

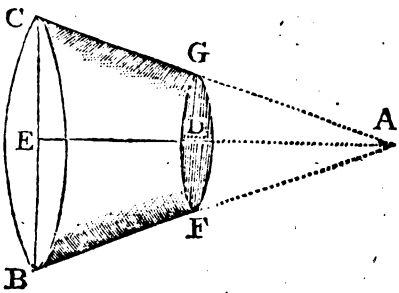
To find the dimenſions of a cylindric buſhel, made of the leaſt quantity of metal, of a given thickneſs.

Here $\frac{2150.4}{3.1416} = 684.492$, and $\overline{684.492}\big|^{\frac{1}{3}} \times 2 = 17.626$ inches $= AB$ the diameter; ∵ the depth $DA = 8.813$ inches.

PROBLEM VI.

To find that fruſtum of a cone, of a given baſe and altitude, which moving in direction of its axis, with its leſſer end againſt the parts of an homogeneous fluid, ſhall ſuffer the leaſt reſiſtance poſſible from it.

THEOREM. $EA = \frac{ED + \sqrt{\overline{ED}^2 + \overline{CB}^2}}{2}$, and by ſimilar triangles, $AE : AD :: BC : FG$.



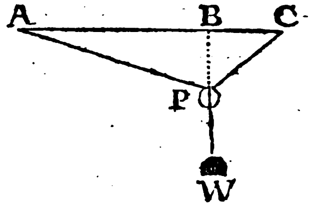
EXAMPLE.

Let the baſe $CB = 6$ and altitude $ED = 8$; then, per *Theorem*, $\frac{\sqrt{64 + 36} + 8}{2} = 9 = EA$; ∵ $9\ (= EA) - 8\ (= ED) = 1 = DA$. Therefore, $9 : 1 :: 6 : \frac{6}{9} = \frac{2}{3} = GF$. Whence the fruſtum CGFB is determined.

PRO-

PROBLEM VII.

Let P be a pulley hanging freely at the end of a cord AP, faſtened at A; and let W be a weight connected to the cord CPW, put over the pulley P, which cord is faſtened at C, ſo that the points A and C lie in the ſame horizontal line AC. Now if the pulley and cords be ſuppoſed to have no weight, it is required to find in what place the pulley will ſettle or come to reſt?



THEOREM.

$$\sqrt{\frac{\overline{8AC^2 + AP^2} \times AP + AP^2}{4AC}} =$$

AB; from which point B, if a perpendicular be let fall, will be a tangent to the pulley P, and paſs through the center of gravity of the weight W, when that and the pulley come to reſt.

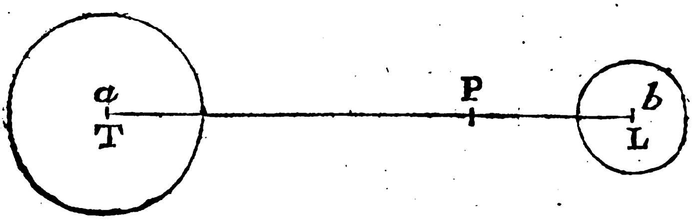
EXAMPLE.

Let $AC = 10$, $AP = 8$, required AB?

Then $\sqrt{\frac{\overline{8 \times 100 + 64} \times 8 + 64}{40}} = 7.478 =$ AB required.

PROBLEM VIII.

To find a point P, in a right line connecting the centers *a* and *b* of two ſpherical bodies T and L, of given diameters and denſities; at which, if a third be placed, it ſhall be the leaſt ſubject to their joint attraction.



THEOREM. Let the quantity of matter in the body T, be to that in the body L, (which will always be found by their given diameter and denſities) as M to 1; when will

will $\frac{ab \times M^{\frac{1}{3}}}{1 + M^{\frac{1}{3}}} = aP$, the distance of the point P from the center a of the greater body T.

EXAMPLE.

Suppose the mean distance of the moon and earth to be equal to 240000 miles, and the quantity of matter in the earth to that in the moon, as 40 to 1; required to find where a body must be placed in a right line connecting their centers, so as to be the least attracted by these two planets?

First, $40^{\frac{1}{3}} = 3.42$ very near; and by the theorem, $\frac{240000 \times 3.42}{4.42} = 185701.3$ miles, the distance of the required point from the earth's center; and, consequently, 54298.7 miles, equal its distance from the moon's.

PROBLEM IX.

The latitude of the place and sun's declination being given, to find what time of the day the shadows of perpendicular objects move the slowest.

THEOREM.

From the natural sine of the given latitude subtract the square root of the difference of the squares of the natural sines of the latitude and declination; divide that remainder by the natural sine of the declination, and the quotient will be the natural sine of the sun's altitude at the time required; from whence the time itself will be readily found.

EXAMPLE.

At what time, on June 10th, 1765, will the shadow of a perpendicular object move the slowest at Spalding * in Lincolnshire?

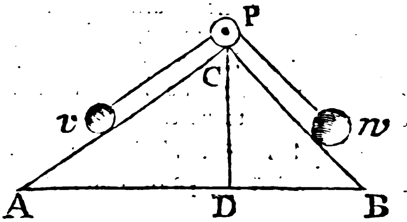
* Latitude 52° 46′ N. 1.

The sun's declination, on June 10, is 23° 5′ N. its nat. sine = .39207, to rad. 1. The nat. sine of 52° 46′, the given lat. is = .796178. Then by the theorem $796178^2 - .39207^2 = .480180522784$, whose square root is = .69295 nearly, and $\frac{796178 - .69295}{.9207} = 2632897$ = the natural sine of 15° 16′ nearly, the Sun's altitude at the required time, whence the time itself is found to be 40 past five in the morning, or 20 past six in the evening, very near.

PRO-

PROBLEM X.

Being the PRIZE QUESTION *in the* MATHEMATICAL MAGAZINE, Numb. IV.



Let AC and CB be two given inclin'd planes, and let the given weight w be suppofed to defcend along CB, whilft v (being connected by a ftring moving parallel to the planes over the pulley P) afcends along AC; it is required to determine the weight of v, fo that its momentum, in thefe circumftances, may be the greateft poffible?

Method of Solution.

Put $CB = a$, $CA = b$, and $CD = c$; then, per Mechanics, $(a : c :: w :)\ \frac{cw}{a}$ will exprefs the force with which the weight w tends to defcend along the plane CB; and $(b : c :: v :)\ \frac{cv}{b}$, that of v along the plane CA; therefore $\frac{cw}{a} - \frac{cv}{b}$ will be as the efficacious force wherewith the weights are accelerated: this divided by $w + v$, their quantity of matter, and there arifes $\frac{bcw - acv}{ba \times \overline{w + v}}$ for their common velocity; which, multiplied by v, gives $\frac{bcwv - acv^2}{ba \times \overline{w + v}}$, a maximum.

In Fluxions it will be $av^2\dot{v} + 2awv\dot{v} - bw^2\dot{v} = 0$, which equation folved, gives $v = \overline{\sqrt{\frac{b + a}{a}} - 1} \times w$.

COROLLARY.

If the inclinations of the planes be equal, then will $v = \overline{\sqrt{2} - 1} \times w = .4142\, w$.

SCHO-

SCHOLIUM.

If w = the weight of the atmosphere, pressing upon the piston in the cylinder of an engine for raising water by fire, and v = the quantity of water raised at one stroke of the great beam; then will $v = \overline{\sqrt{2}-1} \times w$, likewise; or $w : v :: 1 : .4142$, when the engine produces the greatest effect in a given time.

* *

SECT. III.

Of the Investigation of the Sums of certain infinite Series.

I.

PUT $\dot v = \frac{\dot x}{1-x} = \dot x + x\dot x + x^2\dot x + x^3\dot x$ &c. where, taking the fluents $v = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$ &c. multiply each side of the last equation by $x^n\dot x$, and there arises $vx^n\dot x = x^{n+1}\dot x + \frac{x^{n+2}\dot x}{2} + \frac{x^{n+3}\dot x}{3} + \frac{x^{n+4}\dot x}{4}$ &c., whose fluents are $\frac{vx^{n+1}}{n+1}$ — the fluent of $\frac{x^{n+1}\dot v}{n+1}$, or $\frac{vx^{n+1}}{n+1}$ — the fluent of $\frac{x^{n+1}}{n+1} \times \frac{\dot x}{1-x}$, or $\frac{vx^{n+1}}{n+1}$ + the fluent of $\frac{x^{n+1}}{n+1} \times \frac{\dot x}{x-1} = \frac{x^{n+2}}{n+2} + \frac{x^{n+3}}{2.n+3} + \frac{x^{n+4}}{3.n+4} + \frac{x^{n+5}}{4.n+5}$ &c.

But $\frac{x^{n+1}}{n+1} \times \frac{\dot x}{x-1} = \frac{1}{n+1} \times \overline{x^n\dot x + x^{n-1}\dot x + x^{n-2}\dot x + x^{n-3}\dot x \ldots \text{to } \dot x - \dot v}$. Therefore $\frac{vx^{n+1} - v}{n+1} + \frac{1}{n+1} \times \overline{\frac{x^{n+1}}{n+1} + \frac{x^n}{n} + \frac{x^{n-1}}{n-1} + \frac{x^{n-2}}{n-2} \ldots \text{to } x}$, is = the sum of

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the infinite series $\frac{x^{n+2}}{n+2}+\frac{x^{n+3}}{2.\overline{n+3}}+\frac{x^{n+4}}{3.\overline{n+4}}+\frac{x^{n+5}}{4.\overline{n+5}}$ &c.

Suppose, now, $x=1$, and the above series's will become $\frac{1}{n+1}\times\frac{1}{n+1}+\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n-2}$ .. to 1, equal to $\frac{1}{n+2}+\frac{1}{2.\overline{n+3}}+\frac{1}{3.\overline{n+4}}+\frac{1}{4.\overline{n+5}}$ &c. *ad infinitum.*

Therefore, taking $n=0$, or any positive integer, we can obtain the sums of as many infinite series's of this kind as we please. Thus,

Suppose $n=0$, then the sum of the infinite series.

$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\frac{1}{5.6}$ &c. will be equal 1, as observed in Sect. 1.

Taking $n=1$, then the sum of the infinite series $\frac{1}{1.2}+\frac{1}{2.4}+\frac{1}{3.5}+\frac{1}{4.6}+\frac{1}{5.7}$, &c. will be equal $\frac{3}{4}$.

Taking $n=2$, then the sum of the infinite series $\frac{1}{1.4}+\frac{1}{2.5}+\frac{1}{3.6}+\frac{1}{4.7}+\frac{1}{5.8}$ will be $=\frac{11}{18}$.

Again, taking $n=3$, then the sum of the infinite series $\frac{1}{1.5}+\frac{1}{2.6}+\frac{1}{3.7}+\frac{1}{4.8}+\frac{1}{5.9}$ &c. will be $=\frac{25}{48}$.

2.

Assume $\dot{v}=\frac{\dot{x}}{1-x^2}=\dot{x}+x^2\dot{x}+x^4\dot{x}$ &c. then $v=x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}$ &c. Multiplying each side of the last equation by $x^n\dot{x}$, we shall have $vx^n\dot{x}=x^{n+1}\dot{x}+\frac{x^{n+3}\dot{x}}{3}+\frac{x^{n+5}\dot{x}}{5}+\frac{x^{n+7}\dot{x}}{7}$ &c. whose fluents are $\frac{vx^{n+1}}{n+1}$ —the fluent

fluent of $\frac{x^{n+1}\dot{v}}{n+1}$, or $\frac{vx^{n+1}}{n+1}$ + the fluent of $\frac{x^{n+1}}{n+1} \times \frac{\dot{x}}{x^2-1} = \frac{x^{n+2}}{n+2} + \frac{x^{n+4}}{3\cdot\overline{n+4}} + \frac{x^{n+6}}{5\cdot\overline{n+6}} + \frac{x^{n+8}}{7\cdot\overline{n+8}}$ &c.

But $\frac{x^{n+1}}{n+1} \times \frac{\dot{x}}{x^2-1} = \frac{1}{n+1} \times \overline{x^{n-1}\dot{x} + x^{n-3}\dot{x} + x^{n-5}\dot{x} \ldots\ldots \text{ to } \dot{x} - \dot{v}}$. Therefore the fluent of $\frac{x^{n+1}}{n+1} \times \frac{\dot{x}}{x^2-1}$ is $= \frac{1}{n+1} \times \overline{\frac{x}{n} + \frac{x^{n-2}}{n-2} + \frac{x^{n-4}}{n-4} \ldots, \text{ to } x - v}$; consequently, $\frac{vx^{n+1}}{n+1} - v + \frac{1}{n+1} \times \overline{\frac{x^n}{n} + \frac{x^{n-2}}{n-2} + \frac{x^{n-4}}{n-4} \ldots \text{ to } x}$ is = the sum of the infinite series $\frac{x^{n+2}}{n+2} + \frac{x^{n+4}}{3\cdot\overline{n+4}} + \frac{x^{n+6}}{5\cdot\overline{n+6}} + \frac{x^{n+8}}{7\cdot\overline{n+8}}$ &c.

Taking $x = 1$; then will $\frac{1}{n+1} + \overline{\frac{1}{n} + \frac{1}{n-2} + \frac{1}{n-4} \ldots\ldots \text{ to } 1}$ = the sum of the infinite series $\frac{1}{1.\overline{n+2}} + \frac{1}{3\cdot\overline{n+4}} + \frac{1}{5\,\overline{n+8}}$, &c. where n may be any number in this progression, 1, 3, 5, 7, 9, &c.

Suppose $n = 1$, then the sum of the infinite series $\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \frac{1}{9\cdot11}$, &c. will be $= \frac{1}{2}$.

Taking $n = 3$, then the sum of the infinite series $\frac{1}{1\,5} + \frac{1}{3\cdot7} + \frac{1}{5\cdot9} + \frac{1}{7\cdot11} + \frac{1}{9\cdot13}$, &c. will be $= \frac{1}{3}$.

Taking $n = 5$, then the sum of the infinite series $\frac{1}{1\cdot7} + \frac{1}{3\cdot9} + \frac{1}{5\cdot11} + \frac{1}{7\cdot13} + \frac{1}{9\cdot15}$, &c, will be $= \frac{23}{90}$.

Again taking $n = 7$, then the sum of the infinite series $\frac{1}{1\cdot9} + \frac{1}{3\cdot11} + \frac{1}{5\cdot13} + \frac{1}{7\cdot\,5} + \frac{1}{9\cdot17}$, &c. will be $= \frac{22}{108}$.

And proceeding thus, by taking $n = 9, 11, 13$, &c. agreeable to what is above specified, the reader may sum as many series of this kind as he pleases. Moreover, if we assume other values of $\dot{v}$, we can, with equal facility extend this method much further.

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The sum of the infinite series $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9}$, — &c.

($=$ the fluent of $\frac{\dot{x}}{1+xx}$, or that circular arc whose rad. is unity, and tangent x) being given $= y$; to find the sum of the infinite series $\frac{x}{1.2} - \frac{x^3}{3.4} + \frac{x^5}{5.6} - \frac{x^7}{7.8} +$, &c.

By multiplying each side of the given equation by $\dot{x}$, we shall have $y\dot{x} = x\dot{x} - \frac{x^3\dot{x}}{3} + \frac{x^5\dot{x}}{5} - \frac{x^7\dot{x}}{7}$ —, &c. Whose fluents are xy — the fluent of $x\dot{y} = \frac{x^2}{1.2} - \frac{x^4}{3.4} + \frac{x^6}{5.6} - \frac{x^8}{7.8} + \frac{x^{10}}{9.10}$ — &c. But $x\dot{y} = \frac{x\dot{x}}{1+xx}$, whose fluent is the hyp. log. of $\sqrt{1+xx}$. Therefore xy — the hyp. log. of $\sqrt{1+xx} = \frac{x^2}{1.2} - \frac{x^4}{3.4} + \frac{x^6}{5.6} - \frac{x^8}{7.8} + \frac{x^{10}}{9.10}$ — &c. or $y - \frac{1}{x} \times$ the hyp. log. of $\sqrt{1+xx} = \frac{x}{1.2} - \frac{x^3}{3.4} + \frac{x^5}{5.6} - \frac{x^7}{7.8} +$ &c. Q. E. F.

Taking $x =$ $\left\{\begin{array}{l} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \&c \end{array}\right\}$ Then will the sum of the series

Wherefore

$$\left\{\begin{array}{l} \frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \&c. \\ \frac{1}{1.2.2} - \frac{1}{3.4.2^3} + \frac{1}{5.6.2^5} - \frac{1}{7.8.2^7}, + \&c. \\ \frac{1}{1.2.3} - \frac{1}{3.4.3^3} + \frac{1}{5.6.3^5} - \frac{1}{7.8.3^7} + \&c. \\ \frac{1}{1.2.4} - \frac{1}{3.4.4^3} + \frac{1}{5.6.4^5} - \frac{1}{7.8.4^7} \&c. \&c. \end{array}\right\}$$

ad infinitum. $\left\{\begin{array}{l} = \\ = \\ = \\ = \\ \&c \end{array}\right\}$

the arc of 45° — hyp. log. of $\sqrt{2}$.

the arc of 26°. 33′ 54″. — hyp. log. of $\frac{5}{4}$

the arc of 18°. 26′ 6″ — hyp. log. of $\left(\frac{10}{9}\right)^{\frac{3}{2}}$

the arc of 14°. 2′. 10″ — hyp. log. of $\left(\frac{17}{16}\right)^{2}$

&c.

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An example or two by way of illustration, for sake of the learner, in finding the values of the above series's, may not be deemed improper.

Note, If the common (or Briggs's) logarithm of any number be multiplied by 2,3025809, the product will be the hyperbolic logarithm of the same number.

EXAMPLE I.

To find the value of the arc of 45° — hyp. log. of $\sqrt{2}$, which is the sum of the first series.

As 180° : 3.141583 :: 45° : ,785398, the length of the arc of 45°.

And $\sqrt{2} = 1.414214$ nearly, its common log. = ,150516, which multiplied by 2.3025809, produces .346575267 nearly, for the hyp. log. of $\sqrt{2}$. Therefore, 785398 —, 346575267 = ,438822733, the value of the required series $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} +$ &c. *ad infinitum.*

EXEMPLUM II.

Required the value of the arc of 18° . 26′ . 6″ — hyp. log. of $\frac{10^{\frac{3}{2}}}{9}$, which is the sum of the third series.

The common log. of the root $\frac{10}{9}$ is ,045758, and $\frac{3}{2} \times$ .045758 = .068637, which multiplied by 2.3025809, produces .1580423 nearly, for the hyp. log of $\frac{10^{3.}}{9}$ Again, 18°. 26′. 6″ = 18.435°. Then as 180 : 3.141593 : : 18.435 : .32175148 the length of the arc of 18° 26′. 6. Therefore .36175148 — .1480423 = .16370918, the sum of the required series $\frac{1}{1.2.3} - \frac{1}{3.4\,3^3} + \frac{1}{5.6.3^5} - \frac{1}{7.8\,3^7} +$ &c. *ad infinitum.*

COROLLARY.

In general, if x be taken $= \frac{1}{n}$ then will the circle arc. — hyp. log. of $\overline{1 + \frac{1}{n^2}}|^{\frac{n}{2}}$ be equal to the sum of the infinite series $\frac{1}{1.2.n} - \frac{1}{3.4.n^3} + \frac{1}{5.6\,n^5} + \frac{1}{7.8.n^7} +$ &c. where n is supposed a positive integer.

FINIS.

8   8-4 3

24

5

2   15

18   6   105

6 - 2

18   45

14 1

7 14 28

1 2 72

7 14 28

1 1 2

1 1 1